Name : Wasef ullah

ID No : 15391

Programe : BS(c.s)

Paper : Differential Equatio

Exam : Mid term

Teacher : Sir Latif Jan

Semester : 3rd

↔

Q.1 Define differential equatio along with examples:

Ans Differential equations

Differential equation can be defined as an equation that makes one or more than one function and their derivatives

Example # 1:

P.T.O

Find the Solution of $y' = 5$
where $x = 0$, $y = 2$.

Ans: $y' = 5$

$dy = 5\,dx$

Now by integrating both sides

$y = 5x + K$

by applying the bnury condition:

$x = 0$, $y = 2$

$\therefore K = 2$

$$\boxed{y = 5x + 2}$$

Example: 2

Find the particular solution of

$y''' = 0$

where $y(0) = 3$, $y'(1) = 4$, $y''(2) = 6$

As $y''' = 0$

$y'' = A$

where A is Constant

by integrating:

$y' = Ax + B$ $\therefore B$ is Constant too

once again by integrating

$y = \frac{Ax^2}{2} + Bx + c$ $\therefore C$ is Contant

P.T.O

So $y(0)=3,\ y'(1)=4,\ y''(2)=6$

Now $y(0)=3$ gives $C=3$

$y''(2)=6$ gives $A=6$

$y'(1)=4$ gives $B=-2$

$y=3x^2-2x+3$

checking solution by differentiating & substituting:

$$y'=6x-2$$

$$\Rightarrow y'(1)=6(1)-2=4$$

$$y''=6$$

$$y'''=0 \quad \text{Ans.}$$

b) Define a separable differential equation.

Separable D.E:

Separable differential equation is defined as a first order D.E $y'=f(x,y)$ is called a separable equation. If the function $f(x,y)$ can be

P.T.O

factored in to product of two functions of x and y.

$f(x,y) = p(x)\, h(y)$

where $p(x)$ & $h(y)$ are contuous function.

~x~x~

Solve the following intial value Problem (IVP) using separable DE and find the interval of validity solution

(a) $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

Sol:

$$y^{-3}\, dy = x(1+x^2)^{-1/2}\, dx$$

$$\int y^{-3}\, dy = \int x(1+x^2)^{-1/2}\, dx$$

$$\frac{1}{-2y^2} = \sqrt{1+x^2} + c$$

$$\frac{-1}{2} = \sqrt{1} + C$$

$$C = \frac{3}{2}$$

$$\frac{1}{-2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

$$\frac{1}{y^2} = 3 - 2\sqrt{1+x^2}$$

$$y^2 = \frac{1}{3-2\sqrt{1+x^2}}$$

$$y(x) = -\frac{1}{\sqrt{3-2\sqrt{1+x^2}}}$$

by finding interval of validity

$$3-2\sqrt{1+x^2} > 0.$$

$$3 > 2\sqrt{1+x^2}$$

$$9 > 4(1+x^2)$$

$$\frac{9}{4} > 1+x^2$$

$$\frac{5}{4} > x^2$$

$$-\frac{\sqrt{5}}{2} < x < \frac{\sqrt{5}}{2}$$

$x = 2$

is interval of validity

)

$$Y' = e^{-y}(2x-4) \quad Y(5)=0$$

By multiplying e^y & dx.

P.T.O

$e^y \, dy = (2x-4)\, dx$

$\int e^y = \int (2x-4)\, dx$

$e^y = x^2 + 4x + c$

Natural log

$y = \ln(x^2 + 4x + c)$

Now finding c

$y(5) = \ln((5)^2 - 4(5) + c)$

$\ln(5 + c) = 0$

$5 + c = 1$

$c = -4$

$y = \ln(x^2 - 4x - 4)$ — Ans

x x x x x

2 Solve the following IVP using Linear differential method.

1) Explain the steps for solving Linear D.E.

Ans First substitute $y = uv$
Factor the parts involving v.
Put the v term equal to zero.
Solve separation of variables to u.

Substitute u back in to the u ate step 2.
Solve that to find v.

