Day: M T W T F S

Date: __/__/__

Name	Asim Siddiq
ID	7691
Section	A
Subject	Introduction to structural dynamics and Earthquake engineering
Department	BE(C)
Instructor	Engr. Yaseen.

Q No 1.

Given data:

$E = 29,000$ ksi

$I = 150\ in^4$

δ_{st} = Deflection due to 7691 lb static load.

Beam is pulled $\frac{1}{2}$" downwards.

Required data:

- Natural time period of system develope and solve the equation.
- Draw graphs to show the variation of displacement with time and the variation of equilivent static forces with time.

Day. M T W T F S Date: __/__/__

Solution:-

General EOM for SDOF system is :

$$KU + C\dot{U} + m\ddot{U} = p(t)$$

Since system is undamped $(c=0)$ undergoing free vibration $p(t)=0$

Hence general EOM becomes

$$KU + m\ddot{U} = 0 \longrightarrow (1)$$

$$K = \frac{3EI}{L^3} \Rightarrow 3 \times 29000\ K/in^2 \times 150\ in^4$$

$$K = 7.55208\ K/in$$

⇒ In order to eliminate chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m sec.

Scanned with CamScanner

Day. M T W T F S Date: __/__/__

$K = 7.55208 \; K/in \Rightarrow 90625 \; lb/ft$

$m = \frac{w}{g} \Rightarrow \frac{7691}{32.2}$

$m = 238.8509 \; slug$

So;

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\omega_n = \sqrt{\frac{90625}{238.8509}}$$

$\omega_n = 19.4787 \; rad/sec$

and;

$$T_n = \frac{2\pi}{\omega_n}$$

$$T_n = \frac{2\pi}{19.4787}$$

$T_n = 0.3227 \; sec$

put m & K in eq^n ①

Day: M T W T F S Date: __/__/__

$$90625u + 238.8509\,\ddot{u} = 0$$

where K is in lb/ft & m is in lb sec/ft²

→ General solution to EOM for undamped free vibration is;

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24}\text{ ft} \;\&\; \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.4787) + 0$$

$$= \left(\frac{1}{24}\right) \times \cos(19.4787t)$$

Equivalent static force at anytime 't' is

$$f_s(t) = K.\,u(t) = \frac{90625 \times \cos(19.4787t)}{24}$$

$$= 3776\cos(19.4787t)$$

Day: MTWTFS Date: __/__/__

⇒ Amplitude of dynamic displacement u_o for undamped free vibration is;

