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BSSE- 2nd Semester

Department of Computer Science

Final Term Assignment

Linear Algebra

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BSSE

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Q1 Determine the following system consistant or not:

$$x_1 - 6x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$\left[\begin{array}{ccc:c} 1 & -6 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -6 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 30 & -10 & 10 \end{array}\right] R_3 + 5R_1$$

$$\left[\begin{array}{ccc:c} 1 & -6 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & -5 & 10 \end{array}\right] R_3 + 5R_1$$

It is a consistant

$$x_1 - 6x_2 + x_3 = 0$$
$$+2x_2 - 8x_3 = 8$$
$$-5x_3 = 10$$

Q2 Find the inverse of

$$A=\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \\ 5 & -2 & 7 \end{bmatrix}$$ by adjoint method.

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 3\begin{vmatrix} -1 & 0 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 0 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 3(-7+2) - 4(14-0) + 5(10+5)$$

$$= 3(-5) - 4(14) + 5(15)$$

$$= -15 - 56 + 75$$

$|A| = 4$ Cofactor formula

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 0 \\ -2 & 7 \end{vmatrix} = 1(-7-0) = -7$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 0 \\ 5 & 7 \end{vmatrix} = -1(14-0) = -14$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = 1(-4+5) = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = -1(14+10) = -24$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 1 \end{vmatrix} = 1(3-25) = -22$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -1(-6-20) = 26$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ 1 & 0 \end{vmatrix} = 1(0-5) = -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} = -1(0-10) = 10$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3-8) = -11$$

Find the adjoint matrix becomes

$$\text{Adj } A = \begin{bmatrix} -7 & -14 & 1 \\ 24 & -22 & 26 \\ -5 & 10 & -11 \end{bmatrix}^t$$

$$\text{Adj } A = \begin{bmatrix} -7 & 24 & -5 \\ 14 & -22 & 10 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{4} \begin{bmatrix} -7 & 24 & -5 \\ 14 & -22 & 10 \\ 1 & 26 & -11 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7/4 & 24/4 & -5/4 \\ 14/4 & -22/4 & 10/4 \\ 1/4 & 26/4 & 11/4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -7/4 & 6 & -5/4 \\ 7/2 & -11/2 & 5/2 \\ 1/4 & 13/2 & 11/4 \end{bmatrix}$$ **Ans**

Q3 Solve the following of linear equation by Guass Jordan Method.

$$2x+2y+4z=18$$
$$x+3y+2z=13$$
$$3x+2y-3z=14$$

By use Guass Jordan Method

$$\begin{bmatrix} 2 & 2 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 13 \\ 14 \end{bmatrix}$$

$$A_b = \left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 2 & 2 & 4 & 18 \\ 3 & 2 & -3 & 14 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 3 & 2 & -3 & +12 \end{array}\right] R_3-2R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & -7 & -9 & -12 \end{array}\right] R_3-3R_1$$

$$\underset{\sim}{R}\left[\begin{array}{ccc|c} 1 & 3 & 2 & 13 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & 9/7 & 12/7 \end{array}\right] -\frac{R_3}{7}$$

Which is echelon form

From $\underline{R_3}$:-

$$0x+0y+\frac{9}{7}z=\frac{12}{7}$$

$$z=\frac{12}{7}\times\frac{7}{9}$$

$$z=\cancel{84/63}\ 28/21 \qquad \boxed{z=\frac{28}{21}}$$

From $\underline{R_2}$:-

$$0x-4y+0z=-8$$

$$\boxed{y=2}$$

From $\underline{R_1}$:-

$$x+3y+2z=13$$

$$x+3(2)+2\left(\frac{28}{21}\right)=13$$

$$x+6+56/21=13 \qquad \boxed{x=91/21}$$

Sol-set $=\{(28/21, 2, 91/21)\}$

Q4 Show that this matrix is Diagonsiable:-

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$|A - x I| = 0$$

$$\begin{vmatrix} 4-x & 2 & -2 \\ -5 & 3-x & 2 \\ -2 & 4 & 1-x \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\begin{vmatrix} -1-x & 2 & -2 \\ -1-x & 3-x & 2 \\ -1-x & 4 & 1-x \end{vmatrix} = 0$$

$$(-1-x)\begin{vmatrix} 1 & 2 & -2 \\ 1 & 3-x & 2 \\ 1 & 4 & 1-x \end{vmatrix} = 0$$

$$R_2 - R_1$$

$$R_3 - R_1$$

$$-(1+x)\begin{vmatrix} 1 & 2 & -2 \\ 0 & 1-x & 4 \\ 0 & 2 & 3-x \end{vmatrix}$$

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Expanded w.r.t to C_1

$$-(1+x)\left[(1-x)(3-x)-8\right]=0$$

$$x=-1, 1, 3, -8$$

Eigen value ~~and~~ of A are $=-1, 1, 3, -8$

$$Ax=\lambda x$$

$$(A-\lambda I)x=0$$

$$\begin{bmatrix} 4-x & 2 & -2 \\ -5 & 3-x & 2 \\ -2 & 4 & 1-x \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ——— (1)

The rank of the matrix $=1$

The rank of the matrix$=3$

$$\begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2/5 \quad R_2-2R_1$

$$\begin{bmatrix} 3 & 2 & -2 \\ 1 & -2/5 & -2/5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x + 2y + 2z = 0$$

$$3x + 2y - 2z = 0$$

Ans.

Q5 Determine the following homogenous has a non-terival solution. Then

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

$$A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_b = \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & 8 & 0 \end{array}\right]$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & 20 & 0 & 0 \\ 6 & 1 & 8 & 0 \end{array}\right] \quad R_2 + R_1$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 20 & 0 & 0 \\ 6 & 1 & 8 & 0 \end{array}\right] \quad R_1/3$$

$$\underset{\sim}{R} \left[\begin{array}{ccc|c} 1 & 5/3 & -4/3 & 0 \\ 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 - 6R_1$$

From R_2:-

$$20x_2 = 0$$
$$x_2 = 0$$

From R_1:-

$$x + \frac{5}{3}x_2 - 4/3\,x_3 = 0$$
$$x = t$$

S.S $= \{t, -t, t\}$

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Q6 =

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad R_2 - 3R_1$$

$$\therefore A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_3 - R_1$$

$$\therefore A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad R_{1/3}$$

~~It is~~ $\therefore A = \begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{matrix} R_1 + 2R_3 \\ R_3 + R_2 \end{matrix}$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_3 - 4R_1$$

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Ans

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