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# ASSIGNMENT # 02

## SUBJECT # DIFFERENTIAL EQUATION

## I,D # 13131

## SUBMITTED TO # SIR LATIF JAN

Q1) Use method of separation of variables to find the general solution to the following differential equations.

a) $x' = \sqrt{x}$

## SOLUTION:-

$$x' = \sqrt{x}$$

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = dt.$$

On integration on b.s, we get

$$\int \frac{dx}{\sqrt{x}} = \int dt$$

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$$\int x^{-1/2}\,dx = \int dt$$

$$-1/2\,x^{-1/2-1} = t + C$$

$$-\frac{1}{2x^{3/2}} = t + C$$ (C is the constant of integration)

required solution

b) $x' = e^{-2x}$

SOLUTION:-

$$x' = e^{-2x}$$

$$\frac{dx}{dt} = e^{-2x}$$

multiply both side by $dt$

$$dx = e^{-2x}\,dt$$

Divide both sides by ~~dx~~ $e^{-2x}$

$$\frac{dx}{e^{-2x}} = dt$$

$$e^{2x}\,dx = dt$$



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integrated b.s

$$\int e^{2x}\,dx = \int dt$$

$$\frac{e^{2x}}{2} = t + C$$

multiply b.s by 2.

$$e^{2x} = 2(t+C)$$

taking natural logarithm from b.S

$$\ln(e^{2x}) = \ln(2t + 2c)$$

$$2x = \ln(2t + C_1)$$

$$x = \frac{\ln(2t + C_1)}{C_1}$$

c) $\acute{y} = 1 + y^2$

SOLUTION:-

$$\acute{y} = 1 + y^2$$

$$\frac{dy}{dx} = 1 + y^2 \qquad \because y' = \frac{dy}{dx}$$

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$$dy = (1+y^2)\,dx$$

$$\frac{dy}{1+y^2} = dx$$

integrating b.s

$$\int \frac{dy}{1+y^2} = \int dx + C \qquad \because \int \frac{dx}{1+y^2} = \tan^{-1}(y)$$

$$\tan^{-1}(y) = x + C$$

$$\boxed{y = \tan(x+C)}$$

d) $u' = \frac{1}{5-2u}$

SOLUTION:-

$$u' = \frac{1}{5-2u}$$

$$\frac{du}{dv} = \frac{1}{5-2u}$$

$$(5-2u)\,du = dv$$

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Now integrating on b.s

$$\int (5-2u)\,du = \int dv$$

$$5u - \frac{2u^2}{2} + C = v + C$$

$$5u - u^2 = v + C$$

Further simplification

$$u^2 - 5u = C - v$$

$$\left[u^2 - 5u + \frac{25}{4} - \frac{25}{4}\right] = C - v$$

$$\left[u - \frac{5}{2}\right]^2 = C - v$$

$$\left[u - \frac{5}{2}\right] = \sqrt{C - v}$$

$$\boxed{u = \frac{5}{2} + \sqrt{C - v}}$$

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e) $x' = au + b, \quad a, b > 0$

SOLUTION:-

$x' = au + b \quad \ldots\ldots (1) \quad a > 0, \; b > 0$

$\frac{dx}{du} = au + b$

by variable separation

$dx = (au + b)\,du$

integration on b.s

$\int dx = \int (au + b)\,du + C$

$\int dx = \int au\,du + \int b\,du + C$

$\int dx = a\int u\,du + b\int du + C$

$$\boxed{x = a\frac{u^2}{2} + bu + C}$$

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f) $\phi' = \frac{\phi}{4+\phi^2}$

# SOLUTION:-

$$\phi' = \frac{\phi}{4+\phi^2}$$

the independent variable be t so thus we have

$$\phi' = \frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{\phi}{4+\phi^2}$$

$$\frac{(4+\phi^2)}{\phi} d\phi = dt$$

integration on b.s

$$\int \frac{(4+\phi^2)}{\phi} d\phi = \int dt$$

$$\int \left(\frac{4}{\phi} + \frac{\phi^2}{\phi}\right) d\phi = \int dt$$

$$\int \left(\frac{4}{\phi} + \phi\right) d\phi = \int dt$$

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$$\int \frac{4}{\theta}\, d\theta + \int \theta\, d\theta = \int dt$$

$$4\int \frac{1}{\theta}\, d\theta + \int \theta\, d\theta = \int dt$$

$$\boxed{4\ln|\theta| + \theta^2/2 = t + C}$$

g) $x' = e^{x^2}$

SOLUTION:

$$x' = e^{x^2}$$

$$1/e^{x^2}\ \frac{dx}{dt} = 1$$

$$1/e^{x^2}\ dx = dt$$

integration on b.s

$$\int 1/e^{x^2}\ dx = \int dt$$

$$\int e^{-x^2}\ dx = \int 1\, dt$$

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Multiply & divided by $\sqrt{\pi}$

$$\int \frac{1}{e^{x^2}}\, dx = \int 1\, dt$$

$$\sqrt{\pi} \int \frac{1}{\sqrt{\pi}} e^{-x^2}\, dx = \int 1\, dt$$

Multiplying & divided by 2

$$\frac{\sqrt{\pi}}{2} \int 2 \frac{1}{\sqrt{\pi}} e^{-x^2}\, dx = \int 1\, dt \quad (1)$$

