

Q3 Find the Eigen values and Eigen Vectors of the below matrix

$$\begin{bmatrix} ID3 & -6 & 2 \\ -6 & ID2 & -4 \\ 2 & -4 & ID4 \end{bmatrix}$$

Sol: $ID1=1, ID2=4, ID3=0, ID4=1, ID5=1$

Putting values in the matrix

$$\begin{bmatrix} 0 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 7 \end{bmatrix}$$

$$(A - \lambda I) x = 0$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 0 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & 4 & 7 \end{bmatrix} = \lambda \begin{bmatrix} 0 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

Putting $\lambda = 0$

$$\begin{bmatrix} 0 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-6y + 2z = 0 \rightarrow \text{eq (1)}$$

$$-6x + 4y - 4z = 0 \rightarrow \text{eq (2)}$$

$$2x - 4y + 7z = 0 \rightarrow \text{eq (3)}$$

Taking equation 2 and 3

$$\frac{x}{28-16} = \frac{-y}{-8+4z} = \frac{z}{24-8}$$

$$\frac{x}{12} = \frac{y}{34} = \frac{z}{16}$$

$$\frac{x}{6} = \frac{y}{17} = \frac{z}{8}$$

Eigen vector = $\begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix}$ correspond
 $\lambda = 0$

$$\begin{bmatrix} 0-\lambda & -6 & 2 \\ -6 & 4-\lambda & -4 \\ 2 & -4 & 7-\lambda \end{bmatrix} = 0$$

$$\Rightarrow -\lambda[(4-\lambda)(-7-\lambda) - (-4 \times -4)] + 6[(-6)(7-\lambda) - (2 \times 4)] + 2[+6 \times (-4) - (2(4-\lambda))]$$

$$\Rightarrow -\lambda^3 - 3\lambda^2 + 84\lambda - 172$$

$$\lambda(-\lambda^2 - 3\lambda + 84 - 172) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 3\lambda - 88$$

$$-\lambda^2 - 8\lambda - 111 - 88$$

$$-\lambda(\lambda + 8) - 11(\lambda + 88)$$

Eigen value is 0, 8, -11

$$\lambda = 0, \lambda = -8, \lambda = -11$$

Eigen vector

$$[A - \lambda I] [x] = 0$$

$$\begin{bmatrix} 0-\lambda & -6 & 2 \\ -6 & 4-\lambda & -4 \\ 2 & -4 & 7-\lambda \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Q1 Solve the system of equations that corresponds to this augmented matrix.

$$\left[\begin{array}{cccc} 1 & -3 & 4 & -102 \\ 3 & -7 & 7 & -104 \\ -4 & 6 & -1 & 103 \end{array} \right]$$

Sol: $\left[\begin{array}{cccc} 1 & -3 & 4 & -102 \\ 3 & -7 & 7 & -104 \\ -4 & 6 & -1 & 103 \end{array} \right]$

Evaluate.

$$\begin{array}{cccc} 1 & -3 & 4 & -102 \\ 3 & -7 & 7 & -104 \\ -4 & 6 & -1 & 103 \end{array}$$

TRANSPOSE MATRIX

$$\left(\begin{array}{ccc|ccc} 1 & 3 & -4 & & & \\ -3 & -7 & 6 & & & \\ 4 & 7 & -1 & & & \\ -102 & -104 & 103 & & & \end{array} \right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -7 & 20 & & \\ 6 & 1 & 2 & 101 & & \\ 0 & 0 & -\frac{5}{2} & 301 & & \end{array} \right)$$

Sol: according to maj ID = 14071
ID1 = 1, ID2 = 4, ID3 = 0, ID4 = 7
ID5 = 1

Matrix $\begin{bmatrix} 1 & 0 & 8 \\ 2 & 7 & 1 \\ -3 & 0 & 0 \\ 1 & 0 & 16 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 0 & 8 \\ 2 & 7 & 1 \\ -3 & 0 & 0 \\ 1 & 0 & 16 \end{bmatrix} \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 + 3R_1 \\ R_4 \rightarrow R_4 - R_1 \end{array}$

$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 7 & -15 \\ 0 & 0 & 24 \\ 0 & 0 & 8 \end{bmatrix} R_3 \rightarrow R_3 - 3R_4$

$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 7 & -15 \\ 0 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix} R_3 \leftrightarrow R_4$

$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 7 & -15 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ = Now the matrix is in echelon form

$\rho(A) = \text{Total number of Non-zero Rows} = 3$

Q.2 Part (a)

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} = A$$

$$\det(A) \rightarrow 0 \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} - (-1) \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow 0 [0-0] + 1 [0-0] + 0 [0-1]$$

$$\Rightarrow 0 + 0 + 0$$

$$= \det(A) = 0$$

AS we know

if the determinant is 0, then

the matrix has no inverse

B) Find an echelon form from the below matrix using row operations.

$$\begin{bmatrix} 1 & ID3 & 8 \\ 2 & ID4 & -1 \\ -3 & 0 & 0 \\ 1 & ID3 & 16 \end{bmatrix}$$