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Q#01 Consider the following analog signal
$X_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

i) Determine the minimum sampling rate required to avoid aliasing.

According to sampling theorm

$f_1 = 100Hz, \quad f_2 = 200Hz$

$f_s \geq 2f_{max} \qquad f = \frac{\omega}{2\pi}$

f_2 is max (greatter than f_1)

$f_s \geq 2\times100$

$\boxed{f_s = 200Hz}$

ii) Suppose that the signal is sampled at rate $f_s = 100Hz$, what is the descret-time signal obtained after sampling? Also Explain the effect of this sampling rate on the newly generated discrete time signal.

Sol:

$f_s = 100Hz$

$$f = \frac{100}{2}$$

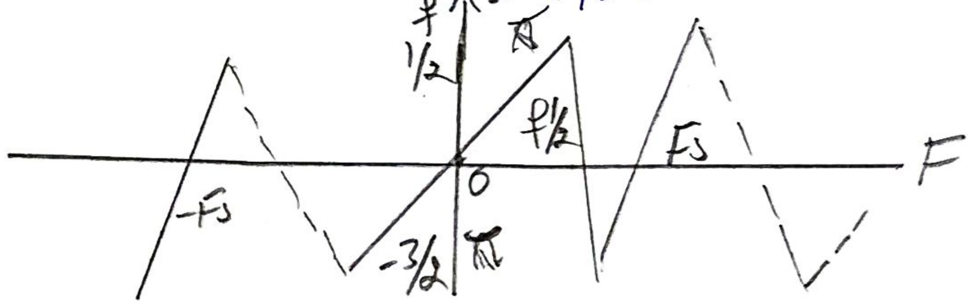
$$f = 50\,Hz$$

This is the max frequency that can be represented uniquely by the Sample signal

As

$$x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos \pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of sampling rate on newely generated discrete time signal is that There will be no display phenomenon mean there will not present unwanted component in Reconstrunction of the signal. The reconstrud orignal signal.

(iii) What is the analog signal $y_a(t)$ we can reconstructed from the samples if we use ideal interpolation =?

$$\text{folding frequency} = F_s/2$$
$$= 100/2$$
$$= 50Hz$$

Both frequency are either equal or greater the folding frequency.

Here for ideal interpolation we can construct orignal signal

$$X_a(t) = 3\cos(100\pi) + 4\sin(200\pi t)$$

Since only the frequency components at 100 Hz are present on sampled signal.

The analog signal we can remove or reconstruct

$$(y_a(t) = 3\cos 100\pi t) \text{ Ans}$$

Q.1
(b)

Consider a discrete time signal which is given by $x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0, & n < 0 \end{cases}$

This signal is sampled at rate Fs 2Hz

(i) Draw the sampled signal.

$$f_s = \frac{1}{T}$$

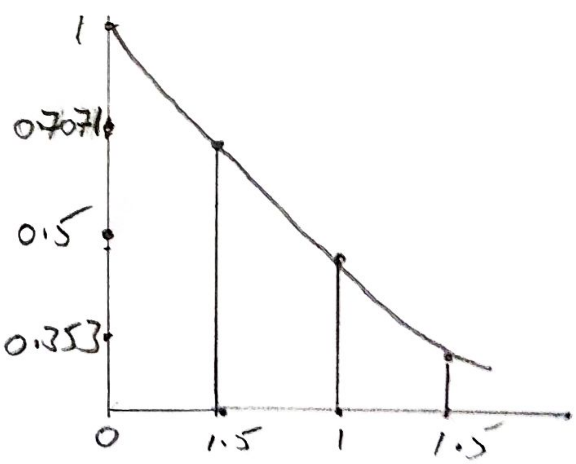
OR

$$T = \frac{1}{f_s}$$

$$T = 1/2$$

$$T = 0.5 \text{ sec}$$

X_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



(5)

(ii) The sample of signal are intended to carry 8 bit ~~perd~~ per sample. Determine the quantization resolution to quantized the sample signal to achived in part (i)