The integral $\int \frac{2}{\sqrt{\pi}} e^{-x^2}\, dx$ is the erf(x)

$$\boxed{\frac{\sqrt{\pi}}{2} \operatorname{erf}(x) = t + C}$$

h) $y' = r(a-y)$

SOLUTION:-

$$y' = r(a-y)$$

$$\frac{dy}{dt} = r(a-y)$$

$$\frac{dy}{(a-y)} = r\, dt$$

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integration on b.S

$$\int \frac{dy}{(a-y)} = \int r\,dt$$

$$-\ln(a-y) = rt + C$$

$$\ln(a-y) = -rt - C$$

$$e^{\ln(a-y)} = e^{-rt-C}$$

$$a-y = e^{-rt-C}$$

$$\boxed{y = -e^{-rt-C} + a}$$

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Q2) Solve $y' = r(a-y)$, where $r$ & $a$ are constants.

SOLUTION:

$$y' = r(a-y)$$

$$\frac{dy}{dx} = r(a-y)$$

$$\frac{1}{a-y}\,dy = r\,dx \quad \text{--- (1)}$$

integration on b.s

$$\int \frac{1}{(a-y)}\,dy = \int r\,dx$$

$$-\ln(a-y) = rx + C$$

$$a-y = e^{-rx-c}$$

$$y = -e^{-(-rx+c)} + a$$

$$\boxed{y = -e^{-(rx+c)} + a}$$

Q 3) In Exercise 1(a) - (b) find the solution to the resulting IVP when $x(0) = 1$.

SOLUTION:-

1 (a)

Consider the initial value problem $\dot{x} = \sqrt{x}, x(0) = 1$

First solve the differential equation

$$\dot{x} = \sqrt{x}$$

$$\frac{dx}{dt} = \sqrt{x}$$

$$\frac{dx}{\sqrt{x}} = dt$$

$$x^{-1/2}\,dx = dt$$

integration on b.s

$$\int x^{-1/2}\,dx = \int dt$$

$$\frac{x^{-1/2+1}}{-1/2+1} = t + C$$

$$\frac{x^{1/2}}{1/2} = t + C$$

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$$\frac{x^{1/2}}{1/2} = t + C$$

$$2\sqrt{x} = t + C$$

initial condition $x(0) = 1$

$$2\sqrt{x} = t + C$$

$$2\sqrt{1} = 0 + C$$

$$2 = 0 + C$$

$$C = 2$$

$$2\sqrt{x} = t + 2$$

$$(2\sqrt{x})^2 = (t+2)^2$$

$$4x = t^2 + 4t + 4$$

$$x = t^2/4 + t + 1$$

Solution of IVP $x' = \sqrt{x}$, $x(0) = 1$ is
$x = t^2/4 + t + 1$

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1(b)

Consider the initial value Problem $x' = e^{-2x}, x(0) = 1$

First solve differential equation

$$x' = e^{-2x}$$

$$\frac{dx}{dt} = e^{-2x}$$

$$\frac{dx}{e^{-2x}} = dt$$

$$e^{2x} dx = dt$$

integration on b.s

$$\int e^{2x} dx = \int dt$$

$$\frac{e^{2x}}{2} = t + C$$

$$\frac{e^{2x}}{2} = t + C$$

$$\frac{e^{2(1)}}{2} = (0) + C$$

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$$C = \frac{e^2}{2}$$

$$\frac{e^{2x}}{2} = t + \frac{e^2}{2}$$

$$e^{2x} = 2t + e^2$$

$$\ln\left(e^{2x}\right) = \ln\left(2t + e^2\right)$$

$$2x = \ln\left(2t + e^2\right)$$

$$\boxed{x = \frac{\ln\left(2t + e^2\right)}{2}}$$

Q4) find general solution $x' = 2x/t+1$

SOLUTION:-

$$x' = \frac{2x}{t+1}$$

$$\frac{dx}{dt} = \frac{-2x}{t+1} \Rightarrow \frac{dx}{2x} = \frac{dt}{t+1}$$

integration on b.s

$$\int \frac{dx}{2x} = \int \frac{dt}{t+1}$$

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$$\frac{1}{2} \log x = \log(t+1) + \log C$$

