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I-D ⇒ 7671

Section ⇒ Senior.

Subject Name ⇒ Mechanics of Solid 2.

Semester ⇒ Senior.

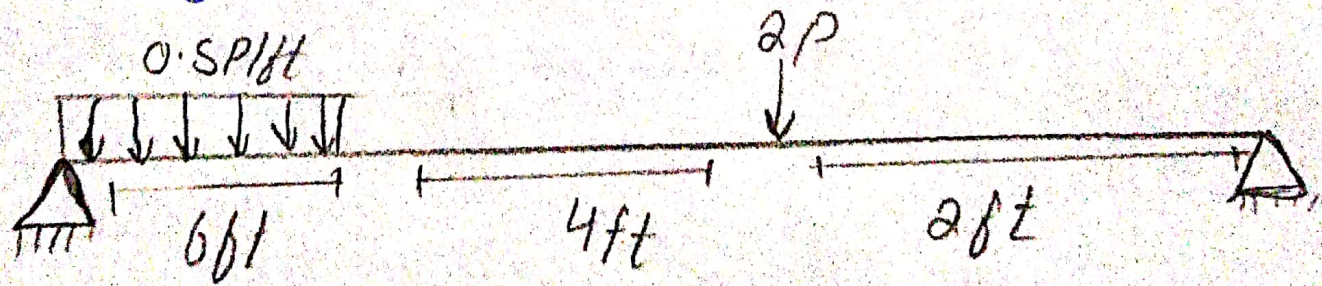
Date ⇒ 16 - April - 2020.

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Construct the Mohr's circle diagram and find the principal stress and maximum in plane shear stress for the stress state of a point 'C' located at the centre of uniform distributed load and 1 inch below the top of fiber of beam cross section shown in figure. However to construct the Mohr's circle it is necessary to draw the shear and flexural stress variation diagram for maximum shear force and bending moment respectively. Compare the result obtained from the Mohr's circle with the stress transformation equation.

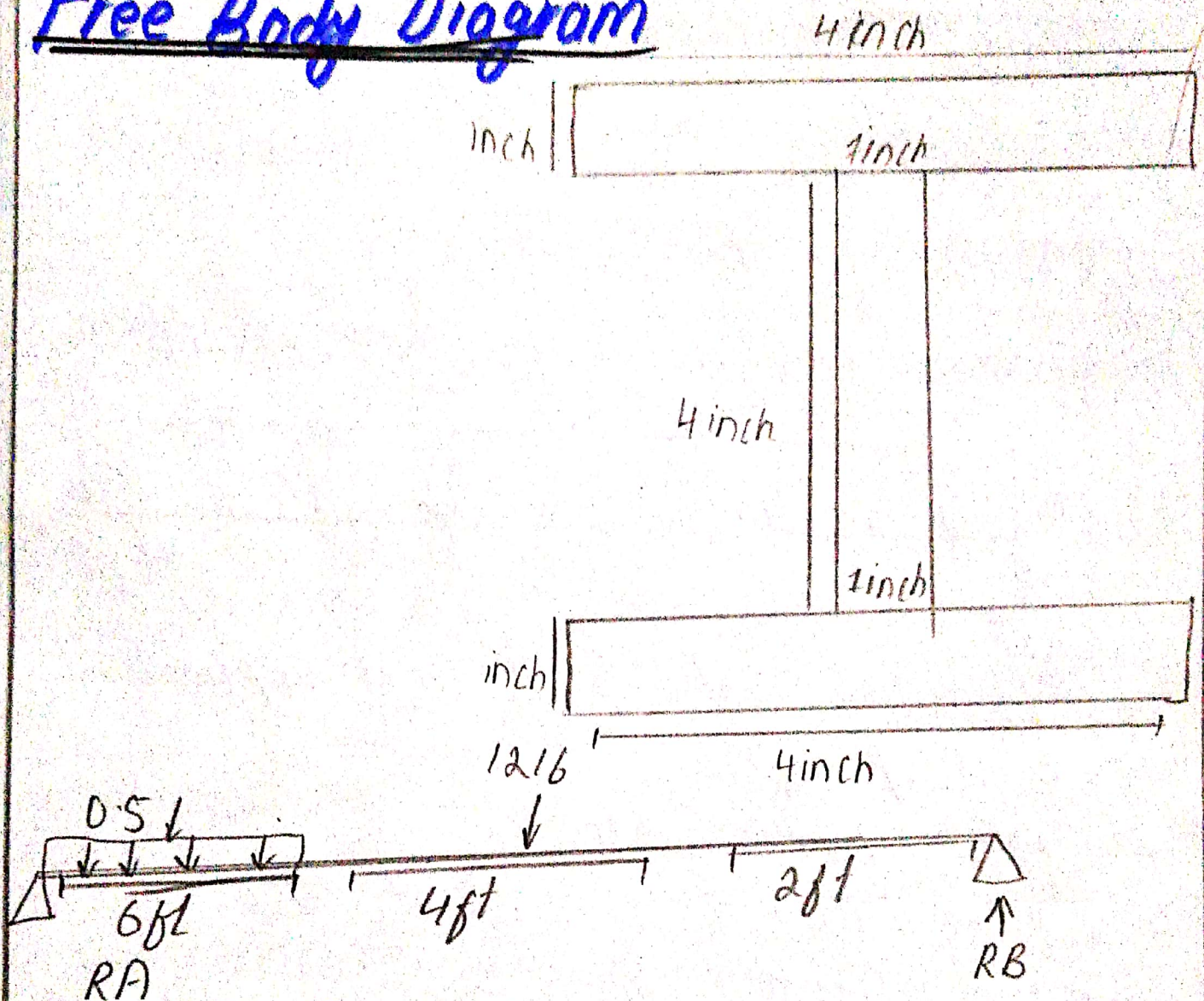
where P is the last two digits of class registration number in points.



