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→ 7986

→ Differential Equ.

→ Final Paper.

→ Section "A"

→ 30 - 06 - 2020

Q No #1 : Part – 1 :-> $w = \sin(x+ct) + \cos(2x+2ct)$.

Answer :->

⇒ **Solution:->** $w = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \quad \text{— (i)}$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2.$$

$$\frac{\partial^2 w}{\partial x^2} = \sin(x+ct) - 4\cos(2x+2ct).$$

$$= [-\sin(x+ct) - 4\cos(x+2ct).$$

Answer:->

$$\boxed{c^2 = \frac{\partial^2 w}{\partial x^2}}$$

※ ——————————————— ※

Q No #1 : Part — II :→

Answer :→

Solution :→ $w = \tan(2x + ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x + ct)$

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x + xt)\right]$$

$$= c2^2 2\sec^2(2x+ct)\tan(2x+ct).$$

Now $\frac{\partial w}{\partial x} = 2\sec^2(2x + ct)$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x + ct)\tan(2x+ct)$$

$$\Rightarrow 4c^2\sec^2(2x+ct)\tan(2x+ct) = 4c^2\sec^2(2x+ct) - \tan(2x+ct).$$

Answer :→ $0 = 0$ (satisfied)

QNo#02: $\longrightarrow$

Answer: $\longrightarrow$

$\Rightarrow$ **Solution:** $\rightarrow$ Given funcation is

$$F(x) = \begin{cases} x \; ; & -\pi < x \leq 0 \\ 2x \; ; & 0 \leq x \leq \pi . \end{cases}$$

We have to find the Fourier co-efficients, a_0, a_n & b_n.

Now:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\Rightarrow \boxed{a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x(\cos nx)\,dx$$

$$= \frac{1}{\pi}\left\{x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left\{x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

(P - T - O) $\Rightarrow$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

so

$$\Rightarrow \boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} & ; \text{ if } n \text{ is add} \\ 0 & ; \text{ if } n \text{ is even} \end{cases}} \longrightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$\Rightarrow \boxed{b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}} \longrightarrow (3)$$

so the required fourier series is:

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

$\Rightarrow$ **Answer:**

※ ——— ※

Q No # 3 :→

Answer :→

Solution :→

$y'' - 4y' + 13y = 8\sin 3x, \; y(0) = 1$ and $y'(0) = 2$ ——— (i)

Associated Homogenous eq of (i) is

$$y'' - 4y' + 13y = 0 \longrightarrow (2)$$

change (2) into Auxiliary equation

Put $y = m$ in (2).

$$m^2 - 4m + 13 = 0$$

use quadratic formula.

$$a = 1, \; b = -4, \; c = 13$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

(P-T-O) ⇒

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$2 \pm 3i$$

$$m_1 = 2+3i$$

$$m_2 = 2-3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x).$$

let $y_c = A \cos 3x + B \sin 3x$ ——→ (*)

Diff w.r.t "x"

$$y_p' = -3A \sin 3x + 3B \cos 3x.$$

Again Diff w.r.t "x".

$$y_p'' = -9A \cos 3x - 9B \sin 3x.$$

Put in (1).

→ $(-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3B \cos 3x) + 13$ —
$(A \cos 3 + B \sin 3x) + B \sin 3x.$

(P-T-O) ⟹

$\Rightarrow -9A \cos 3x - 12B \cos 3x + 13A \cos 3x - 9B \sin 3x + 12A -$

$\sin 3x + 13B \sin 3x - 8 \sin 3x.$

$\Rightarrow (-9A - 12B + 13A) \cos 3x + (-9B + 12A + 13B) \sin 3x = 8 \sin 3x.$

$\Rightarrow (4A - 12B) \cos 3x + (4B + 12A) \sin 3x = 8 \sin 3x.$

Comparing co-effient.

$\sin 3x \Rightarrow 4B + 12A = 8 \longrightarrow (a)$

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$\Rightarrow \boxed{A = 3B} \longrightarrow (b)$

Put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \longrightarrow (c)$

Put c and b

$a = 3/5 \longrightarrow (d)$

Put (c) and (d) in (★)

$y_p = 3/5 \cos 3x + 1/5 \sin 3x \longrightarrow (B)$

The General Solution is

$y = y_c + y_p$

(P-T-O) $\Rightarrow$

$$y = y_c + y_p$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5} \cos 3x +$$
$$+ \frac{1}{5} \sin 3x \quad \text{——— (c)}$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5} \cos 3x + \frac{1}{5} \sin 3x \quad \text{——— (c)}$$

Now we need to find the values of c_1 and c_2 for this.

