NAME

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ID

7782

Subject Introduction to Structure & dynamics Earthquake.

Submitted to

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DATE 29/June/2020.

Q# 01

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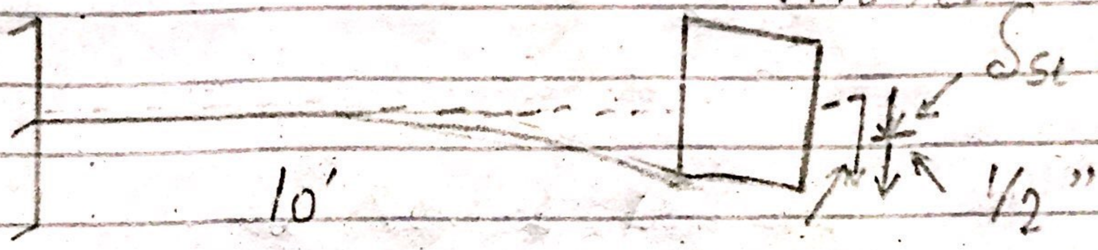


Solution:

The general E.O.M for SDOF System is;

$$Ku + C\dot{u} + m\ddot{u} = p(t)$$

In Our Case System is undamaged (c = 0) undergoing free vibration (p(t) = 0)

Hence general Eom becomes $Ku + m\ddot{u} = 0$ ...(i)

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29000 \text{ K/in}^2 \times 150 \text{ in}^4}{(10 \times 12 \text{ in})^3}$$

$$K = 7.55 \text{ K/in}$$

In Order to Eliminate the Chances of mistake during Calculation, it is more appropriate to use fundamental units like lb, ft sec or Kg, m, sec.

$$K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}$$

$m = \frac{7782}{32.2 \ ft/sec^2}$

$m =$ ~~[crossed out]~~ 241.67 Slug.

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{241.67}}$$

$W_n = 19.36$ rad/sec.

$T_n = 2\pi / W_n = 2\pi / 19.36$

$T_n = 0.327$ Sec.

Substituting the Corresponding Values in Eq - I.

$$90625 \, u + 241.67 \, \ddot{u} = 0$$

Where "K" is in lb/ft and 'm' is in lb sec/ft².

General Solution to the Eom for undamped free vibration is,

$$u(t) = u(0) \cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$$

$u(0) = 1/2'' = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$u(t) = \left(\left(\frac{1}{24}\right) \times \cos(19.36t) + 0 \right)$

$u(t) = (1/24) \times \cos(19.36t)$

Equivalent Static force at time "t" is

$f_s(t) = K \cdot u(t)$

$f_s(t) = \frac{90625 \times \cos(19.36)}{24}$

$f_s(t) = 3776.04 \cos(19.36t)$

Amplitude of dynamic displacement, u_0 for undamped free vibration is.

$u_0 = \sqrt{\left((u(0))^2 + \left(\dot{u}(0)/w_n\right)^2\right)}$

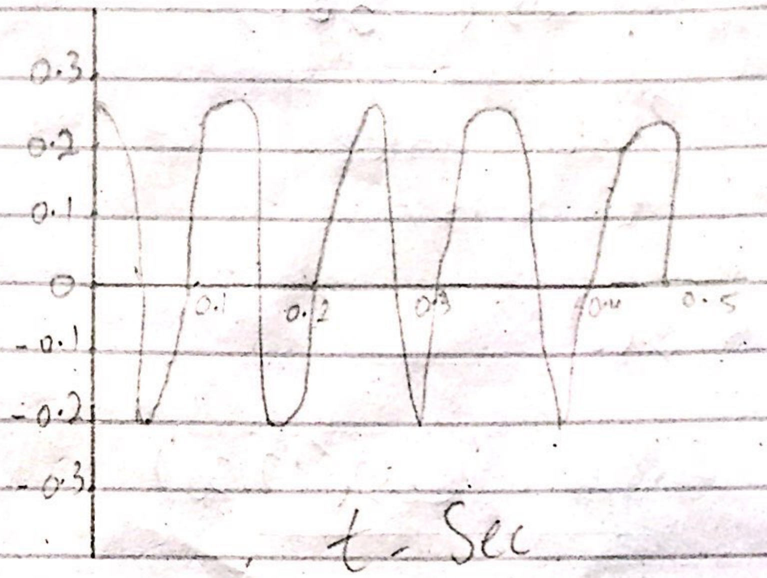
$u_0 = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$

$u_0 = \frac{1}{24}$ ft.

