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Section # B

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Subject # Advance Fluid

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# Question # 01.

## part A:

Define Drag with its components write down the equation for Friction Drag coefficient both in laminar and turbulent boundary Layer.

Ans:

**Drag:-** A body which is wholly immersed in a homogenious Fluid may be subjected to two kind of Forces arising from relative motion between two body and fluid These forces are termed as drag and lift. If the forces parallel to the motion then it is termed as drag force.

There are two components:

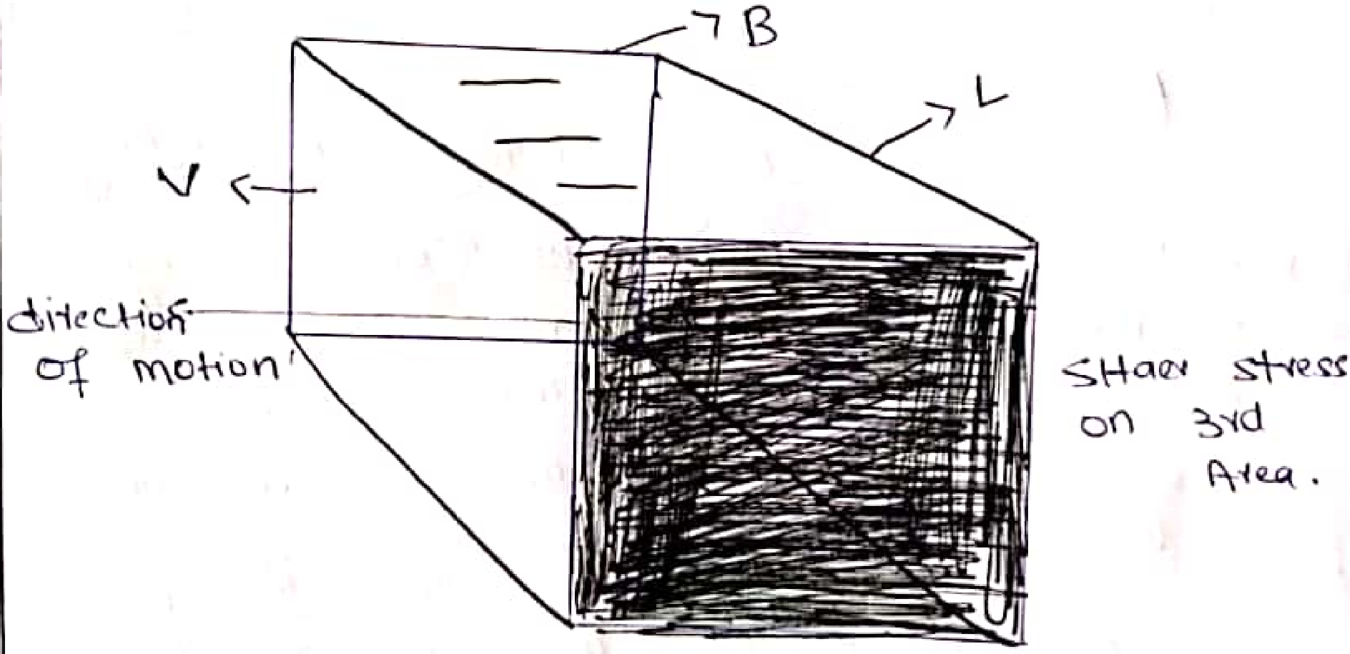
(1) **Pressure Drage (FP):-** It is equel to Integration of components in direction of motion of all pressure forces exerted on surface of body.

$$F_P = C_P \rho \frac{V^2}{2} A$$ ∵ where $C_P$ depends on shape

2) **Friction Drag** :

it is equal to Integration of components of shaer stress along surface of body in direction of motion.

