Name ≠ Asad Shoaib Khalil

ID ≠ 13095

Subject ≠ Probability Methods in Engineering.

Assignment ≠ Sessional Assignment

Teacher ≠ Sir Daud.

Date ≠ 11/7/2020

Date: / /

Question # 1
Part (a).

Solution:-

Given:-

Variance $= \sigma x^2 = 4$.

Mean $= \mu x = 4$.

Find / Required

$n = ?$

$p = ?$

Solution:-

$\mu = np = 4$ — (1)

$\sigma^2 = npq = 4$ — (2)

divide 2 by 1

$$\frac{npq}{np} = \frac{4}{4}$$

$$\boxed{q = 1}$$

As, $p + q = 1$

$p = 1 - q$

$p = 1 - 1 = 0$

Now, $np = 4$

$$n = \frac{4}{p}$$

$$n = \frac{4}{0} \Rightarrow \boxed{\infty}$$

Hence p is 0 and q is ∞

Date: / /201

Question #1.
Part (b).

Find:- Probability.

Solution:- Mean = $\mu = np = 12$. —(i)
SD $= 6 = \sqrt{npq} = 4$ —(2)

Divide (i) by (ii).

$$\frac{\mu}{6} = \frac{np}{\sqrt{npq}} = \frac{\cancel{12}\,3}{\cancel{4}}$$

$$\frac{np}{\sqrt{npq}} = 3$$

Squaring both sides.

$$\frac{(np)^2}{npq} = (3)^2$$

$$\frac{n^{\cancel{2}}p^{\cancel{2}}}{\cancel{np}q} = 9$$

$$\frac{np}{q} = 9.$$

$$np = 9q \quad — 3$$

Now $np = 12$ —(1)

Substracting 3 and 1

$$-np = 9q$$
$$\underline{\overset{+}{-}np = \overset{+}{-}12}$$
$$0 = 9q - 12$$

Date: / /201

$$9q - 12 = 0$$

$$q = \frac{\cancel{12}\,4}{\cancel{9}\,3}$$

$$q = 4/3 > 1$$

The statement is incorrect as q can never be greater than 1.

* Part (C) :-

Critical region:- A critical region, also known as rejection region, is a set of values for the test Statistic for which the null hypothesis is rejected i-e if the observed test statistic is in the critical region then we reject the null hypothesis & accept the alternative hypothesis.

Part (D) :-

* Properties of t- distribution:-

- The distribution shares the bell curve of the Z, but reflects the variability that is inherent with smaller sample size.
- The shape of t-distribution is dependent on the sample size n.
- The standard deviation is greater than 1.
- As the sample size n increases, the shape of the curves approaches the standard deviation.

Part (e) :-

* Analysis of Variance :- Analysis of variance, or ANOVA, is a statistical method that separates observed variance data into different components to use for additional tests. A one-way ANOVA is used for three or more groups of data, to gain information about the relationship b/w the dependent & independent variables.

Part (F) :-

* Define R.B.D :-
RBD stands for "Randomized Block diagram"
" A nuisance factor is used as a blocking factor if every level of the primary factor occurs the same number of times with each level of ~~the~~ nuisance factor".

Part (g) :-

* Define statistical quality control :-

The use of statistical method in the monitoring & maintaining of the quality of Product & services. One method, referred to as acceptance sampling, can be used when a a decision must be made to accept or reject a group of parts or items based on quality found in a sample. A second method, referred to as statistical process control, uses graphical displays know as control charts to determine whether a process should be continued or should be adjusted to acheive the desired quality

Part (h).

* **Chance causes :-**

A cause for variability in a measurement process that occurs randomly and unpredictably and for unknown reasons.

* **Assignable cause :-**

Also known as "special cause" an assignable cause is an identifiable, specific cause of variation in a given process or measurement. A cause of variation that is not random and does not occur by chance is assignable.

Scanned with CamScanner

Part (i)

* Traffic intensity :-

It is a measure of the avarage occupancy of a server or resource during a specific period of time, normally a busy hour. It is measure in traffic units. and defined as the ratio of the time during which a facility is cumulatively occupied to the time this facility is available for occupancy.

traffic intensity is :-

$$\frac{aL}{R}$$

a is avarage arrival rate of packets.
L is average packet length.
R is transmission rate.

Date: / /201

Part (j)

* Queuing System :-

characteristic :-

1. The arrival pattern.
2. The service mechanism.
3. The queue displine.
4. The number of customers allowed in the system
5. The number of service channels.

Date: / /

Question # 2.
Part (a).
Answer:-

* Derive mean & variance of bionomial distribution:-

The binomial distribution:

$$P(X) = \binom{n}{x} p^x (1-p)^{n-x}$$

for $x = 0, 1, 2, 3, \ldots, n$.

$$\mu = np, \quad \sigma^2 = np(1-p)$$

A bionomial random variable can be thought of as the sum of n independant Bernoulli random variables each with mean P and variance $p(1-p)$

let $U_1, \ldots, U_n$ be independant Bernoulli random variables

$$E(U_i) = p \quad \& \quad Var(U_i) = p(1-p)$$

$$X = U_1 + \cdots + U_n$$

$$E(X) = E(U_1 + \cdots + U_n)$$

$$E(X) = E(U_1) + \cdots + E(U_n).$$

let $U_1, \ldots, U_n$, be independent Bernoulli random variable.

$E(U_i) = P$ and $Var(U_i) = P(1-P)$

$X = U_1 + \cdots + U_n$

$Var(X) = Var(U_1 + \cdots + U_n)$

$Var(x) = Var(U_1) + \cdots + Var(U_n)$

The binomial theorem:

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(x) = \sum_{x} x p(x)$$

$$= \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!} \times p^{x-1} (1-p)^{(n-1)-(x-1)}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

Recall: $\sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y} = (a+b)^m$

$$Var(X) = E[(X-\mu)^2]$$
$$= \sum_x (x-\mu)^2 p(x)$$

$$E[(X-\mu)^2] = E(x^2) - [E(x)]^2$$

$$E(x^2) = \sum_{x=0}^{n} x^2 \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$E[X(X-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$E[X(X-1)] = \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$E[X(X-1)] = n(n-1)p^2 \times$$

$$\sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!((n-2)-(x-2))!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

$$E[X(X-1)] =$$

$$n(n-1)p^2 \times \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$E[X(X-1)] = n(n-1)p^2$$
$$E[X^2 - X] = n(n-1)p^2$$
$$Var(X) = E(X^2) - [E(X)]^2$$
$$= n(n-1)p^2 + np - (np)^2$$
$$= np((n-1)p + 1 - np)$$

Date: __/__/__

Question # 2.

Part (b) Answer:-

Solution:-

→ let x denote number of cars which are hired out per day.

For poisson distribution mean $= m = 1.5$

$$P(X=x) = \frac{e^{-m} \cdot m^{x}}{x!} = \frac{e^{-1.5} \cdot 1.5^{x}}{x!}$$

(i) P(neither car is used)

$$P(X=0) = \frac{e^{-1.5} \cdot 1.5^{0}}{0.2231}$$

(ii) P(Some demand is refused) =

$$P(x>2)$$

$$= 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[\frac{e^{1.5}\ 1.5^{0}}{0!} + \frac{e^{1.5}\ 1.5^{1}}{1!} + \frac{e^{1.5}\ 1.5^{2}}{2!}\right]$$

$$= 1 - e^{1.5}\left[1 + 1.5 + \frac{2.25}{2}\right] = 0.1912$$

∴ Proportion of days on which car is used = $0.2231 = 22.31\%$

And proportion of days on which some demand is refused = $0.1912 = 19.12\%$

//