

4th Semester

Name:-

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ID NO:-

7906

Section:-

A

Subject:-

MOS - II

Teacher:-

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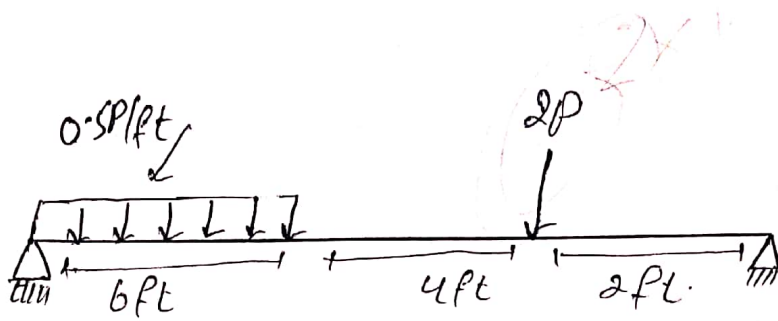
Date .. wed April 15  
2020

## question

Construct the Mohr's circle diagram and find the principal stress and maximum in plane shear stress for the stress state of a point  $c$  located at the centre of uniform distributed load and 2 inches below the top fiber of beam cross section shown in figure. However to construct the Mohr's circle it is necessary to draw the shear and flexural stress variation diagram for maximum shear force and bending moment respectively. Compare the result obtained from the Mohr's circle with the stress transformation equation.

where  $P$  is the last two digits of class registration number in pounds

①

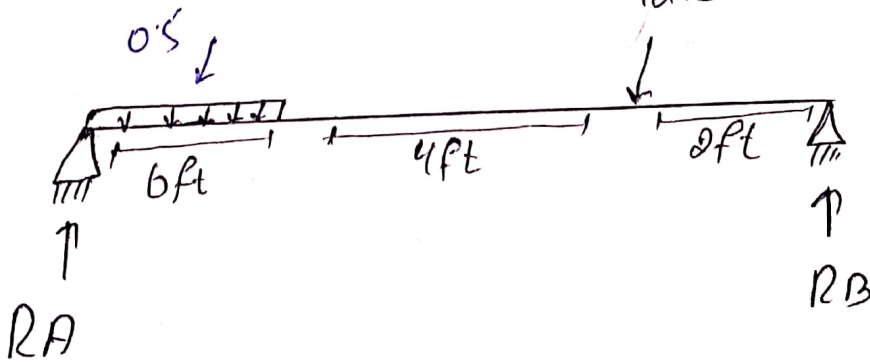
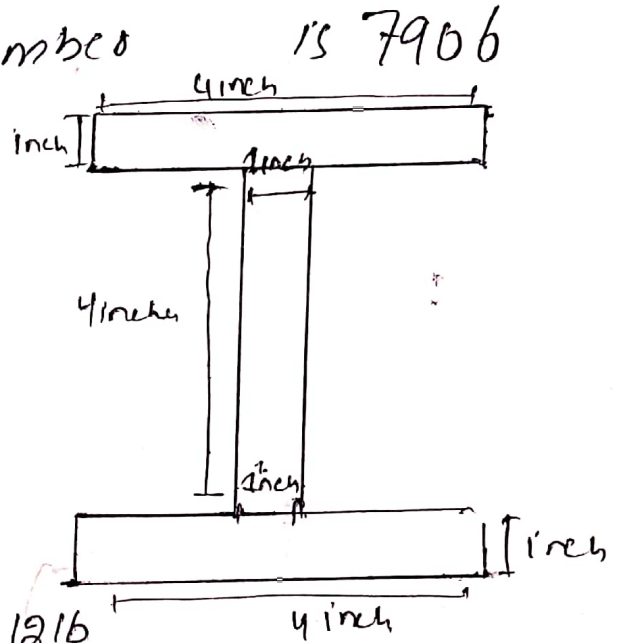


