Date: / /

Final Term

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Subject:- Probality and Statistics

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Degree:- BSSE

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Qno1:-

Q:- A man throw two fair dice what is the ~~contional~~ Conditional probality that the Sum of the two dice will be 7 given th

8- Ans:-

$S = \{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(1,7)(1,8)$
$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(2,7)(2,8)$
$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)(3,7)(3,8)$
$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)(4,7)(4,8)$
$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)(5,7)(5,8)$
$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6)(6,7)(6,8)$
$(7,1)(7,2)(7,3)(7,4)(7,5)(7,6)(7,7)(7,8)$
$(8,1)(8,2)(8,3)(8,4)(8,5)(8,6)(8,7)(8,8),\}$

A = { The Sum is 7 }

B = { The Sum is even }

C = { The Sum is greater then 8 }

D = { The two dise had the Same outcomes }

Now:-

$A = \{(1,6)(2,5)(3,4)(5,2)(6,1)(4,3)\}$

$B = \{(1,1)(1,3)(1,5)(1,7)(2,2)(2,4)(2,6)$
$(2,8)(3,1)(3,3)(3,5)(3,7)(4,2)$
$(4,4)(4,6)(4,8)(5,1)(5,3)(5,5)(5,7)(6,2)$
$(6,4)(6,6)(6,8)(7,1)(7,3)(7,5)(7,7)$
$(8,2)(8,4)(8,6)(8,8)\}$

$C = \{(1,8)(2,1)(2,8)(3,6)(3,7)(3,8)$
$(4,5)(4,6)(4,7)(4,8)(5,4)(5,5)(5,6)(5,7)$
$(5,8)(6,3)(6,4)(6,5)(6,6)(6,7)(6,8)$
$(7,2)(7,3)(7,4)(7,5)(7,6)(7,7)(7,8)$
$(8,1)(8,2)(8,3)(8,4)(8,5)(8,6)(8,7)$
$(8,8)\}$

$D = \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)$
$(7,7)(8,8)\}$

$A \cap B = \{\ \}$ OR ϕ
$A \cap C = \{\ \}$
$A \cap D = \{\ \}$

$P(A) = 6/64$, $P(B) = 32/64$

$P(C) = \frac{36}{64}$, $P(D) = 8/64$

$P(A \cap B) = 0$, $P(A \cap C) = 0$, $P(A \cap D) = 0$

Hence:-

$$P(A/B) = \frac{P(A \cap B)}{B} = 0 \times 32/64$$

$$P(A/B) = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times 36/64$$

$$P(A/C) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times 8/64$$

$$P(A/D) = 0$$

Q no 3:-

Sol:-

(i) exactly 4 games
(ii) at least 4 games
(iii) from 3 to 6 games

Solution:- $p = 2/3$ $n = 8$

$q = 1 - p$

$= 1 - 2/3$ $q = 1/3$

(i) $P(x=4)$

$$= \binom{8}{4}(2/3)^4(1/3)^4$$

$$= \frac{1120}{6561} = 0.1707$$

(ii) $P(x \geq 4)$

$$1 - P(x < 4)$$

$$1 - \sum_{x=0}^{3} \binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[(1/3)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$$

$$= 1 - \frac{1}{6561}\left[1+16+112+448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561-577}{6561}$$

$$= 5984/6561$$

$$= 0.9121$$

(iii) $P(3 \leq x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x}\left(\frac{2}{3}\right)^{x}\left(\frac{1}{3}\right)^{8-x}$$

$$\binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3}$$

$$\binom{8}{6}\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{2}$$

$$\frac{8}{(3)^{8}}\left(56+140+224+224\right)$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

Q no 02:-

Ans:-

Sum of 2 has 1 way 1,1
Sum of 3 has 2 ways 1,2 and 2,1
Sum of 4 has 3 way 1,3 ; 2,2 ; 3,1
5 has 4 ways
6 has 5 ways (Symmetry)
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 way

Those are 15/36 for each side with a sum of 30/36

That leaves a 6/36 = 1/6 ~~Probability~~ probability for sum of 7.

Q no 6:-

Ans:- Binomial Distribution:-

A Binomial distribution can be though of as simply the probality of a success or failure outcome in an experiment or survey that is repeated multiple times

$$p(x-x)\,f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

Binomial frequency Distribution:-

If the Binomial ~~probability~~ probability distribution is multiplied by N the number of experiment or sets, the resulting distribution is known as Binomial frequency Distribution

$$N \binom{n}{x} \left(p^x q^{n-x}\right)$$

Cnoy:-

Ans:- proof:
Since the Ci's from a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / c_i)\, P(c_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/c_i)\, P(B/c_i)\, P(c_i)$$

$\therefore$ (A and B are Conditionally Independent)

$$p(A\cap B) = \sum_{i=1}^{m} p(A|C_i)\,p(B)\,p(C_i)$$

$\because$ (B is independent of all C_i's)

$$p(A\cap B) = p(B)\sum_{i=1}^{m} p(A|C_i)\,p(C_i)$$

$$p(A\cap B) = p(B)\,p(A)$$

$\therefore$ Law of total probability

Hence A and B are independent.

Q no 5 :-

Ans :- The probability function for a Bionomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p if x is a random variable with the probality distribution

$$E(x) = \sum_{x=0}^{n} x\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since $x=0$ let $y=x-1$

and $m=n-1$ Subbing $n=y+1$

and $n=m+1$ into last sum

$$= \sum_{y=0}^{m} \frac{(m+1)}{y!(m-y)} p^{y+1}(1-p)^{m-y}$$

$$= m+1\,p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y}(1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y}(1-p)^{m-y}$$

By Binomial theorem

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Set $a=p$ and $b=1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

$$= (a+b)^m$$

$$= (p+1-p)^m$$

$$= 1$$

So that

$$E(x) = np$$

$y = x-2$ and $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} (p^y (1-p)^{m-y})$$

$$= n(n-1)p^2 (p+(1-p))^m$$

$$= n(n-1)p^2$$

So that the variance of x is

$$(x^2) - \{(x^2) = E(x(x-1)) + E(x)$$

$$E(x^2) = (n(n-1)p^2 + np(np)$$

$$= np(1-p)$$

Q no 7:-

Ans:-

Measure	Data set A	B	C	D
Coefficient of verification	$Cv = \frac{3}{45} \times 100$	$Cv = \frac{11 \times 100}{60}$	$Cv \; 5/50 \times 100$	$Cv = 15/25 \times 100$
	$Cv = 6.07$	$Cv = 18.3$	$Cv = 10$	$Cv = 60$