$L = 2^n$

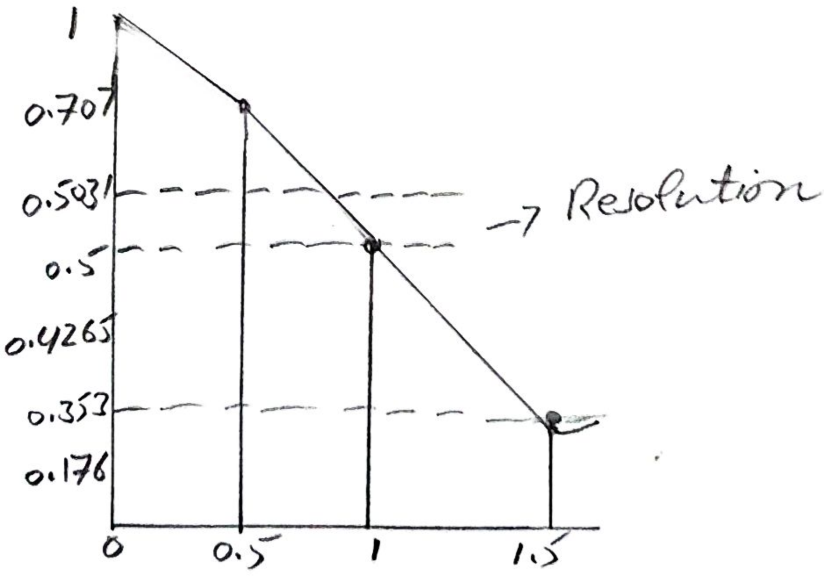
$n = \text{bits} = 3$

$L = 2^8 = 8 \text{ level}$

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1 - 0}{8}$$

$$= 0.125$$



(iii) Perform the process of truncation & rounding off all the values of sampled signal & find the quantization error for each of the sampled data. Express the answer in tabular form.

S/No	Discrete Signal	Truncation	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	0.2
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.1	-0.1

(7)

Q#2 Compute the convolution $y(n)$ of the following signals.

$$x(n) = \begin{cases} a^{n+1} & -3 \le n \le 5 \\ 0 & \text{else wheoe} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & 0 \le n \le 4 \\ 0 & \text{else where} \end{cases}$$

Sol

$$x(n) = x(K) = \{\alpha-2, \alpha-1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, 0, 0 \ldots\}$$

$$h(n) = h(K) = \{\ldots 0, 1, 2, 4, 8, 16, 0 \ldots\}$$

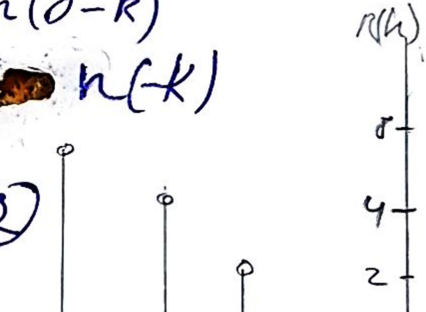
To find $y(n)$:

$$y(n) = \sum_{K=\infty}^{\infty} x(K) h(n-k)$$

for $n=0$ first to find $h(n-K) = h(0-k)$

so by inverting $h(K)$ we get $h(-K)$

$\Rightarrow h(-K) = \{16, 8, 4, 2, 1\}$ — (2)



So $y(0) = \sum_{K=-\infty}^{\infty} x(K) \times h(-K)$

$$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$$

$$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + \alpha + 2$$

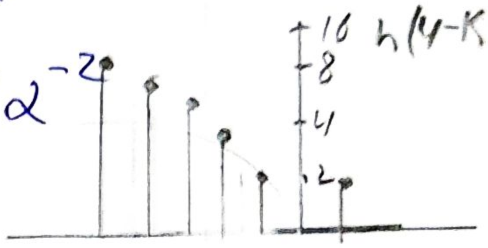
$$y(0) = a.-2 + 4\alpha^{-2} + 4\alpha^{-2}$$

from $= 1 - h(1-k) = \{16, 8, 4, 2, 1\}$

So $y(1) = (\alpha^{-2}\times 16) + (\alpha^{-2}\times 8) + (1\times 4) + (\alpha\times 2) +$
$(\alpha^{2}\times 1)$

