UMAR HADI

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Section # B

Subject # Differential Equation

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QUESTION # 01

(a)

Answer:- $w = \sin(x+ct) + \cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct)+c - \sin(2x+2ct)+2c$$

$$\frac{\partial^2 w}{\partial t^2} = \sin(x+ct)+c^2 - \cos(2x+2ct + 4c^2 \quad -(1)$$

$$\frac{\partial w^2}{\partial x} = \cos(x+ct) - \sin(2x+2ct)+2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

Hence, $c^2 \cdot \partial^2 w / \partial x^2$ Ans.

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

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②

: Wave Equation :

The wave equation is an Important second-order linear partial differential equation for description of wave as they occur in classical physics such as mechanical wave.

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(7)

QUESTION 1 part B

ii. $w = \tan(2x+ct)$

Solutions:- We know that

$$w = \tan(2x+ct)$$

Taking derivative w.r.t 't'

$$\Rightarrow \frac{\delta w}{\delta t} = \frac{\delta}{\delta t} \tan(2x+ct).c$$

$$\Rightarrow \frac{\delta w}{\delta t} = \sec^2(2x+ct).c$$

$$\Rightarrow \frac{\delta w}{\delta t} = c\sec^2(2x+ct$$

Again taking derivaties

$$\Rightarrow \frac{\delta^2 w}{\delta t^2} = c \frac{\delta}{\delta t} \sec^2(2x+ct)$$

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(4)

$$\Rightarrow \frac{\delta^2 w}{\delta t^2} = c \cdot 2\sec(2x+ct) \cdot \sec(2x+ct) \cdot \tan(2x+ct) \cdot c$$

$$\Rightarrow \frac{\delta^2 w}{\delta t^2} = 2c^2 \sec^2(2x+ct) - \tan(2x+ct)$$

Now $w = \tan(2x+ct)$

Taking derivaties w.r.t x

$$\Rightarrow \frac{\delta w}{\delta x} = \frac{\delta}{\delta x} \tan 2x+ct$$

$$\Rightarrow \frac{\delta w}{\delta x} = \sec^2(2x+ct) \cdot 2$$

$$= \frac{\delta w}{\delta x} = 2\sec^2(2x+ct)$$

Again taking derivatiesy

$$\Rightarrow \frac{\delta^2 w}{\delta x^2} = 2 \cdot 2\sec(2x+ct)\sec(2x+ct) \tan(2x+ct) \cdot 2$$

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(5)

$$\Rightarrow \frac{\delta^2 w}{\delta x^2} = 6\sec^2(2x+ct)\tan(2x+ct)$$

$\Rightarrow$ As we already know wave equation

$$\frac{\delta^2 w}{\delta t^2} = c^2 \frac{\delta^2 w}{\delta x^2}$$

putting values in wave equation

$1 \neq 3$

So we have concluded that

$$L.H.S \neq R.H.S$$

∴ "Hence it is not the proof of wave equation ∴"

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QUESTION # 02

Expand the following function in Fourier series

$$F(x) = x, \quad -\pi < x \le 0$$
$$= 2x, \quad 0 \le x \le \pi$$

Solution: Given function is

$$f(x) = \begin{cases} x ; & -\pi \le x \le 0 \\ 2x ; & 0 \le x \le \pi \end{cases}$$

Find the fourier co-efficient a_0, a_n & b_n

Now,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{\cancel{2}}{\pi}\left[\frac{\pi^2}{\cancel{2}} - 0\right]$$

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(3) $a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}$ —(1)

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} x(\cos x \, dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx) \, dx$$

$$= \frac{1}{\pi} \left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right) \right]_{-\pi}^{0}$$

$$= \frac{2}{\pi} \left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right) \right]_{0}^{\pi}$$

$$A_n = \frac{1}{\pi} \left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2} \right] + \frac{2}{\pi} \left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2} \right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{If 'n' is odd} \\ 0 ; & \text{If n is even} \end{cases} \longrightarrow (2)$$

