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Sec : A

Subject : Numerical Analysis

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Q No 1

Given data:

Equation $\Rightarrow$ $x^3 + 3.993 * 10^{-4} = 0.165x^2$

$$x_0 = 0.02$$

Required data:

Root of equation using Newton Raphson method;

Solution:

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

$$f(x) = x^3 - 0.165x^2 + 3.993 \times 10^{-4} \rightarrow (a)$$

As we know that;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \longrightarrow (1)$$

first we take derivative of (a)

$$f'(x) = \frac{d}{dx}\left(x^3 - 0.165x^2 + 3.993 \times 10^{-4}\right)$$

$$f'(x) = 3x^2 - 2(0.165)x + 0$$

$$f'(x) = 3x^2 - 0.33x \longrightarrow (2)$$

By putting eq (a) and eq (b) in eq (1) we get:

$$x_{n+1} = x_n - \frac{x^3 - 0.165x^2 + 3.933 \times 10^{-4}}{3x^2 - 0.33x}$$

For n = 0:

By putting $n = 0$ we get

$$x_1 = x_0 - \frac{x^3 - 0.165x^2 + 3.933 \times 10^{-4}}{3x^2 - 0.33x}$$

$$x_1 = 0.02 - \frac{(0.02)^3 - 0.165(0.02)^2 + 3.933 \times 10^{-4}}{3(0.02)^2 - 0.33(0.02)}$$

$$x_1 = 0.082$$

For n = 1:

$$x_{1+1} = x_1 - \frac{x^3 - 0.165x^2 + 3.933 \times 10^{-4}}{3(0.02)^2 - 0.33(0.02)}$$

$$x_2 = 0.082 - \frac{(0.082)^3 - 0.165(0.082)^2 + 3.933 \times 10^{-4}}{3(0.082)^2 - 0.33(0.082)}$$

$$x_2 = 0.058$$

For $n = 2$:

$$x_{2+1} = x_2 - \frac{x^3 - 0.165x^2 + 3.933 \times 10^{-4}}{3(x)^2 - 0.33x}$$

$$x_3 = 0.058 - \frac{(0.058)^3 - 0.165(0.058) + 3.933 \times 10^{-4}}{3(0.058)^2 - 0.33(0.058)}$$

$$x_3 = 0.062$$

Ans

Q No 2

Given data :- $x_0 = 2$

$x_1 = 2.75$

$x_2 = 4$

Required :- Find lagrauge luterpolation Polynomial for $F(x) = \frac{1}{x}$ at $x = 3$

Solution :- Putting value in the function

$y_0 = 0.5$, $y_1 = 0.364$, $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3.2)(3.4)}{(2.75-2)(2.75-4)}$$

$$\times 0.364 + \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\boxed{y = 0.33}$$

QNO 3 Given data :-

x =	1.0	1.3	1.6	1.9	2.2
y =	0.7651977	0.6200860	0.4554022	0.2818186	0.1103623

	x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i+1}, x_i)$	$f(x_{i-3}, \ldots, x_i)$	$f(x_i, -4 \ldots, x_i)$
x_0	1	0.7651977				
			-0.48370056			
x_1	1.3	0.6200860		-0.108734		
			-0.548946		0.0658785	
x_2	1.6	0.4554022		-0.0494433		-0.0023049
			-0.578612			
					0.0625125	
x_3	1.9	0.2818186		0.006818		
			-0.571521			
x_4	2.2	0.1103623				

i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$

(2) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = 0.548996}$

(3) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

(4) $f(x_3, x_4) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$f(x_3, x_4) = -0.571521$

Second divided difference

$f(x_0, x_1, x_2) = \dfrac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$= \dfrac{-0.548946 - (-0.4837056)}{1.6 - 1}$

$= -0.108734$

$f(x_1, x_2, x_3) = \dfrac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$

$= -\dfrac{0.578612 - (-0.548946)}{1.9 - 1.3}$

$F(x_1, x_2, x_3) = -0.0494433$

$F(x_2, x_3, x_4) = \dfrac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$

$= \dfrac{-0.571521 - (-0.578612)}{2.2 - 1.6}$

$= 0.006818$ Ans

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= \boxed{0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.000818 - (-0.049443)}{2.2 - 1.3}$$

$$\boxed{0.06251255}$$

4th divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= \boxed{0.0028049}$$

$$f x = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x - 1) - 0.4837056 + (x - 1)(x - 1.3)(-0.108734) + (x - 1)(x - 1.3)(x - 1.6)(0.0658785) + (x - 1)(x - 1.3)(x - 1.6)(x - 1.9)(-0.0028049)$$