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^{2}$

$= \alpha^{2} + 2a + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

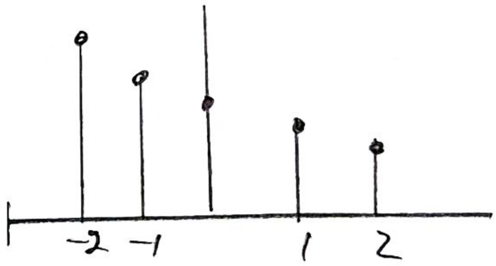


Now for $n = 2$

$n(2-K) = \{16, 8, 4, 2, 1\}$

$y(2) = \{(\alpha^{-3}\times 16) + (1\times 8) + (\alpha\times 4) +$
$(\alpha^{2}\times 2 + \alpha^{3}\times 1)\}$

$= 16\alpha^{-2} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$



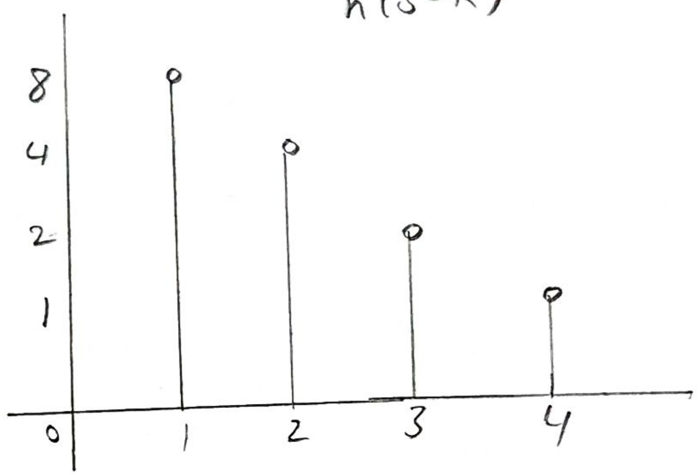
Similarly for $n=3$

$h(3-k) = \{16, 8, 4, 2, 1\}$

$y(3) = (1\times16)+(\alpha\times8)+(\alpha^2+4)+(\alpha^3\times2)+(\alpha^4\times2)$

$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + 2\alpha^4$

$h(3-K)$

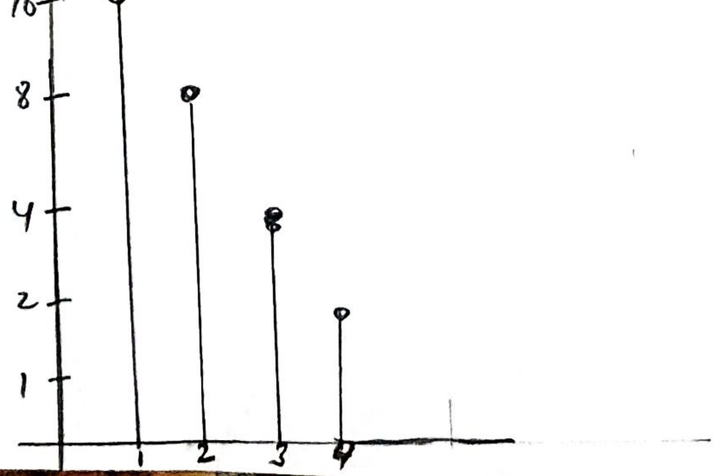


Now $h'(4-K) = \{16, 8, 4, 2, 1\}$

$y(4) = (\alpha\times16)+(\alpha^2\times8)+(\alpha^3\times4)+$
$(\alpha^4\times2)+(\alpha^5\times1)$