$$F_f = C_f \rho \frac{V^2}{2} BL$$




→ **Friction Drag of boundry layer** :-

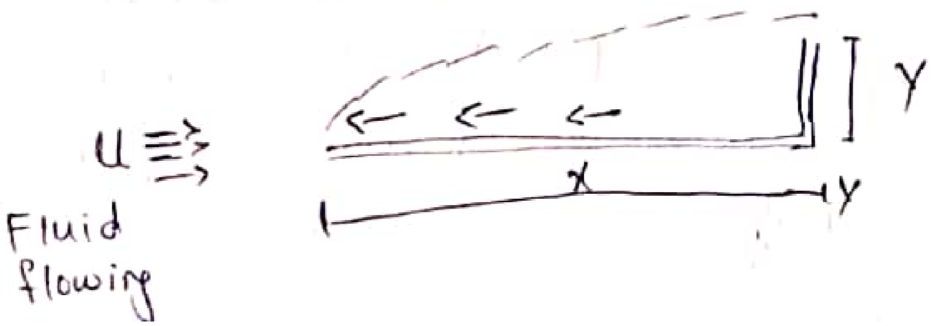
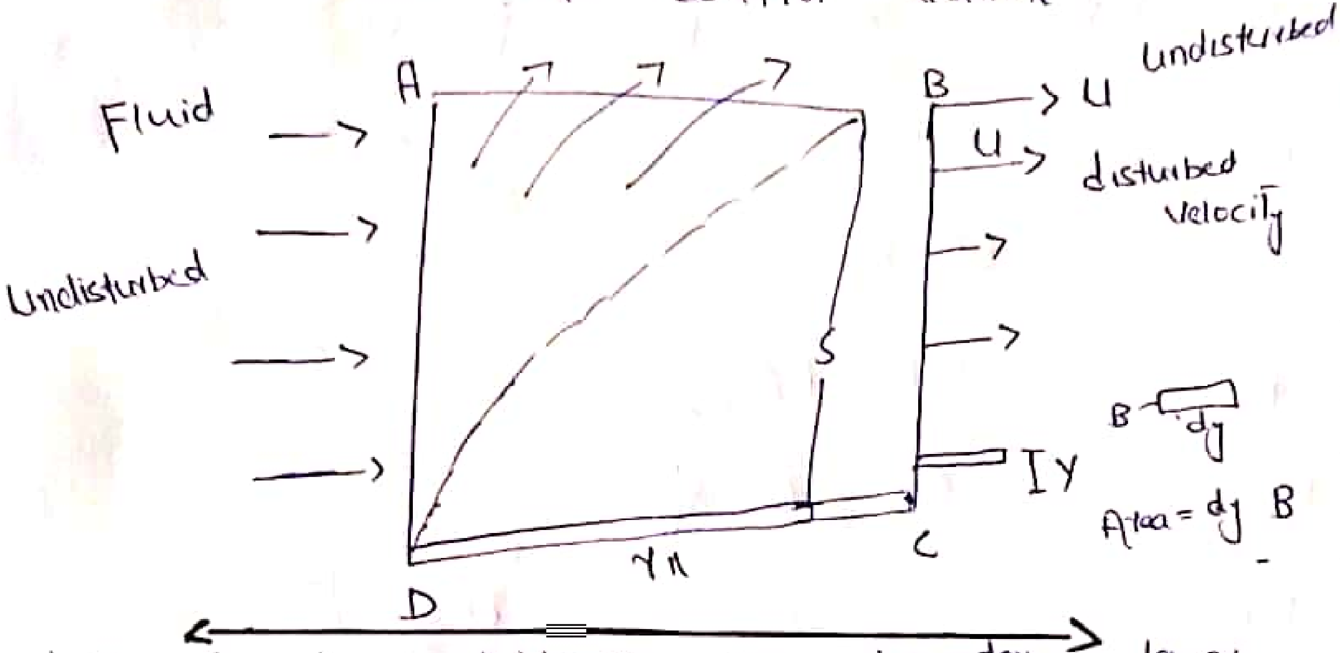


Figure shows growth of boundry layer along one site of smoth plate inside the fluid.

Now consider of control volume.




where $\delta$ is thickness of boundary layer and U is undisturbed Velocity.

Thus $-F_x$ = drag = (rate in momentum in x-direction leaving through BC + rate of momentum ~~entering~~ through AB) − rate of momentum entering through DA)

$\Delta p = P_{out} - P_{in}$

Thus according to momentum.

$$\Sigma f = \frac{d(P)}{dT} = \frac{d\,mov}{dT}$$

$\because$ Where $\frac{dm}{dT} = \rho Q$ Thus

$$F = \rho Q V \xRightarrow{OR} F = \rho A \cdot V \cdot V$$

$$F = \rho A V^2 .$$

Now

$$DA \Longrightarrow \rho u \, (u B \delta)$$

$$BC \Longrightarrow \rho B \int_0^{\delta} u^2 \, dy$$

$$AB \longrightarrow \rho u \left( u B \delta - B \int_0^{\delta} u \, dy \right) .$$

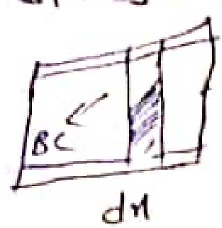
putting value.

$$F x = \rho B \int_0^{\delta} u \, (U - u) \, dy$$

$$F x = \rho B U^2 \delta \alpha$$ $\because$ Where $\alpha$ is Function OF boundary layer.

