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SEMESTER $10^{Th}$

PAPER NUMERICAL ANALYSIS

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## QUESTION #01:

Find a root of the equation $x^3 + 3.993 * 10^{-4} = 0.165x^2$. Use Newton Raphson method with $x_0 = 0.02$.

## GIVEN DATA:

$$x^3 + 3.993 * 10^{-4} = 0.165x^2$$

## REQUIRED:

USE Newton Raphson method.

## SOLUTION:

As we know that

Re-arranging the equation;

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{0.081 - (-0.00016)}{-0.0070}$$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

$x_2 = 0.058$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(x_2) = 0.000030$

$f'(x_2) = -0.0090$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$x_3 = 0.061$

## QUESTION NO 2:

Use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the Lagrange interpolation polynomial for $f(x) = 1/x$ at $x = 3$.

## SOLUTION:

$x_0 = 2$, $y_0 = 0.5$

$x_1 = 2.75$, $y_1 = 0.36$

$x_2 = 4$, $y_2 = 0.25$

As we know that Lagrange Interpolation formula

$$y = \frac{(x - x_1)(x_1 - x_2) \ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_n)} y_0$$

$x_0 = 2$ , $y_0 = 0.5$

$x_1 = 2.75$ , $y_1 = 0.364$

$x_2 = 4$ , $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

putting the values

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\boxed{y = 0.33}$$

QUESTION NO 3:

Complete the divided difference table for the given data, and Construct the interpolation polynomial that uses all this data.

| X = | 1.0 | 1.3 | 1.6 | 1.9 | 2.2 |
|---|---|---|---|---|---|
| Y = | 0.7651977 | 0.6200860 | 0.4554022 | 0.2818186 | 0.1103623 |

| $x_i$ | $f(x_i)$ | $f(x_{i-1}, x_i)$ | $f(x_{i-2}, x_{i-1}, x_i)$ | $f(x_{i-3}, x_i)$ | $f(x_{i-4}, x_i)$ |
|---|---|---|---|---|---|
| $x_0 = 1$ | 0.7651977 | | | | |
| | | −0.4837056 | | | |
| $x_1 = 1.3$ | 0.6200860 | | −0.108734 | | |
| | | −0.548946 | | 0.0658785 | |
| $x_2 = 1.6$ | 0.4554022 | | −0.0494433 | | −0.00280 49 |
| | | −0.578612 | | 0.0625255 | |
| $x_3 = 1.9$ | 0.2818186 | | 0.0625255 | | |
| | | 0.571521 | 0.006818 | | |
| $x_4 = 2.2$ | 0.1103623 | | | | |

i → $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$ .

ii → $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

iii → $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

④ → $f(x_3, x_4) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$\boxed{f(x_3, x_4) = 0.571521}$

Second divided difference;

$f(x_0, x_1, x_2) = \dfrac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$= \dfrac{-0.548946 - (-0.4837056)}{1.6 - 1}$

$= \boxed{-0.108734}$

$f(x_1, x_2, x_3) = \dfrac{f(x_2), x_3) - f(x_1, x_2)}{1.9 - 1.3}$

$f(x_1, x_2, x_3) = \boxed{-0.0494433}$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= \boxed{0.006818}$$

Third divided difference:

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.049443 - (-0.18734)}{1.9 - 1}$$

$$= \boxed{0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$= \boxed{0.06251255}$$

4th divided difference :

$$f(x_0,x_1,x_2,x_3,x_4) = \frac{0.0625255 - 0.0658785}{2.2 - 1}$$

$$= \boxed{0.0028049}$$

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2)$$

$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)$$

$$(x-x_2)(x-x_3) f(x_0,x_1,x_2,x_3,x_4)$$

$$= 0.7651977 + (x-1) - 6.4837056 + (x-1)(x-1.3)$$

$$(-0.108734) + (x-1)(x-1.3)(x-1.6)(0.0658785)$$

$$+ (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049) .$$