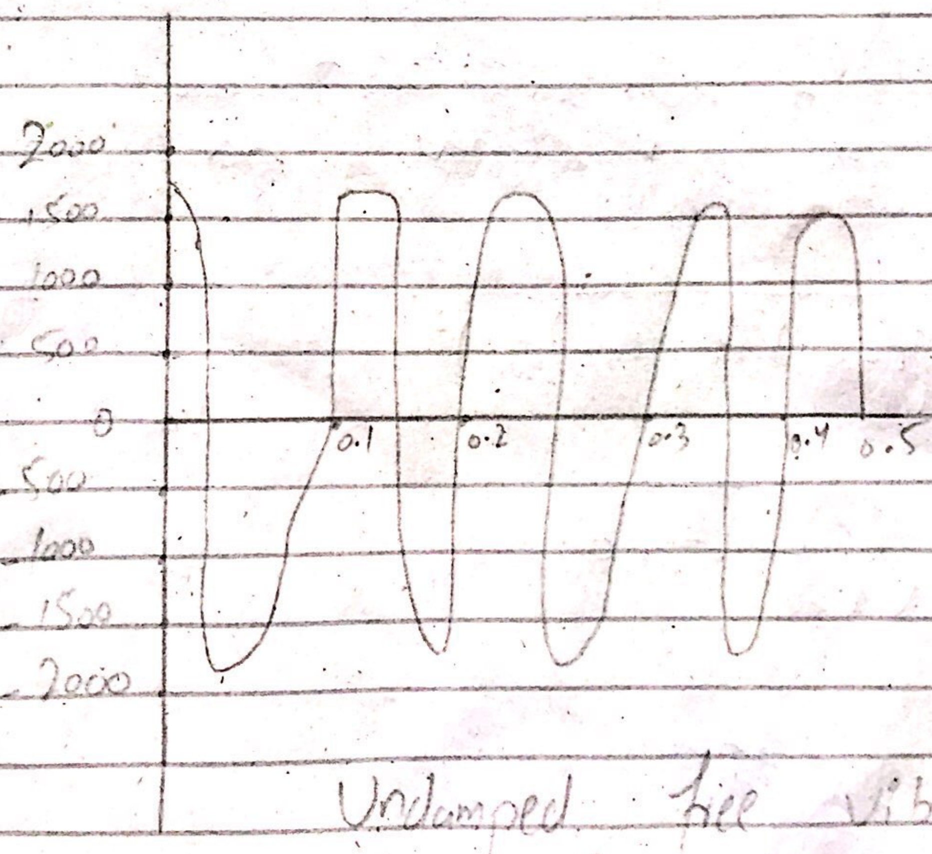
Amplitude of Equivalent Static force.

$K u_0 = 90625 \times 1/24$

$K u_0 = 3776.04 \, lb$

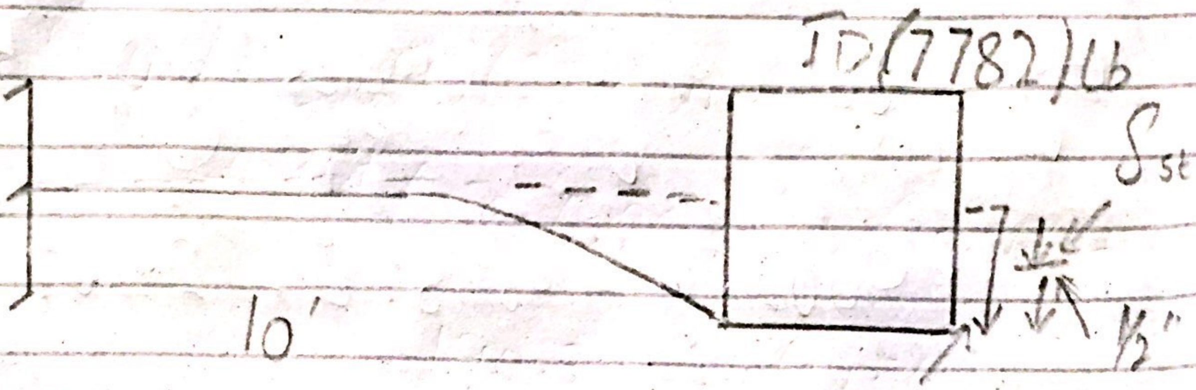


Undamped force vibration



Undamped free vibration.