Here my class registration number is 7906

so  $20 = 2 \times 0.6 = 12 \text{ lb}$

$0.5 \times 0.6 = 3 \text{ lb/ft}$

Free body diagram



Support Reactions

As we know that

$\sum F_y = 0 \quad \uparrow^+ \downarrow^-$

$$R_A + R_B = 15 \text{ lb} \quad (2)$$

Now

$$\sum M = 0 \quad \curvearrowright -$$

$$R_B \times 12 - 12 \times 10 - 12 \times 3 = 0$$

$$12 R_B - 120 - 36 = 0$$

$$12 R_B = 120 + 36$$

$$\frac{12 R_B}{12} = \frac{156}{12}$$

$$\boxed{R_B = 13 \text{ lb}}$$

As

$$R_A + R_B = 15$$

$$R_A + 13 = 15$$

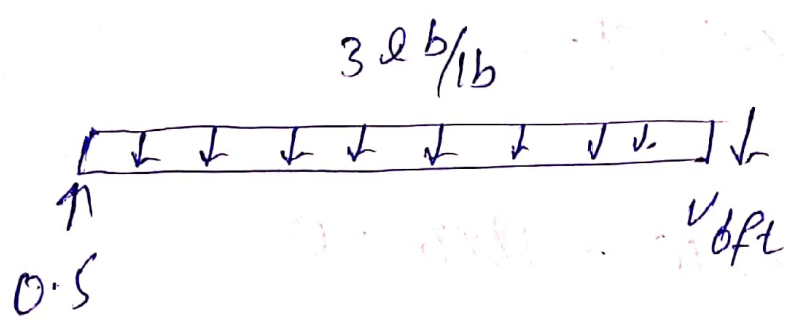
$$R_A = 15 - 13$$

$$\boxed{R_A = 2 \text{ lb}}$$



(3)

Now shear force at change point of beam.



