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PAPER: DIFFERENTIAL EQUATION:—

QUESTION: 1:

Define differential equation with two examples.

ANSWER:-

* DIFFERENTIAL EQUATION:-

A differential equation is an equation which contain one or more terms which involve the derivatives of one variable with respect to the other variable.

$$f(x) = \frac{dy}{dx}$$

* FOR EXAMPLE:-

$$\frac{dx}{dt} = 5x - 3$$

Solution:

$$\frac{dx}{5x-3} = dt$$

Taking integral on b/s

$$\int \frac{dx}{5x-3} = \int dt$$

$$\frac{1}{5} \log|5x-3| = t + C_1$$

$$5x - 3 = \pm \exp(5t + 5C_1)$$

$$x = \pm \frac{1}{5} \exp(5t + 5C_1) + 3/5$$

$$y(x)^2 = \left(\frac{-1}{\frac{7}{4}x^4 + C}\right)^2 = \frac{1}{\left(\frac{7}{4}x^4 + C\right)^2}$$

$$\frac{dy}{dx} = \frac{7x^3}{\left(\frac{7}{4}x^4 + C\right)^2} = 7x^3y^2$$

*

$$3 = \frac{-1}{\frac{7}{4}2^4 + C}$$

$$C = -28\frac{1}{3} = -\frac{85}{3}$$

And final Solution is:

$$y(x) = \frac{-1}{\frac{7}{4}x^4 - \frac{85}{3}}$$ Answer.

Q: 1
PART: B:
Define a Seperable Differential Equation (DE)?

* SEPERABLE DIFFERENTIAL EQUATION:-
A differential equation is said to be seperable if the variables can be seperated into two parts. A seperable equation is one that can be written in the form.

$$F(y)\,dy = G(x)\,dx.$$

1 Solve The following Initial value Problem (IVP) using seperable DE and find the interval of validity of the solution.

(a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$.

Solution :-

$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$ Multiply $\frac{1}{y^3}$ on b/s.

$\frac{1}{y^3} \frac{dy}{dx} = \frac{x\cancel{y^3}}{\sqrt{1+x^2}} \frac{1}{\cancel{y^3}}$ Multiply dx on b/s.

$\cancel{dx}\, y^{-3} \cdot \frac{dy}{\cancel{dx}} = x(1+x^2)^{-1/2} \cdot dx$

Now taking integral on b/s.

$\int y^{-3} \cdot dy = \int x(1+x^2)^{-1/2}$

$\frac{1}{2y^2} = (1+x^2)^{+1/2} + C \quad \text{--} (1)$

put $y = 1$, $x = 0$ in (1)

$\frac{1}{2(1)^2} = (1+0^2)^{1/2} + C$.

$\frac{1}{2} = (1)^{1/2} + C$.

$\frac{1}{2} = \sqrt{1} + C$ $\quad \therefore \sqrt{1} = 1$

$$\frac{1}{2} = 1 + C.$$

$$\frac{1}{2} - 1 = C.$$

$$\frac{1-2}{2} = C$$

$$-1/2 = C.$$

put C in (1)

$$\frac{+1}{2y^2} = \sqrt{1+x^2} - \frac{1}{2}$$ **Answer.**

PART: B:

$$y' = e^{-y}(2x-4) \quad y(5) = 0.$$

Solution :-

Multiplying e^y on b/s.

$$e^y \times \frac{dy}{dx} = \cancel{e^{-y}} \cdot \cancel{e^y}(2x-4)$$

Multiply dx on b/s

$$\cancel{dx}\, e^y \cdot \frac{dy}{\cancel{dx}} = (2x-4) \cdot dx.$$

Integrating both sides.

$$\int e^y \cdot dy = \int 2x - 4 \cdot dx$$

$$e^y = x^2 - 4x + c \rightarrow (1)$$

put $y=0$, $x=5$ in (1), we get

$$e^0 = 5^2 - 4(5) + c$$
$$1 = 25 - 20 + c$$
$$1 = 5 + c$$
$$1 - 5 = c$$
$$\boxed{c = -4}$$

put c in (1)

$$e^y = x^2 - 4x - 4$$ Answer.

QUESTION : 2.

Solve the following IVP for the exact equation and find the interval of validity for the solution.

i) Explain the steps for solving Linear Differential equation.

★ ANSWER :-

Following are the steps for solving linear differential equation:

1. Simplify each side, if needed.
2. Use add./Sub. Properties to move the variable term to one side & all others terms to the other side.
3. Use ~~Add~~ Multiplication/division properties
4. Take integral on b/s.