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(b) $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\frac{-\pi\cos\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos\pi}{n}\right] \Rightarrow -\frac{3\cos n\pi}{n}$$

$$= \frac{+3(-1)^{n+1}}{n} \quad \text{--- (3)}$$

Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

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(a) # QUESTION # 03

Solve the initial value problem

$y'' - 4y' + 13y = 8\sin 3x$, $y(0) = 1$ and $y'(0) = 2$

Solution:- Associated homogenous equation of (1) is

$$y'' - 4y' + 13y = 0 \quad \text{— (ii)}$$

Change (ii) into Auxiliary equation

put $y = m$ in (ii)

$$m^2 - 4m + 13 = 0$$

=> Use quadratic formula

$$a = 1, \quad b = -4, \quad c = 13$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

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$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$m_1 = 2 + 3i$

$m_2 = 2 - 3i$

$$y_c = e^{2x}\left(c_1 \cos 3x + c_2 \sin 3x\right) \rightarrow A$$

let

$$y_p = A \cos 3x + B \sin 3x \; - \; \textcircled{*}$$

$\Rightarrow$ Diff w.r.t x

$$y'_p = -3A \sin 3x + 3D \cos 3x$$

$\Rightarrow$ Again diff

$$y''_p = -9A \cos 3x - 9B \sin 3x$$ put in 1

$$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3 \cos 3x)$$

$$+ 13(A \cos 3x) + B \sin 3x) = 8 \sin 3x$$

$$\Rightarrow (-9A - 12B + 13A) \cos 3x + (-9B + 12A + 13B) \sin 3x$$

$$= 8 \sin 3x$$

$$\Rightarrow (4A - 12B) \cos 3x + (4B + 12A) \sin 3x = 8 \sin 3x$$

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(ii) Comparing co-efficients

$\sin 3x \Rightarrow 4B + 12A = 8 \quad - (a)$

$\cos 3x \Rightarrow 4A - 12B = 0$

$4A = 12B$

$A = 3B \quad - (b)$

Put (b) in (a)

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$B = \frac{1}{5} \rightarrow (c)$

put (c) in (b)

$\Rightarrow A = 3/5 \quad - (d)$

$\Rightarrow$ put "c" and "d" in (*)

$y_p = \frac{B}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$

$\therefore$ General Solution:

$y = y_c + y_p$

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(12) $y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x$

$+ \frac{1}{5}\sin 3x \longrightarrow$ (c)

Now we need to find the values of c_1 and c_2 for

$\Rightarrow$ Put $x=0$, $y=1$ in (c)

$1 = e^{x(2)}(c_1 \cos 3(0) + c_2 \sin 3(0)) + \frac{3}{5}\cos 3(0)$

$+ \frac{1}{5}\sin 3(0)$

$1 = (c_1(1) + c_2(0)) + 3/5(1) + \frac{1}{5}(0)$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5$

$c_1 = 2/5$ — (xx)

Diff "c" w.r.t 'x'

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x$

$+ 3e^{2x}\cos x) - 6/5 + 3/5 \cos 3x$ — (D)

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Put $y' = 2$, $x = 0$ in 1)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - 6/5\,\sin 3x + 3/5\cos 3x.$$

=> Put $y' = 2$, $x = 0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)) + c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)) - 6/5\,\sin 3(0) + 3/5\cos 3(0)$$

$$2 = (1(2)) + c_2(3) - 0 + 3/5$$

$$2 = 2c_1 + 3c_2 + 3/5$$

put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = 7/5 + 3c_2$$

$$3c_2 = 2 - 7/5$$

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(14) $3c_2 = 3/5$

$c_2 = 3/15 \rightarrow$ (xxx)

⇒ Put (xx) and (xxx) in (c)

$$y = e^{2x}\left(2/5 \cos 3x + 3/15 \sin 3x\right) + 3/5 \cos 3x + 1/5 \sin 3x$$

$$y = \frac{2}{5} e^{2x} \cos 3x + \frac{3}{15} e^{2x} \sin 3x + 3/5 \cos 3x + 1/5 \sin 3x$$

Required General Solution.

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(15)

QUESTION # 04

$(D^2 - DD')Z = \cos x \cos 2y$

Solution :

$$(D^2 - DD')Z = \cos x \cos 2y$$

CF is given by

$$CF = \phi_1(y) + \phi_2(y+x)$$

which its PI is given by

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\frac{1}{(-1-2)}\right]$$

$$= \frac{1}{2}\left[\cos(x+2y)\frac{1}{(-1+2)} + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Hence the complete solution of PDE is given by

$$4 = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y).$$

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