So,

Shear force at 0ft from left support

$$\sum F_y = 0 \quad \uparrow^- \quad \downarrow^+$$

$$V_{0ft} - 0.5 + 3 \times 6 = 0$$

$$V_{0ft} - 0.5 + 18 = 0$$

$$V_{0ft} = 0.5 - 18$$

$$V_{0ft} = -17.5 \text{ lb}$$

Now shear force at  $V_{10ft}$

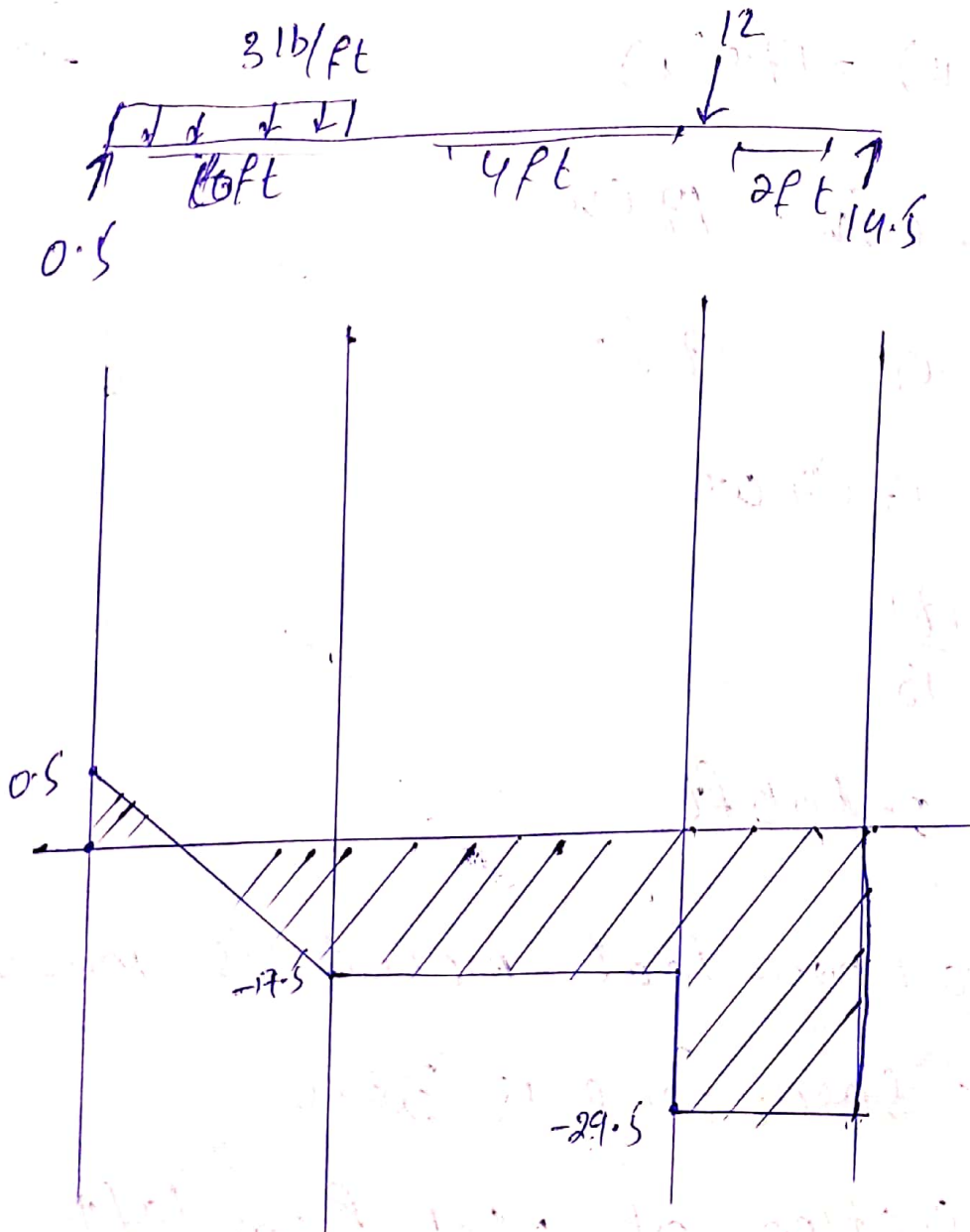
$$\sum F_y = 9.1 \quad \downarrow^+$$

(4)

$$\rightarrow 0.5 + 18 + 12 + V_{10ft} = 0$$

$$V_{10ft} = 0.5 - 18 - 12$$

$$V_{10ft} = -29.5 \text{ Qb}$$

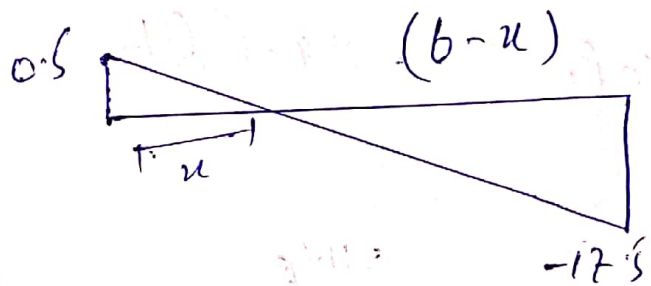


(5)

Now moment at change point

Find Zero Shear Point

$$\frac{0.5}{u} = \frac{17.5}{(b-u)}$$



$$0.5(b-u) = 17.5(u)$$

$$0.5 \times b - 0.5u = 17.5u$$

$$3 = 0.5u = 17.5u$$

$$3 = 17.5u + 0.5u$$

$$\frac{3}{18} = \frac{18u}{18}$$

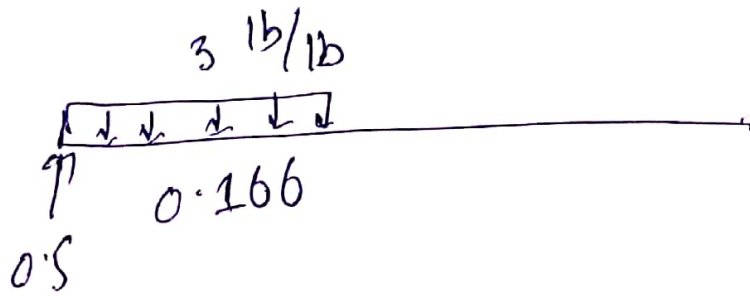
$$u = 0.166 \text{ ft}$$

As we know that moment is maximum

where shear force is zero.