5. Check the answer

ii $\cos(x)\,y' + \sin(x)\,y = 2\cos^3(x)\sin(x) - 1.$

$y\left[\frac{\pi}{4}\right] = 3\sqrt{2}\ ,\ 0 \le x \le \frac{\pi}{2}.$

SOLUTION:-

~~Let $y(x) = \cos(x)y' + \sin(x)y = 2\cos^3(x)\sin(x) - 1$~~

dividing $\cos(x)$ on b/s.

$$y' + \frac{1}{\cos(x)} \cdot \sin(x) = 2\cos^2(x)\sin(x) - \frac{1}{\cos x}.$$

$$y' + \frac{\sin(x)}{\cos(x)} = 2\cos^2(x)\cdot\sin(x) - \sec(x).$$

Finding integrating factors.

$$= \int \tan(x).\,dx = -\ln|\cos(x)| = \ln|\cos(x)|^{-1} = \ln|\sec(x)|$$

Multiply integrating factor Through The differential equation.

$$= \sec(x)y' + \sec(x)\tan(x)\,y = 2\sec(x)\cos^2(x)\cdot\sin(x) - \sec^2(x)$$

$$= (\sec(x)y)' = 2\cos(x).\sin(x) - \sec^2(x).$$

Integrating on both sides.

$$= \int (\sec(x).y(x))'\cdot dx = \int 2\cos(x).\sin(x) - \sec^2(x)\,dx$$

$$= \sec(x).y(x) = \int \sin(2x) - \sec^2(x).\,dx.$$

$$= \sec(x).y(x) = \frac{-1}{2}\cos(2x) - \tan(x) + C$$

QUESTION: 3.

part: 1:

$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0. \qquad y(0) = -3.$$

Solution:

$$M = 2xy - 9x^2 \qquad M_y = 2x.$$
$$N = 2y + x^2 \qquad N_x = 2x.$$

Now, how do we actually find $\psi(x,y)$?

$$\psi_x = M.$$
$$\psi_y = N.$$

$$\psi = \int M dx \quad \text{or} \quad \psi = \int N dy.$$

$$\psi_y = x^2 + h'(y) = 2y + x^2 + 1 = N.$$

$$h'(y) = 2y + 1.$$

$$h(y) = \int 2y + 1 \, dy = y^2 + y + k.$$

$$\psi(x,y) = x^2 y - 3x^3 + y^2 + y + k = y^2 + (x^2+1)y - 3x^2 + k$$

~~$\psi(x,y) = x^2y$ ...~~

$$y^2 + (x^2 + 1)y + 3x^2 + k = C$$

$$y^2 + (x^2+1)y - 3x^2 = C - k$$

$$y^2 + (x^2+1)y - 3x^3 = C$$

initial condition to find C.

$$(-3)^2 + (0+1)(-3) - 3(0)^3 = C \Rightarrow C = 6$$

put the values of C.

$$y^2 + (x^2+1)y - 3x^2 - 6 = 0$$

Quadratic formula

$$y(x) = \frac{-(x^2+1) \pm \sqrt{(x^2+1)^2 - 4(1)(-3x^3-6)}}{2(1)}$$

$$= \frac{-(x^2+1) \pm \sqrt{x^4 + 12x^3 + 2x^2 + 25}}{2}$$

$$-3 = y(0) = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \neq 3$$

$$y(x) = \frac{-(x^2+1) - \sqrt{x^4 + 12x^2 + 25}}{2}$$

$$x^4 + 12x^3 + 2x^2 + 25 = 0.$$

Q: 3 (Part : 2):

$$\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0$$

$$y(5) = 0 .$$

Solution :

$$M = \frac{2ty}{t^2+1} - 2t \qquad M_y = \frac{2t}{t^2+1} .$$

$$N = \ln(t^2+1) - 2 \qquad N_t = \frac{2t}{t^2+1} .$$

Integrate the first one

$$\Psi(x,y) = \int \frac{2ty}{t^3+1} - 2t \, dy = y\ln(t^2+1) - t + h(y)$$

Now differentiate .

$$\Psi_y = \ln(t^2+1) + h'(y) = \ln(t^2+1) - 2$$

$$h'(y) = -2 \Rightarrow h(y) = -2y$$

$$\Psi(t,y) = y\ln(t^2+1) - t^2 - 2y$$

$$y\ln(t^2+1) - t^2 - 2y = c$$

$$c = -25$$

$$y(\ln(t^2+1) - 2) - t^2 = -25 .$$

$$y(t) = \frac{t^2 - 25}{\ln(t^2+1) - 2}$$

$$\ln(t^2+1) - 2 = 0$$

$$\ln(t^2+1) = 2$$

$$t^2 + 1 = e^2$$

$$t = \pm\sqrt{e^2 - 1}$$

Answer.