$$u_o = \sqrt{(u(0))^2 + \left(\frac{v(0)}{\omega_n}\right)^2}$$

$$u_o = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

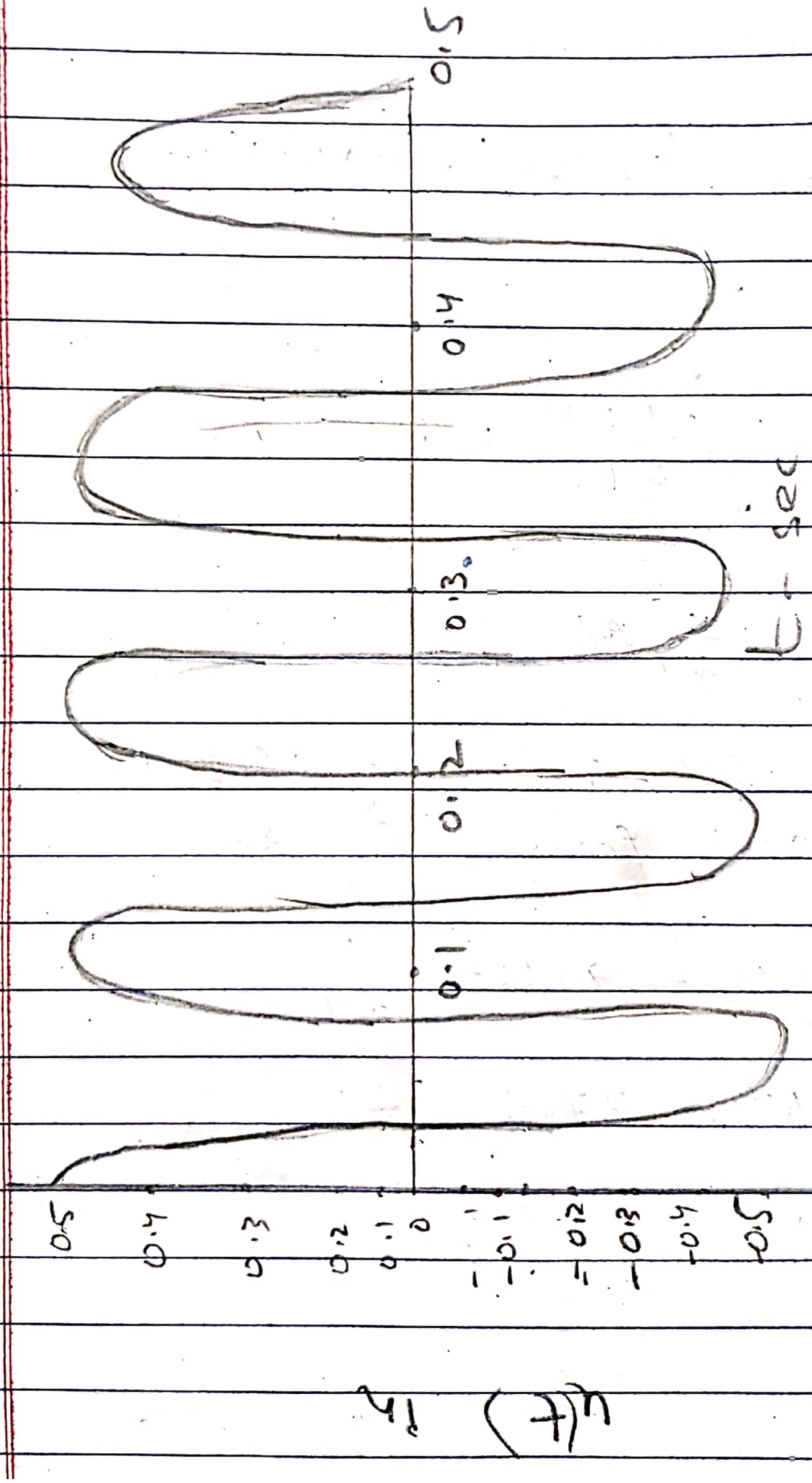
$$u_o = \frac{1}{24} \text{ ft}$$

⇒ Amplitude of equilvent static force, f_{so}

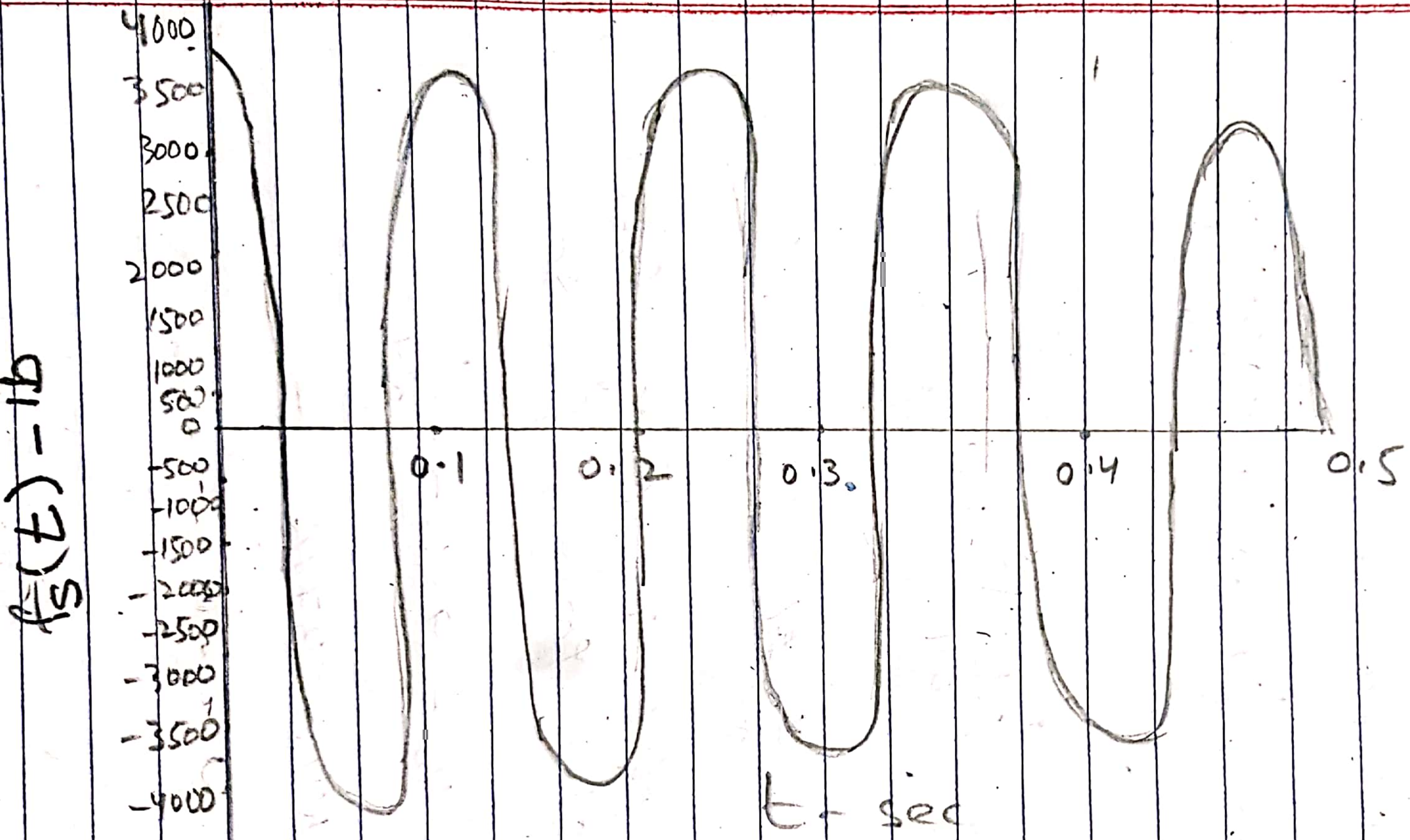
$$Ku_o = 90625 \times \frac{1}{24}$$

$$Ku_o = 3776$$

Day. MTWTFS Date: / /



Variation of displacement with time



Variation of Equivalent static forces with time.

Day: M T W T F S Date: __/__/__

QNO 2

Given data:

ζ (Damping ratio) of Reinforced Concrete with considered cracking = 3–5%

= 3%

Using data of beam given in Question # 1

Required data:

Develop and solve the equation showing variation in equivalent static force with time.

Solution:

EOM for damped free vibration is ;

$$Ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

Day. M T W T F S Date: __/__/__

From Question 1

$K = 90625$ lb/ft

$m = 238.8509 \frac{lb.sec^2}{ft}$

$\omega_n = 19.4787$ rad/sec

$C = \zeta \times 2 m \omega_n \Rightarrow 0.03 \times 2 \times 238.8509 \times 19.4787$

$C = 279.15$ lb.sec/ft

Put the values in eq^n (1)

$90625 u + 279.15 \dot{u} + 238.8509 \ddot{u} = 0$

⇒ Solution to the EOM for damped free vibration is;

$$u(t) = e^{-\zeta \omega_n t} \left[u(0) \cos(\omega_D t) + \frac{1}{\omega_D} \left[\dot{u}(0) + u(0) \zeta \omega_n \right] \right] . \sin \omega_D t$$