Take section of 0.166 from left support and find moment.

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$$\sum M_{0.166} = 0 \quad \curvearrowright +$$

$$M_{0.166} + 0.5 \times 0.166 - 18 \left( \frac{0.166}{8} \right) = 0$$

$$M_{0.166} + 0.083 - 18(0.02083) = 0$$

$$M_{0.166} + 0.083 - 1.494 = 0$$

$$M_{0.166} = 1.494 - 0.083$$

$$M_{0.166} = 1.411 \text{ ft}$$

Now

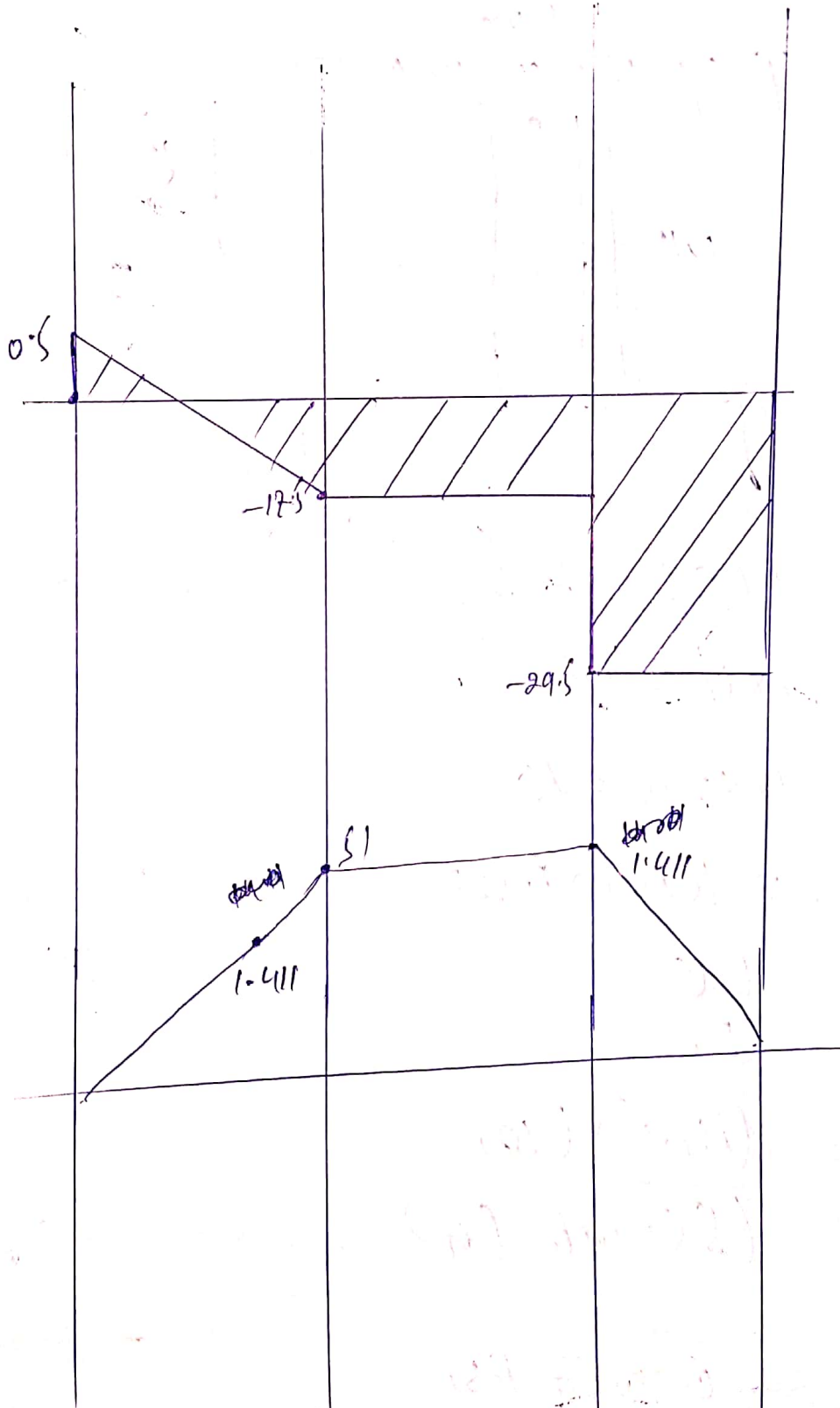
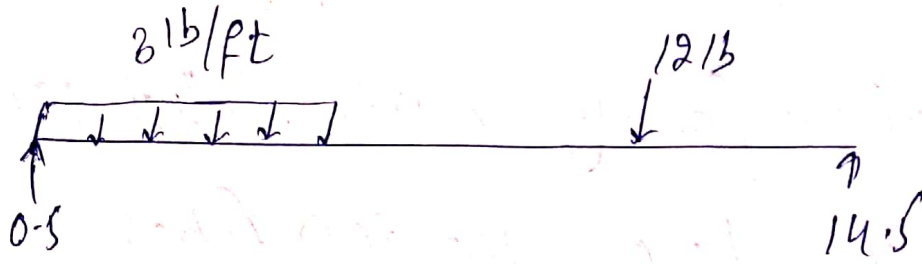
$$\sum m = 0 \quad \curvearrowright +$$

