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FINAL PAPER SUBMISSION.

SUBMITTED TO: "MADAM

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Question 1 :-

Solution :-

$f(t) = 1+t \quad -\pi \leq t \leq \pi$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow (1)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt.$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2} \right) \right)$$

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$$a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\,(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left(\left(1+t\,\frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\,\frac{\sin \cancel{n}t^{0}}{n} - \frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos\ \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t) \sin\, dt$$

$$b_n = \frac{1}{\pi} \frac{((1+t)(-\cos nt)}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{-\cos nt}{n} (1) \right)$$

$$b_n = \frac{1}{\pi} \left(\frac{-(1+t)(\cos nt)}{n} \Big|_{-\pi}^{\pi} + \left(\frac{\sin \cancel{n} t}{\cancel{n}^2} \Big|_{-\pi}^{\pi} \right) \right)$$

$$b_n = \frac{-1}{n\pi} \left((1+\pi)(\cos n\pi) - ((1+(\pi) \cos n\pi) \right)$$

$$b_n = \frac{-1}{n\pi} \left(\cos \cancel{n}\pi + \pi \cos n\pi - \cos \cancel{n\pi}\ \pi + \pi \cos n\pi \right)$$

$$b_n = \frac{-1}{n\pi}\left(2\pi \cos n\pi\right).$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$.

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So equation become.

$$f(t) = \frac{1}{2\pi}\left(2\pi + \pi\right) + 0 + \sum_{n=1}^{\infty}$$

$$\frac{2(-1)^{n+1}}{n} \sin t.$$

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Question 2.

Sol:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}.$$

Eigen values = ?

Sol:-

Step (i)

we have;

$(A - \lambda I)x = 0$ A = Given Matrix.

(Step; 2)

we have; The characteristics equation is given by;

$|A - \lambda I| = 0$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0.$$

Step ; 03

$$\lambda^3 - \left| \begin{matrix} \text{Sum of} \\ \text{Diagonal element} \end{matrix} \right| \lambda^2 + \left| \begin{matrix} \text{Sum of} \\ \text{Diagonal} \\ \text{minor} \end{matrix} \right| \lambda - |A| = 0 \quad \text{--- (B)}$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix}$

$$+ \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$(-6)+(2)+(1)$

$= -6+2+1$

$= -3$

By putting values in eq (B);

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad -(C)$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6+6$

$= 0$

By putting values in (C)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0.$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0.$$

$$\lambda = 0.$$

$$\lambda^2 - 4\lambda - 3 = 0.$$

using Quadratic formula;

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \begin{array}{l} a = 1 \\ b = -4 \\ c = -3 \end{array}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

we have eigen values:-

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Q NO # 03

Given Equation :-

$$5x + 4z + 2m = 3$$
$$x - y + 2x + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 1$$

Writting in Augmented form.

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_2 - 5R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_2 - 5R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_3 + R_2$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 - R_1$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad R_4 + \frac{2}{5} R_2$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -8 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 0 & 0 & -17/5 & 6/5 & -7/5 \end{array}\right] R_3 \leftrightarrow R_4$$

$$\left[\begin{array}{cccc:c} 1 & 1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -17/5 & 6/5 & -7 \\ 0 & 0 & -12 & -7 & -5 \end{array}\right] R_4 - 2R_2$$

$$\left[\begin{array}{cccc:c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & 3 \\ 0 & 0 & -17/5 & 6/5 & -17/5 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\Rightarrow \quad -m = 1$$

$$m = 1$$

$$-\tfrac{17}{5} z + \tfrac{6}{5} m = -\tfrac{17}{5}$$

$$-\frac{17}{5}z = \frac{17}{5} - \frac{6}{5}$$

$$z = \frac{-23}{8} \times \frac{8}{-17}$$

$$z = \frac{23}{17}.$$

$$-5y - 6z - 3m = 3$$

$$-5y - 6\left(\frac{23}{17}\right) - 3(1) = 3$$

$$-5y - \frac{138}{17} = 6$$

$$= \frac{102 + 138}{17}$$

$$y = \frac{240}{17 \times -5}$$

$$y = -\frac{48}{17}$$

$$x + y - 2z + m = 1$$

$$x - \frac{48}{17} - 2\,\frac{(23)}{17} + 1 = 1$$

$$x - \left(\frac{48 - 46}{17}\right) = 0$$

$$x = \frac{2}{17} \quad = 0$$

$$x = \frac{2}{17}$$

Question 4.

Solution:-

$$U(x,t) = \sin(x + rt).$$

Sol:-

$$\frac{\partial^2 u}{\partial t^2} = \frac{c^2 \partial^2 U}{\partial x^2}$$

$U(x t) = \sin(x + rt)$ is

Solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x}$

it will satisfy the above equation.

$$\frac{\partial U}{\partial t} = \cos(x+2t) \cdot \frac{d}{\partial t}$$

$(x+2t)$

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$$\frac{du}{dx} = \cos(x+2t)\,(1+0)$$

$$\frac{du}{dx} = \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\cos(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)\cdot\frac{d}{dx}(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)\,(1+0)$$

$$\boxed{\frac{d^2u}{dx^2} = -\sin(x+2t)}$$

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Eg $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t".

$$\frac{du}{dt} = \frac{d}{dt} \sin(x+2t)$$

$$\frac{du}{dt} = \cos(x+2t)\,(0+2)$$

$$\frac{du}{dt} = 2\cos(x+2t)$$

$$\frac{d^2u}{dt^2} = (2) - \sin(x+2t)\,(0+2)$$

$$\boxed{\frac{d^2u}{dt^2} = -4\sin(x+2t)}$$

we know that one dimensional equation is,

$$\frac{d^2 u}{dt^2} = c^2 \frac{d^2 u}{dx^2}$$

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

for the orbitrary constant $c = \pm 2$.

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0.$$

Then it be verifyed for the orbitrary constant $c = 2$

Ans.