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Question No 1

Find the eigenvalues of A

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

Sol

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} (-\lambda) & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & (8-\lambda) \end{bmatrix} = 0$$

$$(-\lambda)((-\lambda)\times(8-\lambda) - 1\times(-17)) - 1(0\times(8-\lambda) - 1\times 4)$$

$$+ 0(0\times(-17) - (\lambda)\times 4) = 0$$

$$(-\lambda)((-8\lambda + \lambda^2) - (-17)) - 1(0-4) + 0(0-(-4\lambda)) = 0$$

$$(-\lambda)(17 - 8\lambda + \lambda^2) - 1(-4) + (4\lambda) = 0$$

$$(-17\lambda + 8\lambda^2 - \lambda^3) - (-4) + 0 = 0$$

$$(-\lambda^3 + 8\lambda^2 - 17\lambda + 4) = 0$$

$$-(\lambda - 4)(\lambda - 0.26794919)(\lambda - 3.73205081) = 0$$

$$(\lambda - 4) = 0 \text{ or } (\lambda - 0.26794919) = 0 \text{ or } (\lambda - 3.73205081) = 0$$

The eigenvalues of the matrix A are given by $\lambda = 0.26794919, 3.73205081, 4$, Question No

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Sol

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} (-\lambda) & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & (3-\lambda) \end{bmatrix} = 0$$

$$(-\lambda)\left((2-\lambda)\times(3-\lambda) - 1\times 0\right) - 0(1\times(3-\lambda)$$

$$-1\times 1) + (-2)(1\times 0 - (2-\lambda)\times 1 = 0$$

$$(-\lambda)((6-5\lambda+\lambda^2)-0) - 0(3-\lambda)-1) - 2[(0-(2-\lambda)) = 0$$

$$(-\lambda)(6-5\lambda+\lambda^2) - 0(2-\lambda) - 2(-2+\lambda) = 0$$

$$(-6\lambda + 5\lambda^2 - \lambda^3) - 0 - (-4 + 2\lambda) = 0$$

$$(-\lambda^3 + 5\lambda^2 - 8\lambda + 4) = 0$$

$$(\lambda - 1)(\lambda - 2)(\lambda - 2) = 0$$

$$(\lambda - 1) = 0 \text{ or } (\lambda - 2 = 0 \text{ or } (\lambda - 2) = 0$$

The eigenvalues of the matrix A are given by $\lambda = 1, 2,$

→ 2 Eigenvectors for $\lambda = 2$

$v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

The eigenvectors compose the columns of matrix P,

$\therefore P \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

The diagonal matrix D is composed of the eigenvalues.

$D \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Now find P^{-1}

$|P| = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$= -2 \times (1 \times 1 - 0 \times 0) + 0 \times (1 \times 1 - 0 \times 1)$
$\quad -1 \times (1 \times 0 - 1 \times 1)$
$= -2 \times (1 + 0) + 0 \times (1 + 0 - 1 \times (0 - 1)$
$= -2 \times (1) + 0 \times (1) - 1 \times (-1)$
$= -2 + 0 + 1$
$= -1$

Ans

Question . 3

$v_1 = (1, -2, 3)$
$v_2 = (5, 6, -1)$
$v_3 = (3, 2, 1)$

Sol

The vectors x, y, z are linealy depentant if their deteminant is zero $|D| = 0$

$$|D| \begin{bmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= 1 \times \begin{bmatrix} 6 & -1 \\ 2 & 1 \end{bmatrix} + 2 \times \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} + 3 \times \begin{bmatrix} 5 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= 1 \times (6 \times 1) - (-1) \times 2) + 2 \times (5 \times 1 - (-1) = 3 \times (5 \times 2 - 6 \times 3)$$

$$= 1 \times (6 + 2) + 2 \times (5 + 3) + 3 \times (10 - 18)$$

$$= 1 \times (8) + 2 \times (8) + 3 \times (-8)$$

$$= 8 + 16 - 24$$

$$= 0$$

Since $|D| = 0$, So vectors x, y, z are linearly dependent.

Question No 4

(a) if we take $\alpha = 2$ & $\beta = -3$

and $v = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

Then $(\alpha + \beta) v = (2-3) \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = -1 \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$

& $\alpha v + \beta v = 2 \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + -3 \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ -6 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ -2 & 6 \end{bmatrix}$

$\Rightarrow (\alpha + \beta) v \neq \alpha v + \beta v$

Q no :- 4
Part :- B

b $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$a_0 = a_1 = a_2 = a_3 = 0$

$p(x) = 0$

under addition

$p_1(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$p_2(x) = b_0 + b_1x + b_2x^2 + b_3x^3$

$p_1(x) + p_2(x) = (a_0 + b_0) + (a_1x + b_1x)$
$+ (b_2x^2 + b_2x^2) + (a_3x^3 + b_3x^3)$

$= c_0 + c_1x + c_2x^2 + c_3x^3$

scalar multiple

$p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$

$cp(x) = ca_0 + ca_1x + ca_2x^2 + ca_3x^3$

$= d_0 + d_1x + d_2x^2 + d_3x^3$.

Question No 4

Definition: A vector space is a set V on which two operations + and are defined called vector addition and scalar multiplication the operation + (vector addition must satisfy the following conditions)

Closure if u and v are any vectors in V, then the sum $u+v$ belongs to V.

(1) Commutative law: for all vectors u and v in V, $u+v=v+u$

(2) Associative law: for all vectors u, v, w in V, $u+(v+w)=(u+v)+w$

(3) Additive identity The set V contains an additive identity element denoted by 0 such that for any vector v in V $0+v=v$ and $v+0=v$

(4) Distributive law: for all real number c and all vectors u, v in V,
$c \cdot u + c \cdot v$