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Subject :- EMF

Assignment no. :- 02

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Q1:- The value of E at $P(\rho=2, \phi=40, z=3)$ ---- for move a $20\mu C$-charged a distance of $6\mu m$.

Ans:- The direction of a_ρ: increase work is given by $dw = -qE \cdot dl$, where in this case, $dl = d\rho a_\rho = 6\times10^{-6} a_\rho$

Thus $dw = -(20\times10^{-6}C)(100 V/m)(6\times10^{-6}m)$

$= -12\times10^{-9}J \Rightarrow -2 mJ$.

→ The direction of a_ϕ: In this case $dl = 2d\phi 2\phi = 6\times10^{-6} a_\phi$ and so

$dw = -(20\times10^{-6})(300)(6\times10^{-6}) = 24\times10^{-8}J = 24 nJ$.

→ The direction of a_z: Here $dl = dz a_z = 6\times10^{-6}a_z$, $dw = -(20\times10^{-6})(300)(6\times10^{-6}) = 3.6\times10^{-8}J = 36 nJ$.

→ The direction of E: Here $dl = 6\times10^{-6} a_E$,

$$a_E = \frac{100a_\rho - 200a_\phi + 300a_z}{(100^2 + 200^2 + 300^2)^{1/2}}$$

$\Rightarrow 0.267a_\rho - 0.535a_\phi + 0.8082a_z$

Thus

$dw = (20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.267a_\rho - 0.535a_\phi + 0.802a_z]\times(6\times10^{-6})$

$\Rightarrow \boxed{-44.9 nJ}$

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the direction of $G = 2a_x - 3a_y + 4a_z$: in this case, $dL = 6\times10^{-6}\, a_G$.

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]} = 0.371a_x - 0.557a_y + 0.743a_z$$

Now:-

$$dw = -(20\times10^{-6})[100a_\rho - 200a_\phi + 300a_z]\cdot[0.371a_x - 0.557a_y + 0.743a_z](6\times10^{-6}).$$

$$\Rightarrow -(20\times10^{-6})[37.1(a_\rho\cdot a_x) - 55.7(a_\rho\cdot a_y) - 74.2(a_\phi\cdot a_x) + 111.4(a_\phi\cdot a_y) + 222.9](6\times10^{-6}).$$

where at P, $(a_\rho\cdot a_x) = (a_\phi\cdot a_y) = \cos(40) = 0.766$

$(a_\rho\cdot a_y) = \sin(40) = 0.643$, and $(a_\phi\cdot a_x)$

$\Rightarrow -\sin(40^\circ) = -0.643$

subjituting these resultion

$$dw = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9]$$

$$(6\times10^{-6}) = -41.8\ nJ$$

Q No. 2:- let $E = 400 a_x - 300 a_y + 500 a_z$ --- distance 1mm in the direction specified by.

(a):- $a_x + a_y + a_z$ we write

$$dw = -q E \cdot dL$$

$$= -4(400 a_x - 300 a_y + 500 a_z) \cdot \frac{a_x + a_y + a_z}{\sqrt{3}} (10^{-3})$$

$$= -\frac{(4 \times 10^{-3})}{\sqrt{3}} (400 - 300 + 500) = -1.39 j$$

(b):- $-2a_x + 3a_y - a_z$. The computation is similar that of part a, but we change the direction-

$$dw = -q E \cdot dL$$

$$\Rightarrow -4(400 a_x - 300 a_y + 500 a_z) \cdot \frac{(-2a_x + 3a_y + a_z)}{\sqrt{14}} \times (10^{-3}).$$

$$\Rightarrow -\frac{(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = \boxed{2.35 j}$$

QNo. 3:- If $E = 120 a_\rho$ V/m find the incremental ----- change a distance of 2mm form-

(A) P(1,2,3) toward Q(2,1,4) the vector along this direction will be $Q-P=(1,-1,+1)$ form which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$

we now write :-

$$dw = qE \cdot dl$$

$$dw = -(50 \times 10^{-6})\left[120 a_\rho \times \frac{a_x - a_y + a_z}{\sqrt{3}}\right](2 \times 10^{-3})$$

$$dw = -(50 \times 10^{-6})(120)\left[(a_\rho \cdot a_x) - (a_\rho \cdot a_y)\right]\frac{1}{\sqrt{3}}(2 \times 10^{-3})$$

At P: $\phi = \tan^{-1}(2/1) = 63.4$ - Thus $(a_\rho \cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho \cdot a_y) = \sin(63.4) = 0.894$ Substituting these we obtain $dw = 3.1 \mu J$.

(B) Q(2,1,4) toward R(1,2,3) - A little through in order here - Note that the field has only a radial component and does not depends ϕ or z. Note Also that P and Q are the same radius ($\sqrt{5}$) from z-axis - But different ϕ and z coordinates - Two point at the same "z" location and problem would not change - Then moving along a straight line Between P and Q would involve moving along a chord of a circle

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whose radius $\sqrt{5}$. Halfway along this line point of symmetry in field (make a sekch to see this). This means that when starting form either point the intial force will be same. This the answer $dw = 31 \mu j$ as part a. This also found by going through the same procedure as part a, but with the direction (role of P and Q) reversed.

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Q.NO. 4:- Complete the value of G ---
using the path.

(A) Striaght line of segments A(1, -1, 2) to B(1, 1, 2) to P(2, 1, 2) In general we have

$$\int_A^P G \cdot dl = \int_A^P 2y\,dx$$

The change of x occurs when moving Between B and P during which $y = 1$. Thus

$$\int_A^P G \cdot dl = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \boxed{2}$$

(B) straight line segment $A(1,-1,2)$, $C(2,-1,2)$ to $P(2,1,2)$ - In case the change in x occure when moving from A to C, during which $Y = -1$ → Thus

$$\int_A^P G \cdot dl = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \boxed{-2}$$

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Q.No. 5:- For $G = 3xy^3 a_x + 2z a_y$. Now things in that path does matter-

(A) Straight line $Y = x - 1$, $z = 1$ we obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_2^3 2z\,dy = \int_2^4 3x(x-1)^2 dx +$$

$$\int_1^3 2(1)\,dy = \boxed{90}$$

(B) Parabola $6y = x^2 + 2$, $z = 1$ we Obtain

$$\int G \cdot dL = \int_2^4 3xy^2 dx + \int_1^3 2z\,dy$$

$$\Rightarrow \int^4 \frac{1}{12} x(x^2+2)^2 dx + \int_1^3 2(1)\,dy = \boxed{82}$$

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