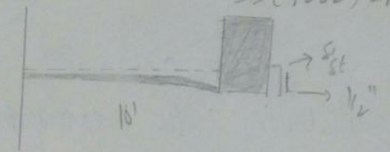
Name	Shah Fahad
ID no	7682
Section	A
Exam	Final Term
Subject	Intro Structural Dynamics & Earthquake Engineering
Semester	8^{th}
Submitted to	Engr. M. Yaseen

Q No 1:

ID(7682) lb



Solution:- The general form for SDOF System is

$$KU + CU + mU = p(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($p(t)=0$) Hence general EOM become

$$KU + mU = 0 \quad \text{—(i)}$$

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29{,}000\ \text{k/in}^2 \times 150\ \text{in}^3}{(10 \times 12\ \text{in})^2}$$

$$K = 7.5\ \text{k/in}$$

in order to eliminate the chance of mistake during calculation, it is more appropriate to use fundemental units like lb, ft, sec or kg, m, sec

$$K = 7.5\ \text{K/in} = 90625\ \text{lb/ft}$$

$$m = \frac{7682\ \text{lb}}{32.2\ \text{ft/sec}^2}$$

$$m = 238.57\ \text{slug}$$

$$\omega_n = \sqrt{K/m}$$

$$\omega_n = \sqrt{\frac{90625}{238.57}}$$

$$\omega_n = 19.48\ \text{rad/sec}$$

$T_n = 2\pi/\omega_n = 2\times3.14/19.48$

$T_n = 0.322$ sec

Substituting the corresprind values in eq ①

$$90625\, u + 238.57\, \ddot{u} = 0$$

where 'k' is in ~~[illegible]~~ lb/ft &

m in lb sec/ft².

General Solution to the EOM for undamped force vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = 1/2'' = 1/24$ ft and $\dot{u}(0) = 0$

$$u(t) = \left((1/24)\times\cos(19.48t) + 0\right)$$

$$u(t) = (1/24)\times\cos(19.48t)$$

Equivalent Static force at any time 't' is

$$f_s(t) = k\cdot u(t)$$

$$f_s(t) = \frac{90625\times\cos(19.48t)}{24}$$

$f_s(t) =$ ~~3776.04 x cos(19.48)~~

$f_s(t) =$ ~~[illegible]~~ 3776.04 cos(19.48t)

Q1

Amplitude of dynamic displacement u_0 for undamped force vibration is

$$u_0 = \sqrt{(C_1 C_2)^2 \times (4/D)/\omega_n)^2}$$

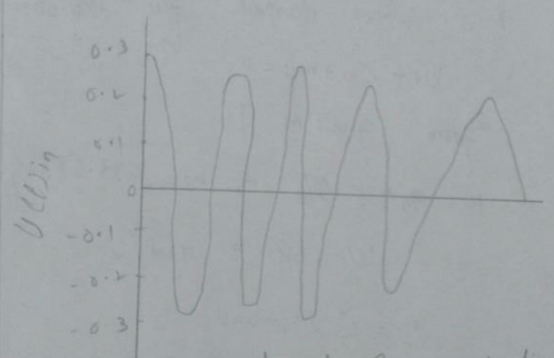
$$u_0 = \sqrt{(1/24)^2 + 0}$$

$$u_0 = 1/24 \text{ ft}$$

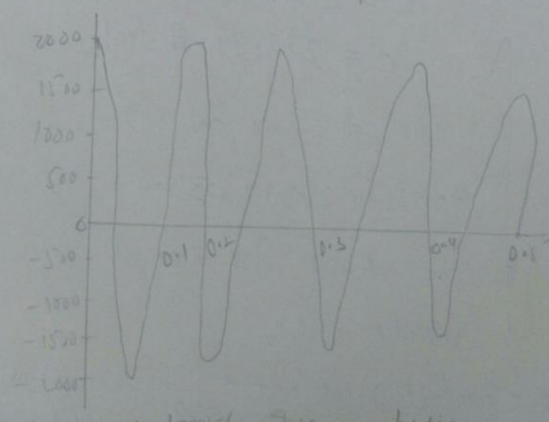
Amplitude of equivalent static force

$$K u_0 = 90625 \times 1/24$$

$$K u_0 = 3776.04 \text{ lb}$$



undamped force vibration



undamped force vibration

Q2:- Given data:-

ζ (Damping ratio) of Reinforced Concrete with considerable Cracking = 3 - 5%
= 3%

Using data of beam given in Question # 1

Required:-

=> Develope & Solve the equation Showing variation in equivalent Static force with time.