Q#2



Solution:

* EOM for damped free vibration is;

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \text{(i)}$$

* it is known from (question 1)

$K = 90625$ lb/ft & $m = 241.67$ slug

$w_n = 19.36$

$C = \xi \times 2m \times w_n = 2 \times 241.67 \times 19.36 \times \xi$

($\xi = 0.03 - 0.05$
with considerable crackings
the damping ratio)

$\Rightarrow C = 2 \times 241.67 \times 19.36 \times 0.05$

$C = 467.87$ lb.sec/ft

By substituting values of K, C & m in eq (1)

$$90625u + 467.87\dot{u} + 241.67\ddot{u} = 0$$

Solution to the E.O.M for damped free vibration

$$u(t) = e^{-\zeta \omega_n t}\left(u(0)\cos(\omega_n t) + \frac{1}{\omega_D}(\dot{u}(0) + u(0)\zeta\omega_n)\cdot \sin(\omega_D t)\right)$$

$\omega_D = 19.36$ rad/sec

$$u(t) = e^{-0.05\times19.36}\left(\frac{1}{24}\times\cos(19.36t) + \frac{1}{19.36}\right) *$$

$$\left(0 + \frac{1}{24}\times0.05\times19.36\times\sin(19.36t)\right)$$

$$u(t) = e^{-0.973t}\left(0.042\times\cos(19.36t) + 0.052\times0.0403\times\sin(19.36t)\right)$$

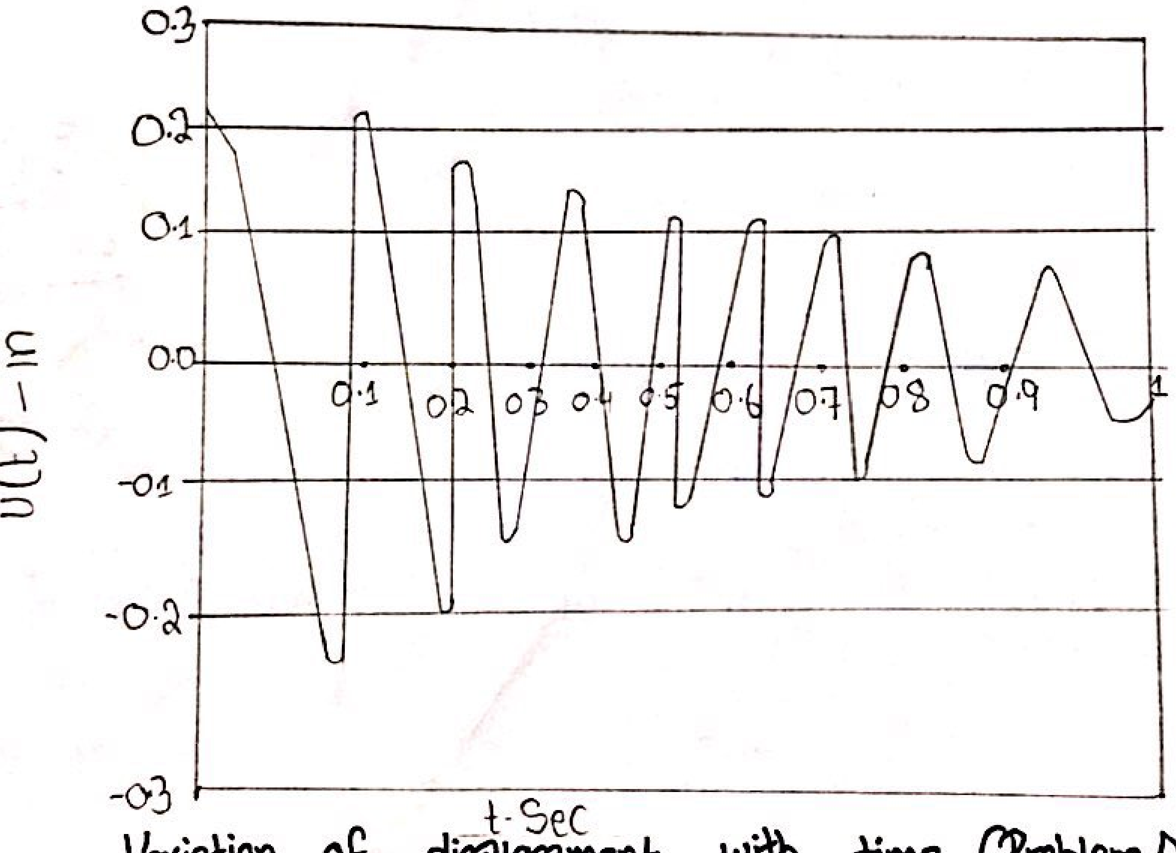
$$u(t) = e^{-0.973t}\left(0.042\times\cos(19.36t) + 0.0021\times\sin(19.36t)\right)$$