$$M_{of} + 0.5 \times 6 - 3 \times 6 \times 3 = 0$$

$$M_{of} = 5 \text{ ft}$$



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Now.

## Shear Stress

As per question the maximum shear stress  $T = \frac{VQ}{It}$  occurs where the

maximum shear force lies in above diagram  
max shear force is  $10 \text{ kN}$

So,

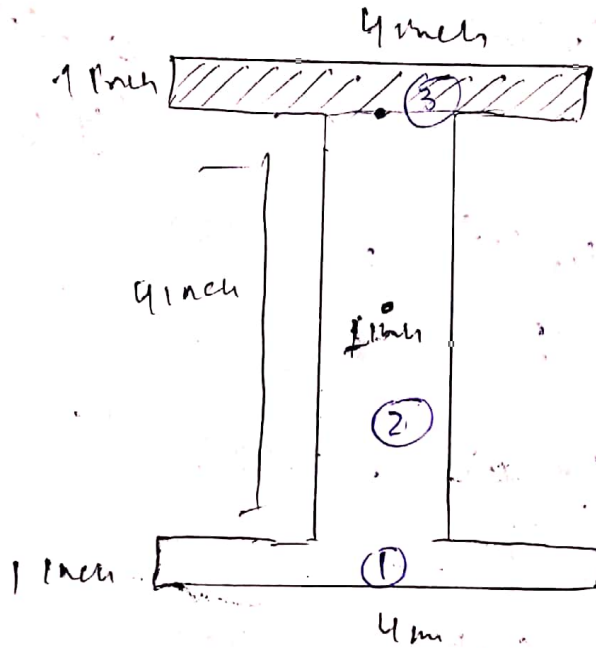
To find the shear stress we have  
the following formulae

$$T = \frac{VQ}{It}$$

we first find the moment of inertia

P.T.O.

(9)



As we know that to find Centroid we have the following formula

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$A_1 = 4 \times 1 = 4$$

$$A_2 = 4 \times 1 = 4$$

$$A_3 = 4 \times 1 = 4$$

$$\bar{y} = \frac{4 \times 0.5 + 4 \times 3 + 4 \times 5.5}{4 + 4 + 4}$$

$$\bar{y} = 3''$$

now moment of inertia.