Now to find local wall shear stress.

$$\tau_0 = \frac{d f x}{B \cdot dx - \text{area}}$$

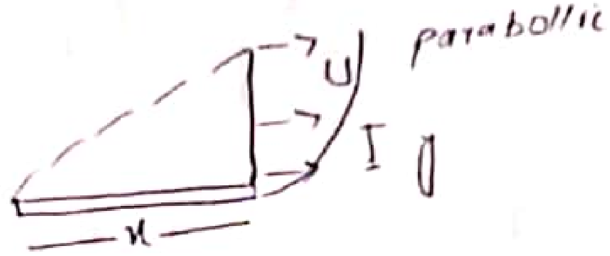


$F_x = \int B u^2 \rho \alpha$

$\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$ in general equation of shear stress.

→ <u>Laminar Boundry layer</u>:-

$\frac{u}{V} = F(y/\delta)$




Assume:

$\eta = y/\delta$ or $y = \eta\delta$

Thus; $\frac{u}{V} = f(\eta)$ or $u = uf(\eta)$

In case: $\frac{u}{V} = f(\eta)$ or $u = uf(\eta)$

$\tau_0 = \mu (du/dy) = \frac{\mu}{\delta}\left(\frac{du}{d\eta}\right)$

$= \frac{\mu u}{\delta}\left[\frac{df(\eta)}{d\eta}\right]$

Now solving the equation.

$\tau_0 = \frac{\mu u B}{\delta} \longrightarrow (1)$

As General equation is $\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$

Equating both equation.

$$\frac{\mu U B}{\delta} = \rho U^2 \alpha \frac{d\delta}{dx} \quad \text{OR} \quad \delta\, d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating The equation:

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x \quad \text{OR} \quad \delta = \sqrt{\frac{2\mu B}{\rho U \alpha}} \quad \text{OR} \quad \sqrt{\frac{2B}{\alpha}} \sqrt{\frac{\mu x}{\rho U}}$$

xing and ÷ing by $x$ :

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu \cancel{x}}{\rho U}} \cdot \frac{x}{\sqrt{\cancel{x}} \cdot \sqrt{x}}$$

where $\alpha = 0.135$, $B = 1.63$ $\quad R_x = \frac{\rho U x}{\mu}$

$$= \delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}$$

Now; $\quad \tau_0 = \frac{\mu U B}{\delta}$

Thus putting value.

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$$\boxed{\tau_0 = 0.332 \frac{\mu u}{x}\sqrt{R_x}}$$

Where $R_x$ is local Reynold number :

Now
$$F_g = B\int_0^L \underset{\text{stress}}{\tau_0\, dx}$$

putting values.

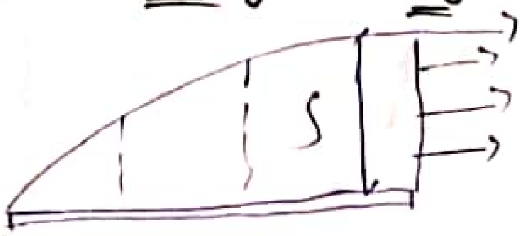
$$F_g = 0.664 B\sqrt{\rho \mu L U^3}$$

As General equation is

$$F_f = C_f \rho \frac{V^2}{2} BL \rightarrow \text{equating both equation.}$$

$$\boxed{C_f = 1.328\sqrt{\frac{\mu}{\rho L U}} = 1.328/\sqrt{R}}$$

⇒ Turbulent boundary layer :-




resistance less so curve become straight

Figure show that velocity distribution in turbulent boundary layer shows a much steeper gradient near wall and flatter through out remaining layer.

The shear stress is greater in turbulent layer than in laminar layer.

As we know

$$\tau_0 = f \rho V^2/8$$ where V donate average velocity of pipe.

now we Have obtained an approximate relation between V and U using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}} :$$

using Friction Factor of 0.028 from chart which is middle critical value.