Finally, Substitute u & v into $y = uv$ to get our solution =

$$\cos(x)y' + \sin(x)y = 2\cos^3(x)$$

$$\sin(x) - 1 \quad y\left[\frac{\pi}{4}\right] = 3\sqrt{2}, \quad 0 \le x \le \frac{\pi}{2}$$

5 $$y' + \frac{\sin(x)}{\cos(x)}y = 2\cos^3(x)\sin(x) - \frac{1}{\cos x}$$

$$y' + \tan(x)y = 2\cos^2(x)\sin x - \sec(x).$$

$$u(t) = e^{\int \tan(x)dx} \cancel{\int \tan(x)dx} = e^{\ln \sec(x)} = e^{\ln \sec(x)}$$

$$\sec(x)y^3 + \sec(x)\tan(x)y = 2\sec(x)\cos^2(x)\sin(x) - \sec^2(x)$$

$$[\sec(x)y]^2 = 2\cos(x)\sin(x) - \sec^2(x)$$

$\int$ on both sides

$$\int[\sec(x)y(x)]\,dx = \int 2\cos(x)\sin(x) - \sec^2(x)$$

$$\sec(x)y(x) = \int \sin(2x) - \sec^2(x)\,dx$$

P.T.O

$\sec(x)y(x) = -\frac{1}{2}\cos(2x) - \tan(x) + c$

$y(x) = \frac{-1}{2}\cos(x)\cos(2x) - \cos(x)\tan(x) + c\cos(x)$

$= \frac{-1}{2}\cos(x)\cos(2x) - \sin(x) + c\cos(x)$

Put the value of "y" & "x"

$3\sqrt{2} = y\left(\frac{\pi}{4}\right) = \frac{1}{2}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)$
$+ c\cos(x)$

$3\sqrt{2} = -\frac{\sqrt{2}}{2} + c\,\frac{\sqrt{2}}{2}$

$c = 7$

$y(x) = \frac{-1}{2}\cos(x)\cos(2x) - \sin(x) + 7\cos(x)$

3 Solve the following IVP for the exact equation and find the interval of validity for the solution.

1 $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \; y(0) = -3$

P. T. O

Ans. $M = 2xy - 9x^2$, $M_y = 2x$

$N = 2y + x^2$, $N_x = 2x$

Now, how do we actually find $\psi(x,y)$?

$$\psi_x = M$$
$$\psi_y = N$$

$$\psi = \int M dx \text{ or } \psi = \int N dy$$

$\psi_y = x^2 + h(y) = 2y + x^2 + 1 = N$

$$h'(y) = 2y + 1$$

$h(y) = \int 2(y) + dy = y^2 + y + k$

$\psi(x,y) = x^2 y - 3x^2 + y^2 + y + k = y^2 + (x^2 + 1) y - 3 + k$

$y^2 + (x^2 + 1) y + 3x^2 + k = c$

$y^2 + (x^2 + 1) y - 3x^2 = c - k$

$y^2 + (x^2 + 1) y - 3x^2 = c$

Initial Condition to find c

$(-3)^2 + (0+1)(-3) - 3(0)^3 = c$ $\therefore c = 6$

Now put the value of c.

P.T.O

$y^2+(x^2+1)y-3x^2-6=0$

By using Quadratic Formula

$y(x) = \frac{-(x^2+1) \pm \sqrt{(x^2+1)^2-4(1)(-3x^2-6)}}{2(1)}$

$= \frac{-(x^2+1) \pm \sqrt{x^2+12x^3+2x^2+25}}{2}$

$-3 = y(0) = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \pm 3, 2$

$y(x) = \frac{-(x^2+1) - \sqrt{x^4+12x^2+25}}{2}$

$x^4+12x^3+2x^2+25=0$

x x

$\frac{2ty}{t^2+1} - 2t - (2-\ln(t^2+1))y' = 0 \quad y(5) = ($

1/2 $M = \frac{2ty}{t^2+1} - 2t \qquad M_y = \frac{2t}{t^2+1}$

$N = \ln(t^2+1) - 2 \qquad N_t = \frac{2t}{t^2+1}$

$\int$ the first one

$\psi(x,y) = \int \frac{2ty}{t^2+1} - 2t \, dy = y \ln(t^2+1) - + h(y)$

P.T.O

Now by differentiating

$\Psi_y = \ln(t^2+1) + h'(y) \quad \ln(t^2+1) - 2$

$h'(y) = -2 \Rightarrow h(y) = -2y$

$\Psi(t,y) = y\ln(t^2+1) - t^2 - 2y$

$y\ln(t^2+1) - t^2 - 2y = c$

$c = -25$

$y(t) = \dfrac{t^2 - 25}{\ln(t^2+1) - 2}$

$\ln(t^2+1) - 2 = 0$

$\ln(t^2+1) = 2$

$t^2 + 1 = e^2$

$t = \pm\sqrt{e^2 - 1}$

Ans