$\omega_D = 19.4787$ rad/sec

Day. M T W T F S Date: __/__/__

$$u(t) = e^{-0.03\times19.4787t}\left[\frac{1}{24}\times\cos(19.4787) + \frac{1}{19.4787}\times\left[0 + \frac{1}{24}\times0.03\times19.4787\times\sin(19.4787t)\right.\right.$$

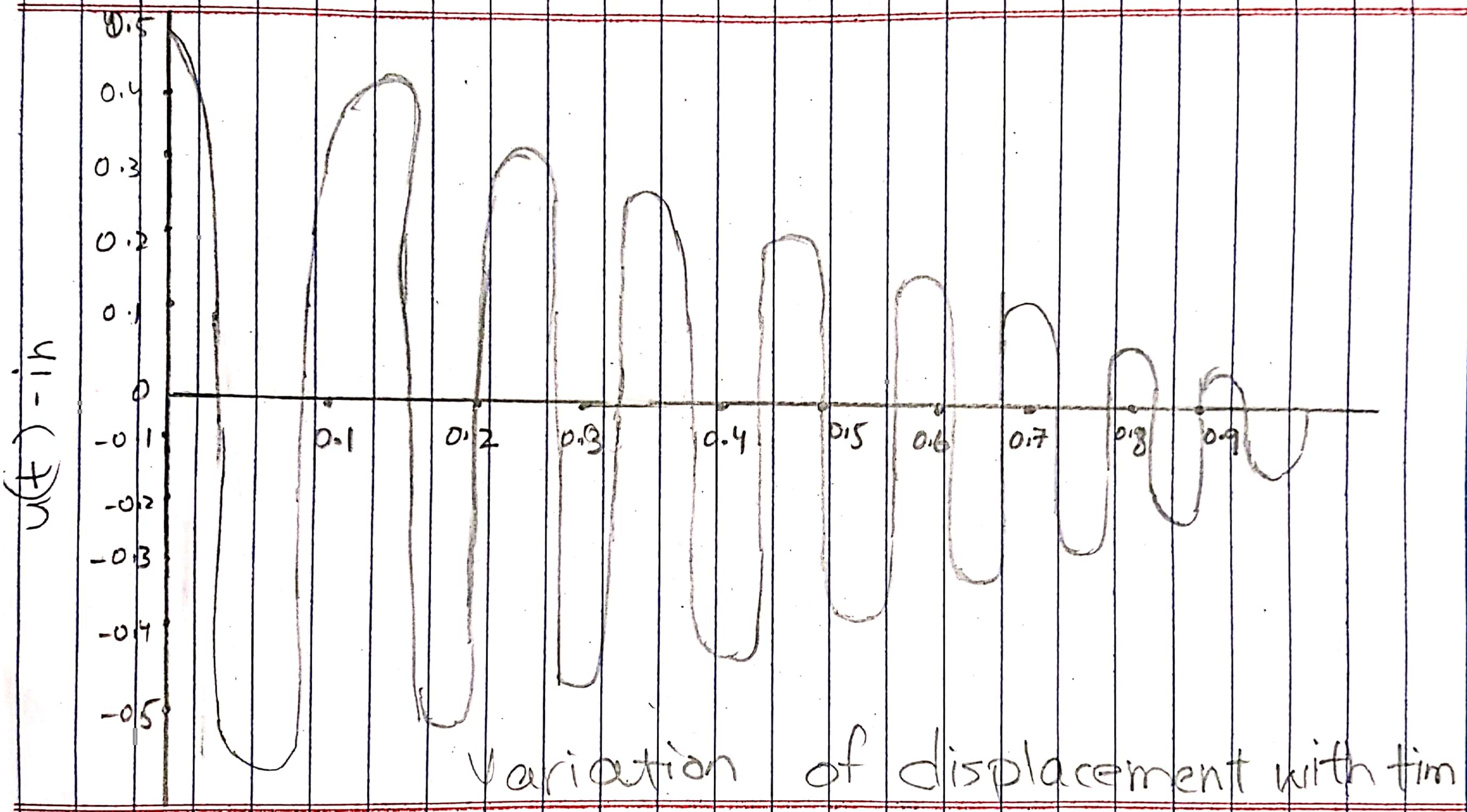
$$u(t) = e^{-0.03\times19.4787t}\left[0.041\times\cos(19.4787t) + 0.00125\times\sin(19.4787t)\right.$$