So $$\boxed{U = 1.235V}$$

Now we Have

$$\tau_0 = f \rho \frac{V^2}{8}$$

As we know:

$$f = 0.316/R^{0.25}$$

Thus $$\tau_0 = 0.316/\left(\frac{DV}{\nu}\right)^{1/4} \rho V^2/8$$

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Where $V = \frac{u}{1.235}$ Thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(u/1.235\right)\right)^{1/4}} \cdot \rho/8 \left(\frac{V}{1.235}\right)^2$$

$$D = 2\delta$$

Thus

$$\tau_0 = \frac{0.023 \rho V^2}{\left(\delta u/\nu\right)^{1/4}}$$

As We know

$$\tau_0 = \rho u^2 \alpha \, d\delta/dx$$

Equating and integrating both for boundry condition of

$$x = 0 \quad , \delta = 0$$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{ux}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\boxed{\delta/x = 0.377/(R_x)^{1/5}}$$

putting value in equation

$$\tau_0 = 0.0587\, \rho\, \frac{u^2}{2} \left( \frac{\nu}{ux} \right)^{1/5}$$

Now $F_f = B \int_0^L \tau_0\, dx$

$$F_f = 0.0735\, \rho\, \frac{U^2}{2} \left( \frac{\nu}{UL} \right) BL$$

As $F_f = C_f\, \rho\, \frac{V^2}{2}\, BL$

equating both

$$\boxed{C_f = \frac{0.0735}{R^{1/5}}}$$

$\because$ R is less than $10^7$
for $500{,}000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log R)^{2.58}}}$$

Ans

# PART B

As Specific Energy $E = y + V^2/2g$

The Flow Q per unit Width "b" canbe expressed as;

↳ $q = \frac{Q}{b}$

Now average velocity will be

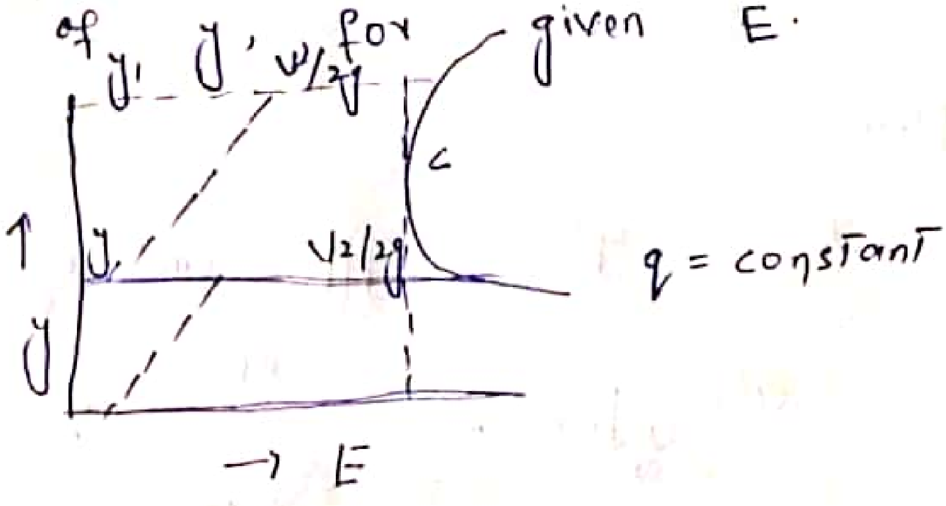
↳ $V = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}$

Thus

$$E = y + V^2/2g \Rightarrow y + 1/2g\,[q^2/y^2]$$

$$[E-y] = 1/2g\,[q^2/y^2] \quad \text{or} \quad (E-y)\,y^2 = q^2/2g$$

Thus plot OF E vs Y will be parabollic For particullar q, There will be two kind of possible values of y, for given E.



The equation is with Three roots being negative point c represents dividing point between two regime of Flow Thus for given $q$, and value of E in minimum and flow of that point is intial flow Depth OF flow at that point is critical The $y_c$ and velocity at that point is critical $V_c$.

