Department of Electrical Engineering
Final Term Assignment Spring 2020
B.tech(E)
Date: 22/06/2020

Course Details

Course Title:	Electromagnetic Fields	**Module:**	4th
Instructor:	Engr.Perniya Akram	**Total Marks:**	50

Student Details

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Note: Attempt all of the following questions.

Q1.	State the differences and similarity between gradient and divergence providing relevant example.	Marks 10
Q2.	Find gradient of function F at point(1,1,2) for $F=x^3+y^3z$.	Marks 10
Q3.	Compute $\mathbf{div}\vec{F}$ and $\mathbf{curl}\vec{F}$ for $\vec{F} = x^2y\,\vec{i} - (z^3 - 3x)\,\vec{j} + 4y^2\vec{k}$.	Marks 10
Q4.	State the relationship between electric potential and potential difference with examples.	Marks 10
Q5.	Find the expression for moving a point charge Q from one position to another by using Line integral.	Marks 10

Answer Sheet

Q1:

State the differences and similarity between gradient and divergence providing Relevant example.

Ans:

There are many differences between a gradient and a divergence.
To start with, the gradient is a differential operator that operates on a scalar field, while the divergence is a differential operator that operates on a vector field (just as the curl is also a differential operator that operates on a vector field).

The result of a gradient is a vector field, while the result of a divergence is a scalar field.

The gradient is a vector field with the part derivatives of a scalar field, while the divergence is a scalar field with the sum of the derivatives of a vector field.

As the gradient is a vector field, it means that it has a vector value at each point in the space of the scalar field. Any given vector has a direction (any given vector points towards a given direction): at each given point in the space of the scalar field, the gradient is the vector that points towards the direction of greatest slope of the scalar field at each point.

The divergence of a vector field is a scalar field that measures the net flow of the vector field at each given point in the space of said vector field.

For Example:

The gradient of the distance from a given point is a vector field of unit length vectors pointing away from the given point.

Whereas the divergence is the measure of the amount of flow out of a given volume minus the amount of flow into a given volume:

For Example:

The divergence of a flow with no source or sink is 00. If there is a net source, the divergence is positive and if there is a net sink the divergence is negative.

Q2:

Find gradient of function F at point(1,1,2) for $F=x^3+y^3z$.

Ans:

Solution:

We know that gradient of function F is

Grad F= $\nabla F = \frac{aF}{ax}\vec{i} + \frac{aF}{ay}\vec{j} + \frac{aF}{az} + \vec{k}$

Apply F is equal to x^3y^3z in equation (1)

Grad F =$\nabla F = \frac{a(x^3y^3z)}{ax}$ i^+ $\frac{a(x^3y^3z)}{ay}$ –j^ $\frac{a(x^3y^3z)}{az}$ k^

Now apply partial differential
So

Grad F = ∇F = ($3x^2$+o)i^ +(o+3$3y^2z$)j^ +(o+$y^3(1)$) K^

Grad F + ∇F = $3x^2 i^\wedge + 3y^2zj^\wedge + y^3k^\wedge$

Now

At point(1,1,2) put x=1 ,y=1 ,z=2

So grad F =∇F = $3(1)^2$ i^ + 3 $(1)^2(2)^2$j^ +$(1)^2$K^

Grad F = ∇F +- 3i^+ 6j^+K^

Q3:

Compute $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$ for $\vec{F} = x^2 y\,\vec{i} - (z^3 - 3x)\,\vec{j} + 4y^2\vec{k}$.

SOLUTION:

Let's compute the divergence first and there isn't much to do other than run through the formula.

$$\text{Div } \vec{F} = \nabla \cdot F = \frac{a}{ax}(x^2y) + \frac{a}{ay}(3x - z^2) + \frac{a}{a_z}(4y^2) = 2xy$$

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{a}{ax} & \frac{a}{ay} & \frac{a}{az} \\ x^2y & 3x - z^3 & 4y^2 \end{vmatrix}$$

$$= \frac{a}{ay}(4y^2)\vec{i} + \frac{a}{az}(x^2y)\vec{j} + \frac{a}{ax}(3x - z^3)\vec{k} - \frac{a}{ay}(x^2y)\vec{k} - \frac{a}{ax}(4y^2)\vec{j} \frac{a}{az}(3x - z^3)\vec{i}$$

$$= 8y\vec{i} + 3\vec{k} - x^2\vec{k} + 3z^2\vec{i}$$

$$= (8y + 3z^2)\vec{i} + (3 - x^2)\vec{k} \quad .$$

Q4:
State the relationship between electric potential and potential difference with examples.