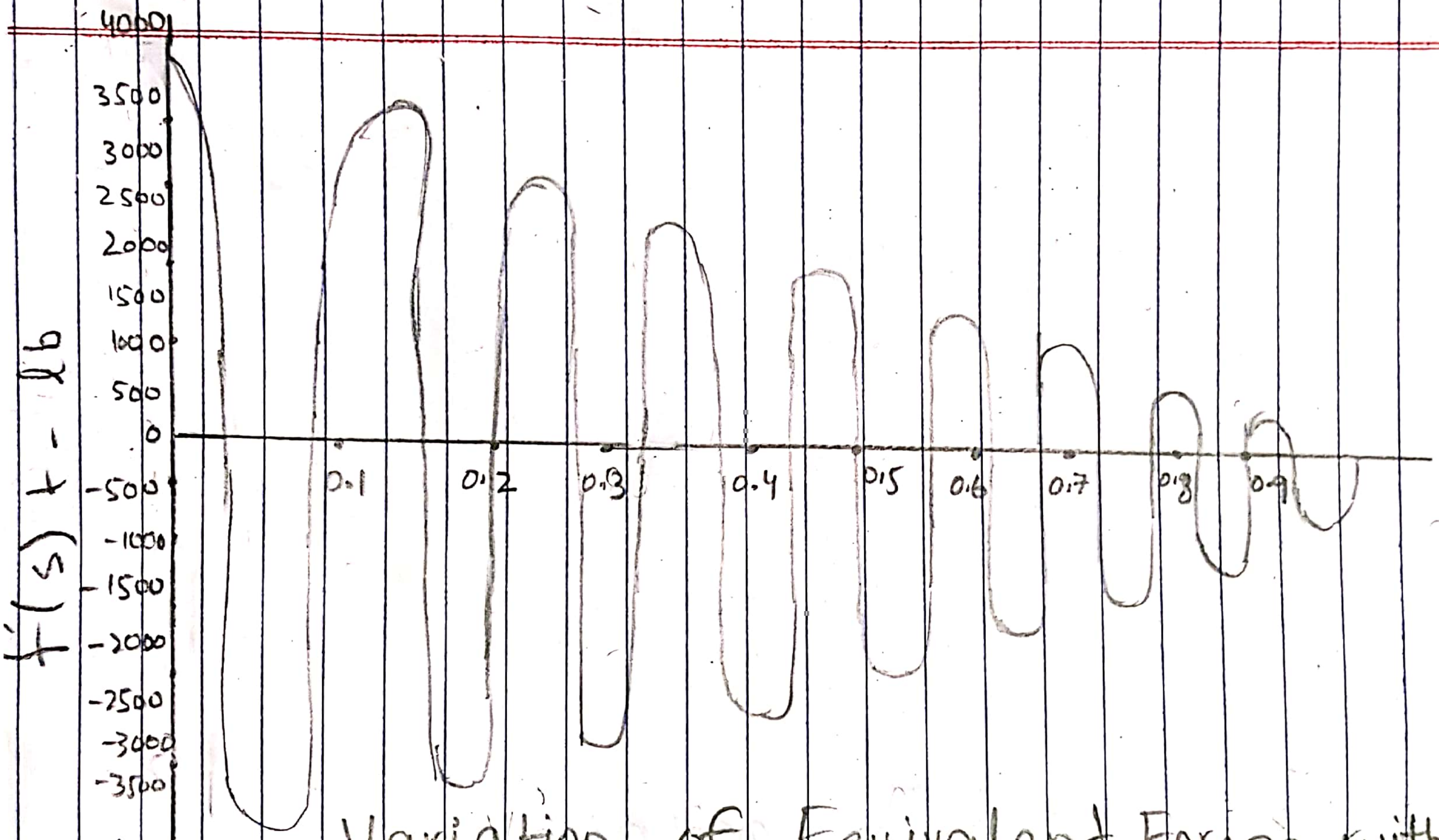
$$f_s(t) = K.u(t) \Rightarrow 90625\times u(t)$$