(10)

NO	A(in <sup>2</sup> )	I <sub>x</sub> (in <sup>4</sup> )	d = (ȳ - y <sub>i</sub> ) (ȳ - y <sub>1</sub> )(ȳ - y <sub>2</sub> )
①	4	$\frac{4 \times (1)^3}{12} = 0.333$	
②	4	$\frac{1 \times (4)^3}{12} = 5.333$	
③	4	$\frac{4 \times (1)^3}{12} = 0.333$	

(now "d")

$$\text{① } d = (\bar{y} - y_1) = (3 - 0.5) = 2.5$$

$$\text{② } d = (\bar{y} - y_2) = (3 - 3) = 0$$

$$\text{③ } d = (3 - 5.5) = -2.5$$

(now Ad<sup>2</sup>)

$$\text{① } 4 \times (2.5)^2 = 25$$

$$\text{② } 4 \times (0)^2 = 0$$

$$\text{③ } 4 \times (-2.5)^2 = 25$$

now

(11)

$$I_u = I_u + Ad^2$$

$$(1) 0.333 + 25 = 25.333$$

$$(2) 5.333 + 0 = 5.333$$

$$(3) 0.333 + 25 = 25.333$$

Total

$$I = I_{u1} + I_{u2} + I_3$$

$$I = 25.333 + 5.333 + 25.333$$

$$I = 55.999 \text{ in}^4$$

now shear stress

$$\tau = \frac{VQ}{Ib}$$

$$v_{max} = 14.5$$

$$Q = \bar{y} A$$

b = breath of that fiber

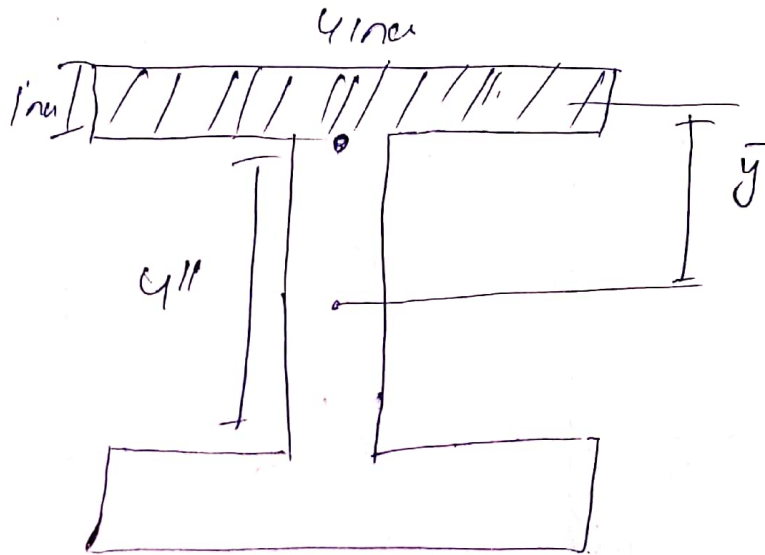
$$R_A =$$
$$R_B = 10$$



shear

⑧ ⑫

Stress at point 'c' located at  
Centre of uniformly distributed load  
and 1 inch below the top fiber.



$$y = 2 + 0.5 = 2.5$$

$$A = 1 \times 4 = 4$$

$$Q = 4 \times 2.5 = 10$$

As we know that:

$$t = \frac{VQ}{Ib}$$

$$t = \frac{(14.5)(10)}{(55.996)(4)}$$

$$t = 0.647 \text{ Psi}$$

Now Flexural Stress Analysis

$$\sigma = \frac{MY}{I}$$

where M is maximum moment in BMD

$$M = 51$$

$$\sigma = \frac{51 \times 2}{55.996}$$

$$\sigma = 1.82155 \text{ PSI}$$

So, shear stress at point "C" is

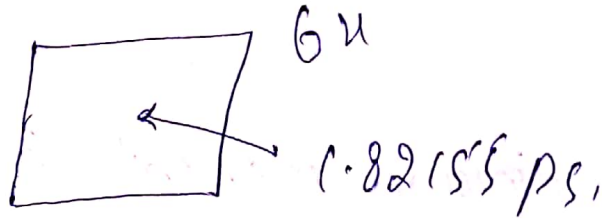
$$\tau = 0.647 \text{ PSI}$$

Flexural stress at point C

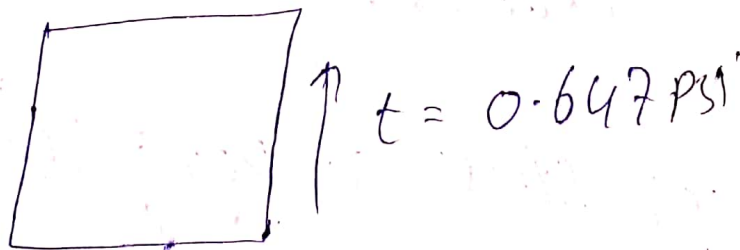
$$\sigma = 1.82155 \text{ PSI}$$

(14)

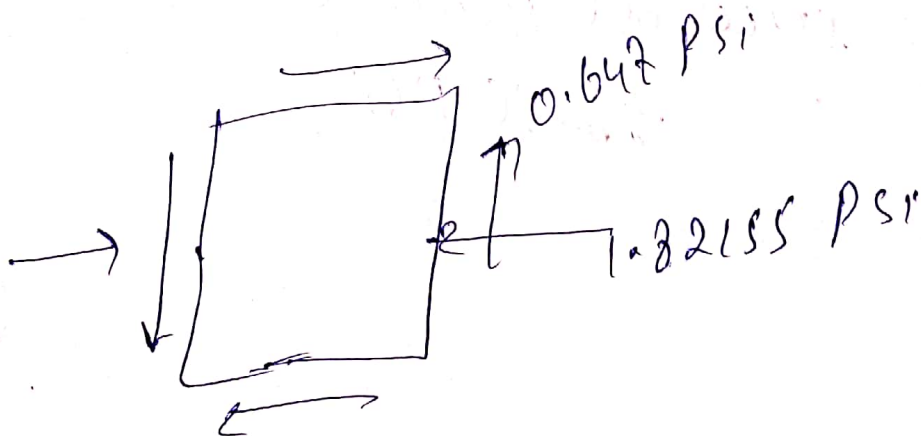
Now consider "C" is a planar element.



$1.82155 \text{ psi}$  is compressive because point C lies in compressive zone of beam now.



Combine stress on 2D element.



Now we can find the stress state condition of point  $C'$  at a degree of  $20^\circ$  clockwise orientation. (15)

Solution.

Given stress state

$$\sigma_x = -1.82155 \text{ psi}$$

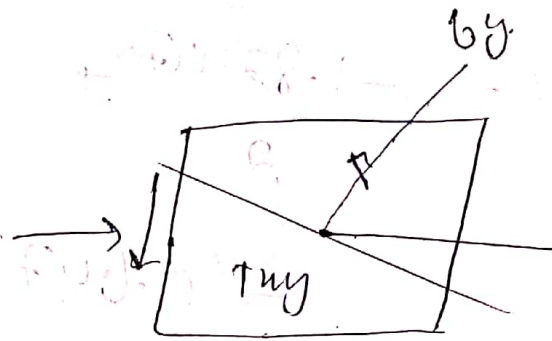
$$\sigma_y = 0$$

$$\tau_{xy} = 0.647 \text{ psi}$$

$$\sigma_{x'} = ?$$

$$\sigma_{y'} = ?$$

$$\tau_{x'y'} = ?$$



As we derived the following formulae, equation for stress transformation.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