Ans:

ELECTRIC POTENTIAL:
The electric potential energy per unit charge is.

$$V = \frac{U}{q}.$$

Since U is proportional to q, the dependence on q cancels. Thus, V does not depend on q. The change in potential energy ΔU is crucial, so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_B - V_A = \frac{\Delta U}{q}.$$

POTENTIAL DIFFERENCE:
The electric potential difference between points A and B , $V_B - V_A$ is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta

$$1\ \text{V} = 1\ \text{J/C}$$

The familiar term voltage is the common name for electric potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor. It is worthwhile to emphasize the distinction between potential difference and electrical potential energy.

The relationship between potential difference (or voltage) and electrical potential energy is given by:

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V.$$

Example:

Calculating Energy

You have a 12.0V motorcycle battery that can move 5000C of charge, and a 12.0V car battery that can move 60,000C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy: To say we have a12.0V battery means that its terminals have a 12.0V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0v, and the charge is given a change in potential energy equal to $\Delta U = q\Delta V$. To find the energy output, we multiply the charge moved by the potential difference.

Solution:

For the motorcycle battery, q=5000C and $\Delta V = 12.0\ \text{V}$. The total energy delivered by the motorcycle battery is

$$\Delta U_{\text{cycle}} = (5000\ \text{C})(12.0\ \text{V}) = (5000\ \text{C})(12.0\ \text{J/C}) = 6.00 \times 10^4\ \text{J}.$$

Similarly, for the car battery, q=60,000C and

$$\Delta U_{\text{cycle}} = (60,000\ \text{C})(12.0\ \text{V}) = 7.20 \times 10^5\ \text{J}.$$

Significance

Voltage and energy are related, but they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. A car battery has a much larger engine to start than a motorcycle. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a depleted car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

CHECK YOUR UNDERSTANDING

How much energy does a 1.5V AAA battery have that can move 100C?

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B), as shown in Figure. The change in potential is $\Delta V = V_B - V_A = +12\ \text{V}$ and the charge q is negative, so that $\Delta U = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

Figure.

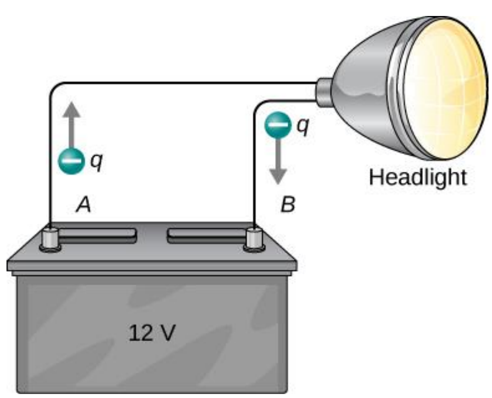

Figure:

A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative terminal. Inside the battery, both positive and negative charges move.

Q5:

Find the expression for moving a point charge Q from one position to another by using
Line integral.

Ans:

The integral expression for the work done in moving path a point charge Q from one position to another. an example of a line integral, which in vector analysis notation always taken the form of the integral along some prescribe path of the dot product of a vector field and a differential vector path length dl without using vector analysis we should have to write

$$W= -Q \int_{initial}^{final} El\ dl$$

Where El = component of E along dL
a line integral is like many other integral which appear in advanced analysis including the surface integral appearing in gauss's law in that it is essentially descriptive we like to look at it much more than we like to work it out it tells us to choose a path break it up into large number of very small segments multiply the component of the field along each segment by length of the segment and then add the result for all segments this summation of cause and the and the integral is obtained exactly only when the number of segment becomes infinite.

Where path has been chosen from an initial position B to a final position A and uniform electric field selected foe simplicity, the path is divided into six segment

$\Delta l_1, \Delta l_2$ ……….Δl_6 and the components is moving a charge Q to B to A then approximately

$$W= -Q(E_{l1}\Delta l_1 + E_{l2} + \dots + E_{L6}\Delta_{L6}$$

Or using vector notation

$$W= -Q(E_1 . \Delta L_1 + E_{L2} . \Delta L_2 + \dots + E_6 \Delta L_6$$

The final position is given the designation A to correspond with the convention for potential difference, as discussed in the following section.