Thus

$$E = y + \frac{1}{2g}\left(\frac{V^2}{y^2}\right)$$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus

↳ $$\frac{dE}{dy} = 1 - \frac{\partial}{\partial y}\left(\frac{q^2}{y^3}\right) = 0$$

$$\frac{q^2}{gy^3} \Rightarrow 1 \Rightarrow q^2 = gy^3$$

↳ $$\frac{q^2}{g} = y^3 \Rightarrow \left[\frac{q^2}{g}\right]^{1/3} = y_{cr}$$

Now

↳ $$q^2 = gy^3 \text{ and } q = Vy \Rightarrow V^2y^2 = gy^3$$

OR

$$V^2 = gy_{cr}$$

OR

$$\boxed{V_{cr} = \sqrt{gy_{cr}}}$$

Question # 02

Given data :

Water flows at rate $Q = 3.5 \ m^3/s$

Bed slope , $S_o$ = 0.0008

$n$ = 0.0219

Width of bed in student ID= 7775 mm

= 7.775 m.

Required :

Depth of Rectangular channel = ?

critical depth , $y_c$ = ?

Critical Velocity , $V_c$ = ?

Flow is critical or sub critical = ?

Solution :

Manning equation

$$Q = \left( \frac{1}{n} R_h^{2/3} S_o^{1/2} \right) A \longrightarrow 1.$$

Area = $7.755 \times d$

Perameter = $d + 7.755 + d$

Hydraulic radius $R_\eta$ = Area/parameter

$$R_\eta = \frac{7.755\,(d)}{2d + 7.755}$$

so We can put The value of "$R_n$" in eq (1)

$\hookrightarrow$ $Q = \left( \frac{1}{n} R_\eta^{2/3} S_o^{1/2} \right) A$

putting values.

$$35 = \left( \frac{1}{0.0219} \times \left( \frac{7.755\,d}{2d + 7.755} \right)^{2/3} \times \left( 0.0008 \right)^{1/2} \times 7.755d \right)$$

$$\Rightarrow \frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left( \frac{7.755\,d}{2d + 7.755} \right)^{2/3} \times \left( 7.755\, d \right)$$

$$\left( \frac{3.5 \times 0.0219}{\sqrt{0.0008}} \right)^{3/2} = 60.14\, d^2 / 2d + 7.755$$

$$= 4.461 \left( 2.d + 7.755 \right) = 60.14\, d^2$$

$$= 8.922d + 34.59 = 60.14\, d^2$$

$60.14d^2 - 8.922d - 34.59 = 0$

$$\boxed{d = 0.673}$$

↳ so The depth of channel is 0.673

Now

q: $q$ = discharge per unit width

$$q = \frac{Q}{b} \Rightarrow q = 3.5/7.755$$

$$\Rightarrow \boxed{q = 0.451 \ m^2/s}$$

↳ Critical depth $y_{cr}$

using equation:

$$y_{cr} = (q^2/g)^{1/3}$$

$$= y_{cr} = \left( \frac{(0.451)^2}{9.81} \right)^{1/3}$$

↳ Critical Velocity, $V_{cr}$

using equation.

$$V_{cr} = \sqrt{g y_{cr}}$$

$V_{cr} = \sqrt{(9.81)(0.274)}$

$$V = \frac{Q}{A} = \frac{35}{7.755 \times 0.673}$$

$\boxed{V_{cr} = 1.63 \text{ m/s}}$

$V = 0.670$ m

↳ $y = 0.673$m , $y_{cr} = 0.274$m , $V_{cr} = 1.63$ m/s

* As $y > y_{cr}$

AND

$V < V_{cr}$

So The flow is "Sub critical flow"

# QUESTION # 03

## GIVEN DATA:

Width OF Smooth plate $B = 200mm = 0.2m$

Length OF smoth plate $L = 800mm = 0.8m$

oil With specific gravity, $S = 0.89$

Undisturbed Velocity, $U = 5 m/s$

Kinematic Viscosity $= \nu = 0.93 \times 10^{-4} m^2/s$

Required Data :-

Friction drag on one side of a smooth plate, $F_f$ = ?

## Solution:-

→ CHeck The Flow

As $\nu = 0.93 \times 10^{-4} m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500,000$$

Thus laminar ~~is~~ Flow.

↳ Now

$$Cf = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{4301075}}$$

$$Cf = 6.403 \times 10^{-3}$$

$$\boxed{Cf = 0.0064}$$

$$\Rightarrow Ff = Cf \, \rho \, V^2/2 \, BL$$

$$= (0.0064)(soil \times \gamma water) \times \left[\frac{(5)^2}{2} \times (0.2)(0.8)\right]$$

$$Ff = \left[(0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)\right]$$

$$\boxed{Ff = 11.392\,N}$$

↳ So the Fraction drag on one side of smooth plate is 11.392N.