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Assignment Mathematics 1
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Summer Final Term

Q^I Part (b)

(b) 17 $z_1 = 8+3i$ and $z_2 = 9-2i$ then find $z_1 z_2$

Given Data:

$z_1 = 8+3i$

$z_2 = 9-2i$

Required:

$z_1 z_2$

Solution:

$z_1 z_2 = (8+3i)(9-2i)$

$\Rightarrow 8(9-2i) + 3i(9-2i)$

$\Rightarrow 72 - 16i + 27i - 6i^2$

$\Rightarrow 72 + 11i - 6(-1)$

$\Rightarrow 72 + 11i + 6$

$\Rightarrow 78 + 11i$

$\boxed{z_1 z_2 = 78+11i}$ Ans.

Q# Find the slope of the Circle $x^2+y^2=25$ at point $(3,-4)$

Ans Solution:

We know that we find the slope we take Differentiate the function.

So differentiate the function w.r.T x^2

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}25$$

$$\Rightarrow \quad 2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

Divide B.S by $2y$

$$\frac{2y}{2y}\frac{dy}{dx} = -\frac{2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Putting Point

$$\left(\underset{x}{3}, \underset{y}{-4}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{+(3)}{+4}$$

So the slope of Circle are $\frac{3}{4}$.

Q# Evaluate $\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right) dx$

Ans Solution::

$$\int \frac{1}{x^2}\,dx - \int x^2\,dx - \frac{1}{3}\int dx$$

$$\Rightarrow \int x^{-2}dx - \int x^2\,dx - \frac{1}{3}\int dx$$

$$\Rightarrow \frac{x^{-2+1}}{-2+1} - \frac{x^{2+1}}{2+1} - \frac{1}{3}x$$

$$\Rightarrow \frac{x^{-1}}{-1} - \frac{x^3}{3} - \frac{1}{3}x$$

$$\Rightarrow \quad \frac{-1}{x} - \frac{1}{3}x^3 - \frac{1}{3}x + C$$

Ans.

Q3 Estimate the the angle between $A = t - 2j - 2k$ and $B = 6t + 3j + 2k$

Ans Solution:-

$A = j - 2j - 2k$

$B = 6j + 3j + 2k$

We know that

$$\tan\theta = \frac{A \cdot B}{|A||B|} \quad \text{——} \star$$

$$A \cdot B = (j - 2j - 2k)(6j + 3j + 2k)$$

$$(1 \times 6) - (2 \times 3) - (2 \times 2)$$

$\Rightarrow$ $6 - 6 - 4$

$\Rightarrow$ $\boxed{A \cdot B = -4}$ —— (i)

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2}$$

$\Rightarrow$ $|A| = \sqrt{1 + 4 + 4}$

$|A| = \sqrt{9} = \sqrt{3^2} \Rightarrow \boxed{|A| = 3}$ —— (ii)

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$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$

$$|B| = \sqrt{36 + 9 + 4} \Rightarrow \sqrt{49}$$

$$|B| = 7 \quad \text{(iii)}$$

putting (i) (ii) & (iii) in ★

$$\text{Tan}\theta = \frac{-4}{3 \times 7} = \frac{-4}{21}$$

$$\text{Tan}\theta = \frac{-4}{21}$$

$$\theta = \tan^{-1}\left(\frac{-4}{21}\right)$$

$$\theta = -10.7579 \quad \text{Ans.}$$

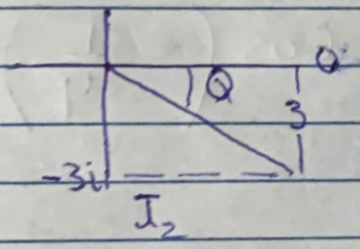
Q I Part (a)

(a) Represent $z = 3-3i$ in polar form.

Ans **Step 1 ::**

$$z = 3-3i$$

Find θ (help to plot z on complex place).



z is in Qudrent 4

θ is b/w $-\frac{I}{2}$ and 0

$$\text{Tan}\,\theta = \frac{-3}{3} = -1$$

$$\theta = -\frac{I}{4}$$

Step II ::

Find Model |z|

$$|z| = \sqrt{3^2 + (-3)^2} = \sqrt{18} = \sqrt{9}\sqrt{2}$$

$\Rightarrow \quad 3\sqrt{2}$

Step 3 :

Write z on polar form

$$z = 3\sqrt{2}\left(\cos\left(\frac{I}{4}\right) + i\,\sin\left(-\frac{I}{4}\right)\right)$$

also know as

$$3\sqrt{2}\,\cos\left(-\frac{I}{4}\right).$$

Q5 Evaluate $\int \sin^2 x \, dx$.

Ans We know that

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

So:

$$\cos^2 x = 1 - 2\sin^2 x$$

$$\cos^2 x = 1 - 2\sin^2 x$$

$$\Rightarrow 2\sin^2 x = \frac{1}{2} - \frac{\cos 2x}{2}$$

So:

$$\int \sin^2 x \, dx = \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{2} \times \frac{1}{2} + C$$

$$\Rightarrow \frac{x}{2} - \frac{\sin 2x}{4} + C \quad \text{Ans.}$$