(16)

$$b_{y'} = \frac{b_x + b_y}{2} - \frac{b_x - b_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

for  $b_{u'}$

$$b_{u'} = \frac{-1.82155 + 0}{2} - \frac{1.82155}{2} \cos(2(-20)) + (0.647) \sin(2(-20))$$

$$b_{u'} = -0.910 + 0.697 - 0.4158$$

$$b_{u'} = -2.0228 \text{ psi } \underline{\text{Compression}}$$

For  $b_{y'}$

$$b_{y'} = \frac{-1.82155 + 0}{2} - \frac{(-1.82155) - 0}{2} \cos(2(-20)) - (0.647) \sin(2(-20))$$

$$b_{y'} = -0.910 + 0.697 + 0.4158$$

$$b_{y'} = 0.2028 \text{ psi } (\text{Tension})$$



(17)

$$\tau_{x'y'} = - \frac{6x \cdot -6y}{2} \sin \theta + \tau_{xy} \cos 2\theta$$

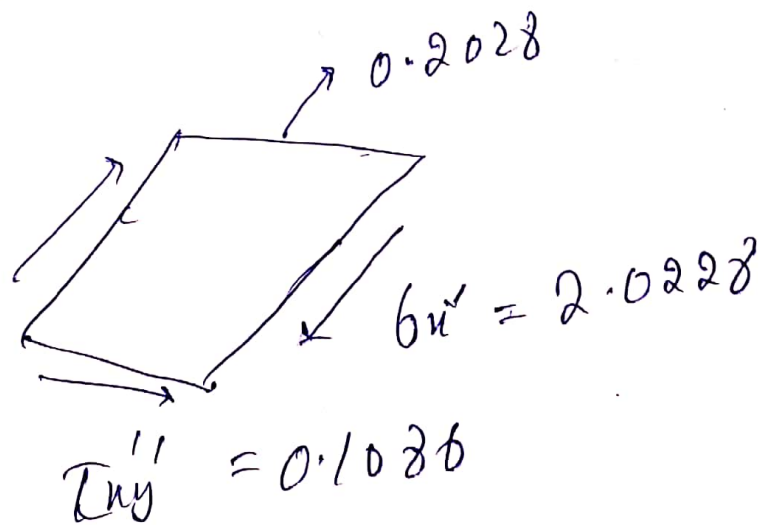
$$= - \frac{(-1.82155 - 0)}{2} \sin 2(-20)$$

$$+ 0.647 \cos 2(-20)$$

$$= -0.5854 + 0.694$$

$$\tau_{x'y'} = 0.1086 \text{ psi}$$

Now the new stress state after the  $20^\circ$  clockwise rotation is



(18)

Find the principle stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-1.85 + 0}{2} \pm \sqrt{\left(\frac{-1.85 - 0}{2}\right)^2 + (0.64)^2}$$

$$= -0.910 \pm \sqrt{0.8295 + 0.413}$$

$$= -0.910 \pm 1.169$$

$$\sigma_y = \sigma_1 = -0.910 + 1.169 = 0.2069$$

$$\sigma_x = \sigma_2 = -0.910 - 1.169 = -1.89215$$

now

Max in Plane Shear Stress

(19)

$$\tau_{xy} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-1.82155 - 0}{2}\right)^2 + (0.647)^2}$$

$$= \sqrt{0.8293 + 0.418}$$

$$\tau_{xy} = 1.1169 \text{ psi}$$

main plan  
shear str

TO Draw Mohr's circle for the  
Given Problem

Solution

As we know that to draw  
the circle we need the coordinates  
of circle as well as radius.

(20)

The coordinates of circle can be find by this

$$\left( \frac{bu + by}{\theta}, 0 \right)$$

Centre coordinates

$$(h, k) = \left( -\frac{1.82155}{\theta}, 0 \right)$$

$$= (-0.91077, 0)$$

Now,

Radius of moon's circle

$$r = \sqrt{\left( \frac{bu + by}{\theta} \right)^2 + (uy)^2}$$

$$r = \sqrt{\left( \frac{-1.82155 - 0}{\theta} \right)^2 + (0.647)^2}$$

$$r = \sqrt{0.8295 + 0.418}$$

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$|r| = 1.1169$