$f_s(t) = K.u(t) = 90625 \times u(t)$.

$f_s(t) = e^{-0.973t}(3806.25 \times \cos(19.36t$

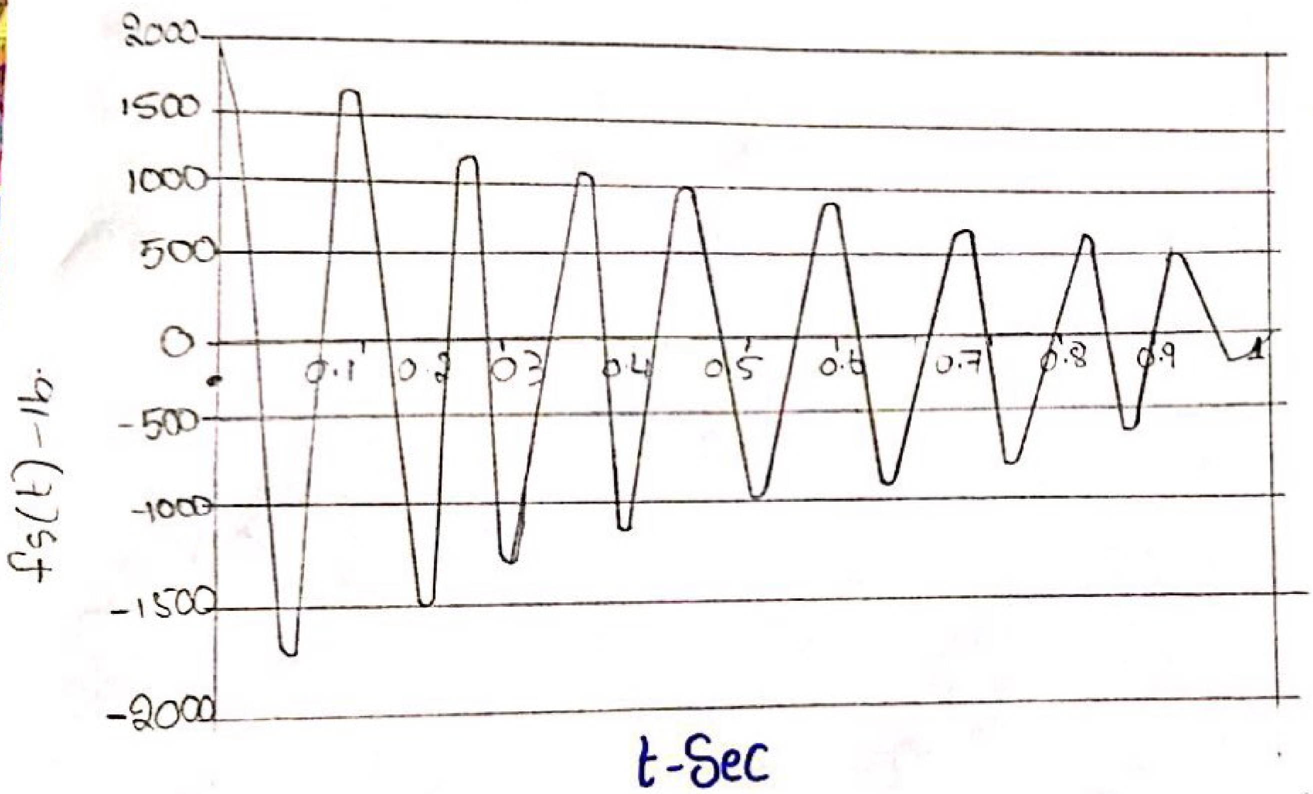
$190.3 \times \sin(19.36t)$.