$$\log x^{1/2} = \log[(t+1) \cdot C]$$

$$x^{1/2} = C(t+1)$$

$$x = C^2(t+1)^2$$

$$\boxed{x(t) = C(t+1)^2}$$

b) $\theta' = t\sqrt{t^2+1}\ \sec\theta$

SOLUTION:-

$$\theta' = t\sqrt{t^2+1}\ \sec\theta$$

$$\frac{d\theta}{dt} = t\sqrt{t^2+1} \times \sec\theta$$

$$\frac{d\theta}{\sec\theta} = t\sqrt{t^2+1}\ dt$$

$$\cos\theta\ d\theta = t\sqrt{t^2+1}\ dt \qquad \because \cos\theta = \frac{1}{\sec\theta}$$

integration on b.s

$$\int \cos\theta\ d\theta = \int t\sqrt{t^2+1}\ dt + C$$

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$$t^2 = x$$

$$2t\,dt = dx$$

$$t\,dt = \frac{dx}{2}$$

$$\int \cos\theta \, d\theta = \int \frac{1}{2}\sqrt{x+1} \times dx + C$$

$$\int \cos\theta \, d\theta = -\frac{1}{2}\int (x+1)^{1/2} \, dx + C$$

$$\sin\theta = \frac{1}{2} \times \frac{(x+1)^{3/2}}{3/2} + C$$

$$\boxed{\sin\theta = \frac{1}{3}(x+1)^{3/2} + C}$$

c) $(2u+1)u - (t+1) = 0$

SOLUTION:-

$$(2u+1)u - (t+1) = 0$$

$$u' = \frac{(t+1)}{2u+1}$$

$$u' = \frac{t}{2u+1} + \frac{1}{2u+1}$$

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$$\frac{dv}{dt} - \frac{t}{2v+1} = \frac{1}{2v+1} \quad \text{--- (1)}$$

Clearly eq (1) is a linear differential equation with $P = \frac{-1}{2v+1}$, $Q = \frac{1}{2v+1}$

Now integrating factor is

$$= +e^{\int -1/2v+1 \, dv}$$

$$= -e^{\int 1/2v+1 \, dv}$$

$$= e^{-1/2 \ln(2v+1)}$$

$$= \frac{1}{\sqrt{2v+1}}$$

Multiplying both side of equation (1)

$$\frac{d}{dv}\frac{(1t)}{\sqrt{2v+1}} = (2v+1)\frac{1}{\sqrt{(2v+1)}}$$

integration on b.s

$$\int \frac{d}{dv}\left(\frac{t}{\sqrt{2v+1}}\right) = \int \sqrt{2v+1} \, dv$$

$$\frac{t}{\sqrt{2v+1}} = \frac{1}{2}\frac{(2v+1)^{1/2+1}}{1/2+1} + C$$

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$$\frac{t}{\sqrt{2u+1}} = \frac{1}{2} + \frac{2}{3}(2u+1)^{3/2} + C$$

$$\frac{t}{\sqrt{2u+1}} = \frac{1}{3}(2u+1)^{3/2} + C$$

$$t = \frac{1}{3}(2u+1)^{3/2}\sqrt{(2u+1)} + C\sqrt{2u+1}$$

$$\boxed{t = \frac{1}{3}(2u+1)^2 + C\sqrt{2u+1}}$$

d) $R' = (t+1)(R^2+1)$

**SOLUTION:-**

$$R' = (t+1)(R^2+1)$$

$$\frac{dR}{dt} = (t+1)(R^2+1)$$

$$\frac{dR}{R^2+1} = (t+1)dt$$

integration on b.S

$$\int \frac{dR}{R^2+1} = \int (t+1)\,dt$$

$$\int \frac{dR}{R^2+1} = \int t\,dt + \int 1\,dt$$

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$$\boxed{\tan^{-1}(R) = +\frac{2}{2} + t + C}$$

e) $y' + y + 1/y = 0$

SOLUTION

$$y' + y + 1/y = 0$$

$$y' = -y - 1/y$$

$$\frac{dy}{dx} = \frac{-y^2 - 1}{y}$$

$$\frac{y}{-(y^2+1)} dy = dx$$

integration on b.s

$$\int \frac{-y}{(y^2+1)} dy = \int dx$$

Put $y^2+1 = t$

$$2y\, dy = dt$$

$$y\, dy = 1/2\, dt$$

$$-1/2 \ln|t| = x + C$$

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$$\boxed{-\tfrac{1}{2} \ln|y^2+1| = x + C}$$

f) $(t+1)\, x' + x^2 = 0$

SOLUTION:-

$$(t+1)x' + x^2 = 0$$

$$(t+1)\frac{dx}{dt} = -x^2$$

$$(t+1)\, dx = -x^2\, dt$$

$$\frac{dx}{-x^2} = \frac{1}{t+1}\, dt$$

integration on b.s

$$\int \frac{dx}{-x^2} = \int \frac{1}{t+1}\, dt$$

$$-\int x^{-2}\, dx = \int \frac{1}{t+1}\, dt$$

$$-\frac{x^{-2+1}}{-2+1} = \ln(t+1) + C$$

$$\boxed{1/x = \ln(t+1) + C}$$

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