$y(4) = 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

$h(1-K)$

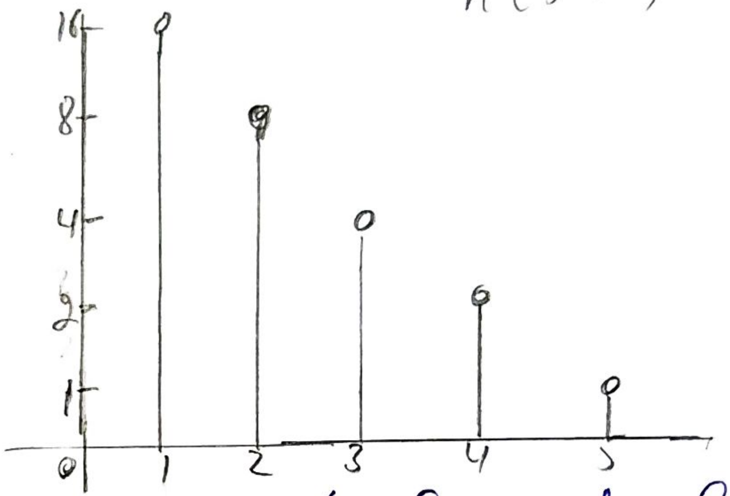


$h(5-K) = \{0, 16, 8, 4, 2, 1\}$

$y(5) = (\alpha \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 8) + (\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$

$y(5) = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$

$h(5-K)$

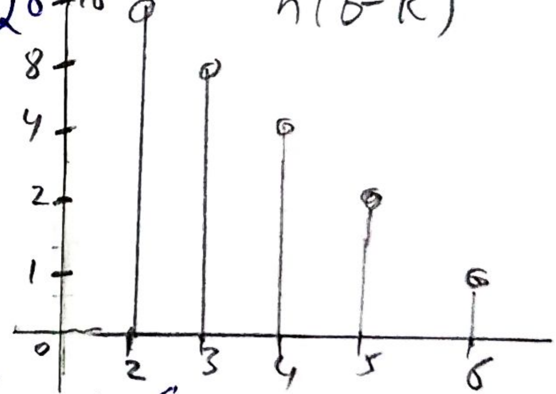


Similarly if we calculate for rest of the values of n up till There are any common value we get

$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

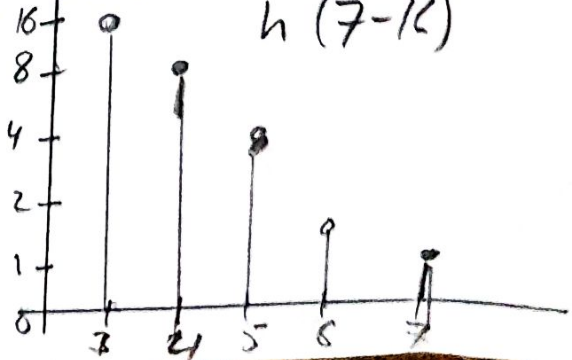
$h(6-K)$



$y(7) = 0 + 0 + 0 + 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

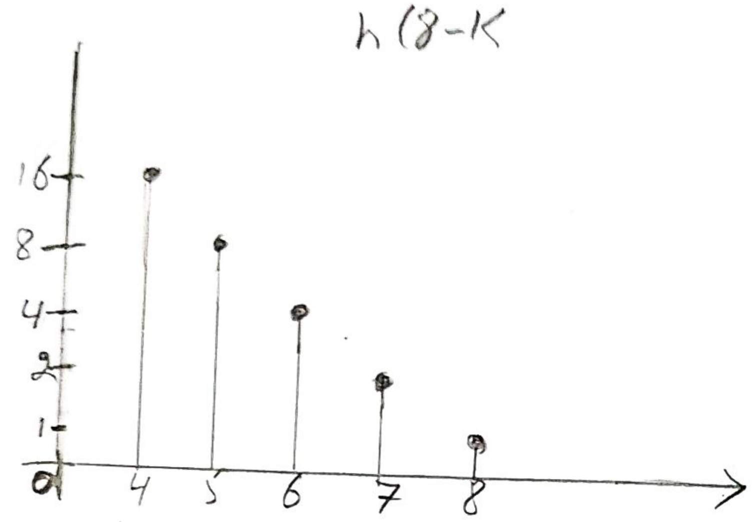
$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