Solution:-

E.O.M for damped free variation

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{--- (i)}$$

from question 1

$K = 90625$ lb/ft & $m = 238.57$ slug ($\frac{lb \cdot sec^2}{ft}$)

$$\omega_n = 19.48 \text{ rad/sec}$$

$$c = \zeta \times 2m\omega_n$$

$$c = (0.03) \times 2(238.57)(19.48)$$

$$c = 278.84 \text{ lb.sec/ft}$$

put values in eq (i)

$$90625 + 278.84 + 238.57 = 0$$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right] - \sin\omega_n t\right.$$

Q2:

$\omega_1 = 19.48$ rad/sec

$$u(t) = e^{-0.03\times19.48t}\left[\frac{1}{24}\times\cos(19.48t) + \left(\frac{1}{19.48}\right)\times\left[\frac{0+1}{24}\times0.03\times19.48\times\sin(19.48t)\right]\right]$$

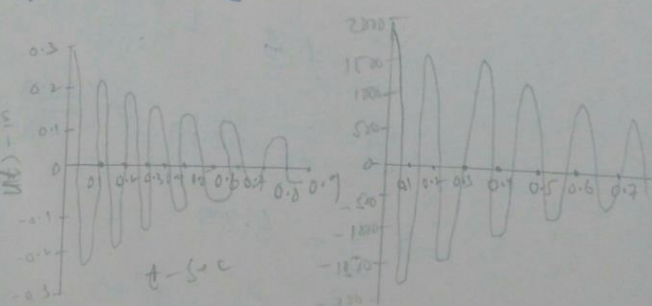
$$u(t) = e^{-0.584t}\left[0.041\times\cos(19.48t) + [0.0513]\times[0.0143]\right]$$

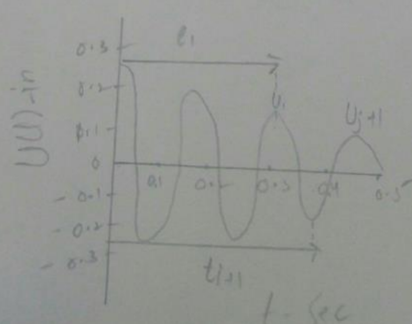
$$u(t) = e^{-0.584t}\left[0.041\times\cos(19.48t) + 0.00125\times\sin(19.48t)\right]$$

$$fs(t) = K\cdot u(t) \Rightarrow 90625\times u(t)$$

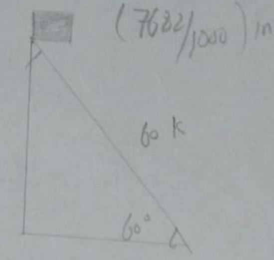
$$fs(t) = e^{-0.584t}\left((90625)\times0.041)\cos(19.48t) + (0.00125\times90625)\sin(19.48t)\right)$$

$$fs(t) = e^{-0.584t}\left(3715.62\cos(19.48t) + 113.28\sin(19.48t)\right)$$





Q3



Sol:

$u_1 = \frac{7682}{1000} = 7.68$

After $j = 7$, $u_{j+1} = u_0 = 2.286$ cm $= 0.9''$

a) ξ = Damping ratio = ?

$\Rightarrow j = \frac{1}{2\pi\xi} \ln\left(\frac{u_i}{u_{j+1}}\right)$

$\Rightarrow 7 = \frac{1}{2\pi\xi} \ln\left(7.68/0.9\right)$

$\Rightarrow \xi = 0.068 = 6.8\%$

b) $T_n = ?$

7 cycles of vibrations are completed in 3.57 sec

Time required to complete one cycle $= 3.57/7$

$T_D = 0.5$ sec

Now

$\omega_D = \omega_n \left(\sqrt{1-\xi^2}\right.$

$\Rightarrow T_D = \frac{2\pi}{\omega_n} = 2\pi / (\omega_n \sqrt{1-\zeta^2})$

$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$

$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$

$\Rightarrow T_n = 0.59 \times \sqrt{1-(0.068)^2}$

$\Rightarrow T_n = 0.5888 = 0.51$ sec

c) $k = ?$

$k = \frac{60 \cos 60^\circ}{7.68} = 3.82$ k/in

$k = 46332$ lb/ft

D) Weight of the tank, $w = ?$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{k}{w/g}} = \sqrt{\frac{k \times g}{w}}$

$\Rightarrow \omega_n^2 = k \times g / w$

$\Rightarrow w = k \times g / \omega_n^2$

Also

$\omega_n = 2\pi / T_n$

$w = \frac{46332 \times 32.2 \times (0.51)^2}{4\pi^2}$

$w = 9,841.25$ 7

$w = 9.84$ k

Q3 e) $c = ?$

It is known that $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$

$\Rightarrow c = 0.068 \times 2 \times 2 \times \left(\frac{\pi}{0.5}\right)\left(\frac{9841.2}{32.2}\right)$

$\Rightarrow c = 0.068 \times 2 \times 2 \times 6.28 \times 305.62$

$\Rightarrow c = 522.04$ lb.sec/ft

f) No of cycles to reduce displacement amplitude from 7.68 in to 0.5", $j = ?$

$j = \frac{1}{2\pi\zeta} \ln\left(\frac{U_i}{U_{j+1}}\right)$

$\Rightarrow j = \frac{1}{2 \times 3.14 \times 0.068} \ln\left(\frac{7.68}{0.5}\right)$

$\Rightarrow j = 6.39$

$\Rightarrow j = 7$ cycles