Put $x = 0$ and $y = 1$ in (c)

$$1 = e^{x(0)}(c_1 \cos 3(0) + c_2 \sin 3(0) + \frac{3}{5} \cos 3(0) +$$
$$+ \frac{1}{5} \sin (3)(0).$$

$$1 = (c_1(1) + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0).$$

$$1 = c_1 + \frac{3}{5}$$

$$c_1 = 1 - \frac{3}{5}$$

$$\Rightarrow \boxed{c_1 = \frac{2}{5}} \longrightarrow (\star\star)$$

Now Diff "c" with . r . "x"

$$y' = c_1(2e^{2x} \cos 3x - 3e^{3x} \sin 3x) + (2(2e^{3x} \sin 3x +$$
$$+ 3e^{2x} \cos 3x) - \frac{6}{5} \sin 3x + $$
$$\Leftarrow (P-T-O) \qquad + \frac{3}{5} \cos 3x \quad \text{——— (D)}$$

$$y' = c_1(2e^{2x}\cos 3x - 3x - 3e^{3x}\sin 3x) + c_2(2e^{3x}\sin 3x + 3e^{2x} - \cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \longrightarrow (D)$$

(9)

Put $y' = 2$, $x = 0$ in (D)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x.$$

Put $y' = 2$, $x = 0$

$$c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) - + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - - \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0).$$

$$2 = 1(2) + c_2(3) - 0 + \frac{3}{5}$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

Put $c_1 = \frac{2}{5}$

$$2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = \frac{3}{5}$$

$$\Longrightarrow \boxed{c_2 = \frac{3}{15}} \longrightarrow (***)$$

(P-T-O) $\Rightarrow$

Put (**) and (***) in (C)

$$y = e^{2x}\left(\frac{2}{5}\cos^{3x} + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.}$$

Answer:

Q NO # 4 :→

Answer :→

Solution :→

Solve $(D^2 - DD')z = \cos x \cos 2y$ ——— (1)

As it is already in symbolical from

as $(D^2 - DD')z = \cos x \cos 2y$

Put A.E $D^2 - DD' = 0$

$\Rightarrow m^2 - n = 0$ $\quad \because \frac{D}{D'} = m_1 - e,\ D = m,$

$\Rightarrow m = 0, 1$ $\quad D' = 1.$

$$C.F = f_1(y) + f_2(y + x)$$

From eq (i)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$\frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

$\therefore 2\cos A \cos B = \cos(A+B) + \cos(A-B).$

$$C.F = f_1(y-x) + x f_2(y-x)$$

$$P.I = \frac{1}{D^2 + 2DD' + D^2} [2(y-x) + \sin(x-y)]$$

(P-T-O) ⇒

$$= \frac{1}{(D+D')^2}\left[2(y-x)+\sin(x-y)\right]$$

$\Rightarrow$ General Method here $m=-1$, $y-x=c$.

$$= \frac{1}{D+D'}\int \left(2c+\sin(-c)\right) dx$$

$$= \frac{1}{D+D'}\left[2cx-(\sin c)x\right]$$

$\Rightarrow$ Replacing (c by $y-x$)

$$= \frac{1}{D+D'}\left[2x(y-x)-x\sin(y-x)\right]$$

Again $m=-1$

Put $y-x=c$

$$= \int (2xc - x\sin c)\, dx = cx^2 - \frac{x^2}{2}\sin c$$

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y).$$

Therefore the required solution of (1) is given by.

P-T-O $\Rightarrow$

$$z = CF + P.I = f_1(y-x) + x f_2(y-x)$$
$$+ x^2y - x^3 + \frac{1}{2}x^2 \sin(x-y).$$

Answer:->

$$\boxed{z = CF + P.I = f_1(y-x) + x f_2(y-x) + x^2y - x^3 + \frac{1}{2}x^2 \sin(x-y).}$$

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