$$f_s(t) = e^{-0.584t}\left[(90625\times0.041)\cos(19.4787t) + (90625\times0.00125)\times\sin(19.4787t)\right]$$

$$f_s(t) = e^{-0.584t}\left[3715.62\cos(19.4787t) + 113.28\sin(19.4787t)\right.$$

Ans


u(t) - ui
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Variation of displacement with tim



Variation of Equivalent Forces with time

Day: M T W T F S Date: __/__/__

Q No 3.

Given data:

Force = 60 kips

Displacement of tank = $\left(\frac{ID}{1000}\right)''$

$= \left(\frac{7691}{1000}\right)'' = 7.691''$

Time taken to complete 7 cycles = 3.57 sec

Amplitude of displacement = 2.286 cm

$= 0.9''$

Required data:-

a. Damping ratios, ζ

b. Natural period of undamped vibration.

c. Stiffness of structures

d. Weight of tank.

e. Damping co-efficient.

f. Number of cycles to reduce the displacement amplitude to 0.5".

Solution:

→ Displacement of tank, $u_1 = 7.691''$

→ After 7 cycles, i-e After $j = 7$, $u_{j+1} = u_8 = 0.9''$

a. Damping ratio, $\zeta = ?$

Damping ratio, is find as,

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.691''}{0.9}\right]$$

$$\zeta = 0.0488 = 4.88\%$$

b- Natural period of undamped vibration, $T_n = ?$

As the 7 cycles of vibrations are completed in 3.57 sec.

$\Rightarrow$ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51\,sec$

Now;

$$w_D = w_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{w_D} = \frac{2\pi}{w_n\sqrt{(1-\zeta^2)}}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\zeta)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094 = 0.51$$

$$\boxed{T_n = 0.51\,sec}$$

Day: M T W T F S Date: __/__/__

So, the Natural period of undamped vibration,
$T_n = 0.51 sec.$

c- Stiffness of structure, $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.691}$$

$K = 3.90 \ k/in$

$K = 3.90 \ k/in = 46800 \ lb/ft$

d- Weight of tank:-
Weight of tank, W is find as,

$$\omega_n = \sqrt{\frac{K}{m}} \Rightarrow \sqrt{\frac{K}{\frac{W}{g}}} \Rightarrow \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{K \times g}{W}$$

$$W = \frac{K \times g}{\omega_n^2}$$

Day: M T W T F S Date: __/__/__

Also;

$$W_n = \frac{2\pi}{T_n}$$

$$W = \frac{K.g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = K.g \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46800\ lb}{ft} \times \frac{32.2\ ft}{sec^2}\right] \times \frac{(0.51 sec)^2}{4\pi^2}$$

$$W = 9928.47\ lb = 9.93K$$

e- Damping Co-efficient, $C = ?$

It is known that;

$$\zeta = \frac{C}{2mw_n}$$

$$\Rightarrow C = \zeta \times 2m \times w_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$C = (0.0488) \times 4 \times \pi \times \left(\frac{9928.47}{32.2}\right)$$

0.51

$$\boxed{C = 370.75 \text{ lb. sec/ft}}$$

f- Number of cycles to reduce the displacement Amplitude to 0.5", j

$$j = \frac{1}{2\pi \zeta} \ln \left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0488} \ln \left[\frac{7.691}{0.5}\right]$$

$$\boxed{j = 8.91 \text{ or } 9 \text{ cycles}}$$