$h(7-K)$



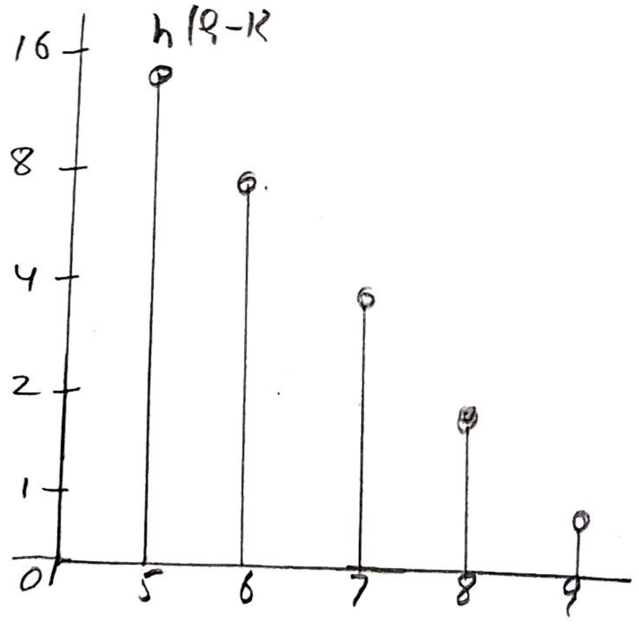
$y(8) = 0+0+0+0+16\alpha^5+8\alpha^6$

$\Rightarrow 16\alpha^5+8\alpha^6$



$y(9) = 0+0+0+0+0+16\alpha^6$

$= 16\alpha^6$



Q.3 Determine the z-transform of the following signals & also sketch its region of convergence (ROC)

i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^n, & n < 0 \end{cases}$

Sol:

As we know

z - transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z + \sum_{n=\infty}^{0} (1/3)^n z^n - 1$$

Using geometric series

$$\Rightarrow \frac{1}{1 - \frac{1}{4} z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1$$

$$\Rightarrow \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{1}{3}} - 1$$

$$= \frac{1 - \frac{1}{\frac{1}{4} z^{-1}} + 1 - \frac{1}{3} - 1}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z^{3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{1} - \left(1 - \frac{1}{4} z^{-2}\right)\left(1 - \frac{1}{3} z\right)}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

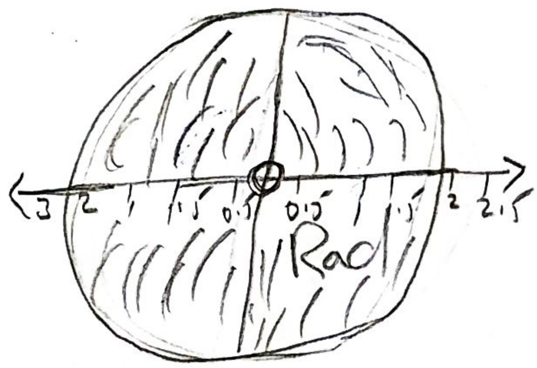
(13)

$$= \frac{1 - \cancel{\frac{1}{3}z} + 1 - \cancel{\frac{1}{4}z^{1}} - 1 + \cancel{\frac{1}{3}} + \cancel{\frac{1}{4}z} + \frac{1}{12}}{\left(1-\frac{1}{4}z^{3}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{1}{12}}{\left(1-\frac{1}{4}z^{1}\right)\left(1-\frac{1}{3}z\right)}$$

Hence The ROC is $\frac{1}{4} < |z| < 3$.

The sketch is below



(ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Sol :-

using z transform pair eq

ie $x(n) = \alpha^n u(n) \longleftrightarrow x(z) = \frac{1}{1 \alpha z^{-1}}$ eq (B)

By putting values

$$X_1(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) z^{n} - n - \sum_{n=0}^{\infty} 2^{n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-3}} - \frac{1}{1-2z^{1}}$$

$$= \frac{\frac{-5}{2} z^{-1}}{\left(1-\frac{1}{2}z^{1}\right)\left(1-2z^{-1}\right)}$$

As seen the ROC use $|z| > 2$

The sketch are