Damped Free Vibration



Variation of displacement with time (Problem M4.3)

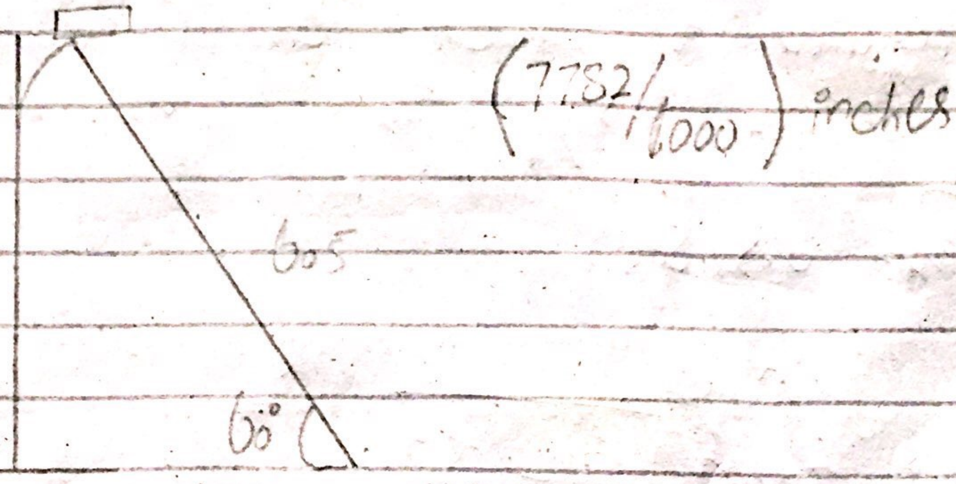
Damped Free Vibration



Variation of Equivalent static forces with time
(Problem M4.3)

Scanned with CamScanner

Q#3



Solution:

$$u_1 = \frac{7782}{1000} = 7.78''$$

After $j = 7$, $u_{j+1} = u_8 = 2.286\,cm$
$= 0.9''$

a) ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.78/0.9)$$

$$\zeta = 0.0490 \quad \text{or} \quad 4.9\%$$

b) T_n = ?

7 cycles of vibrations are completed in 3.57 sec.

Time required to Complete one Cycle.

$$= 3.57/7 = T_D$$

T_D = 0.51 Sec.

Now

$$W_D = W_n\sqrt{(1-\delta^2)}$$

$$2\pi/W_D = 2\pi/(W_n\sqrt{1-\delta^2})$$

$$\Rightarrow T_D = \frac{T_n}{1-\delta^2}$$

$$T_n = T_D \times \sqrt{1-\delta^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.049)^2}$$

$$T_n = 0.5093 = 0.51 \text{ Sec.}$$

(C) $K = ?$

$$K = \frac{60 \times \cos 60}{7.78} = 3.85 \text{ K/in}$$

K = ~~46850~~ ~~lb/ft~~ 46200 lb/ft

(D) Weight of the tank $w = ?$

$$W_n = \sqrt{K/m} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \times g}{W}}$$

$$= W_n^2 = K \times g / W$$

$$W = K \times g / W_n^2$$

Also

$$W_n = 2\pi / T_n$$

$$W = Kg\left(4\pi^2 / T_n^2\right) = Kg \times T_n^2 / 4\pi^2$$

~~46860~~

$$W = \frac{46200 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$$W = 9801.18\ lb$$

$$W = 9.8\ K$$

e) $C = ?$

It is Know that $\delta = C / 2mW_n$

$$C = \delta \times 2mW_n = \delta \times 2m \times (2\pi / T_n)$$

$$C = 0.044 \times 2 \times 2 \times \left(\frac{\pi}{0.51}\right)\left(\frac{9801.18}{32.2}\right)$$

$$C = 367.50\ lb.sec/ft$$

(F) No of Cycles to reduce displacement amplitude from 7.78 in to 0.5" $j = ?$

$$j = \frac{1}{2\pi\delta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2\times\pi\times 0.049} \ln\left(\frac{7.78}{0.5}\right)$$

$\Rightarrow j = 8.89$ or 9 cycles.