Here my class registration number is 7906

$$\text{So } 2P = 2 \times 06 = 12 \text{ k}$$

Free Body Diagram



Support Reactions:

As we know that

$$\sum F_y = 0 \quad \uparrow + \quad \downarrow -$$

$$RA + RB = 12.5$$

NOW

$$\sum M = 0 \quad \curvearrowright + \quad \curvearrowleft -$$

$$RB \times 12 - 12 \times 10 - 3 \times 3 = 0$$

$$12 RB - 120 - 9 = 0$$

$$12 RB = 120 + 9$$

$$\frac{12 RB}{12} = \frac{129}{12}$$

$$R_B = 10.75$$

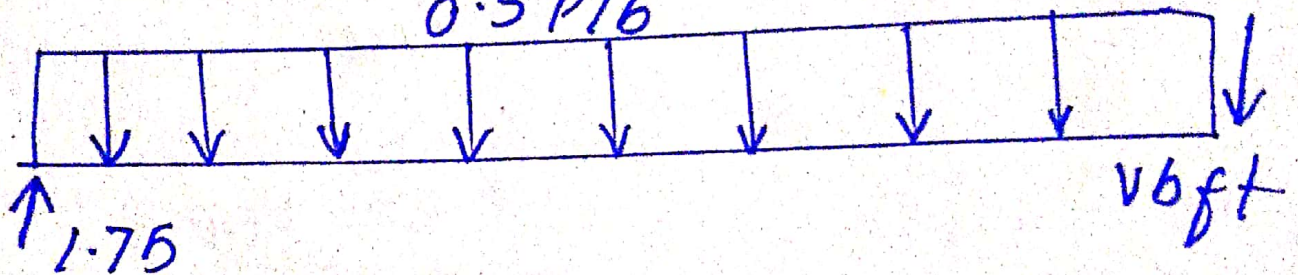
$$R_A + R_B = 12.5$$

$$R_A + 10.75 = 12.5$$

$$R_A = 12.5 - 10.75$$

$$R_A = 1.75$$

Now shear force at change point of beam



So, shear force at $b/4$ from left support

$$\sum F_y = 0 \quad \uparrow \quad \downarrow$$

$$+ V_{sft} - 1.75 + 0.5 \times b = 0$$

$$+ V_{sf} - 1.75 + 3 = 0$$

$$+ V_{sf} - 1.75 + 3 = 0$$

$$V_{bf} = +1.75 - 3$$

$$V_{bf} = 25 - 1.25/b$$

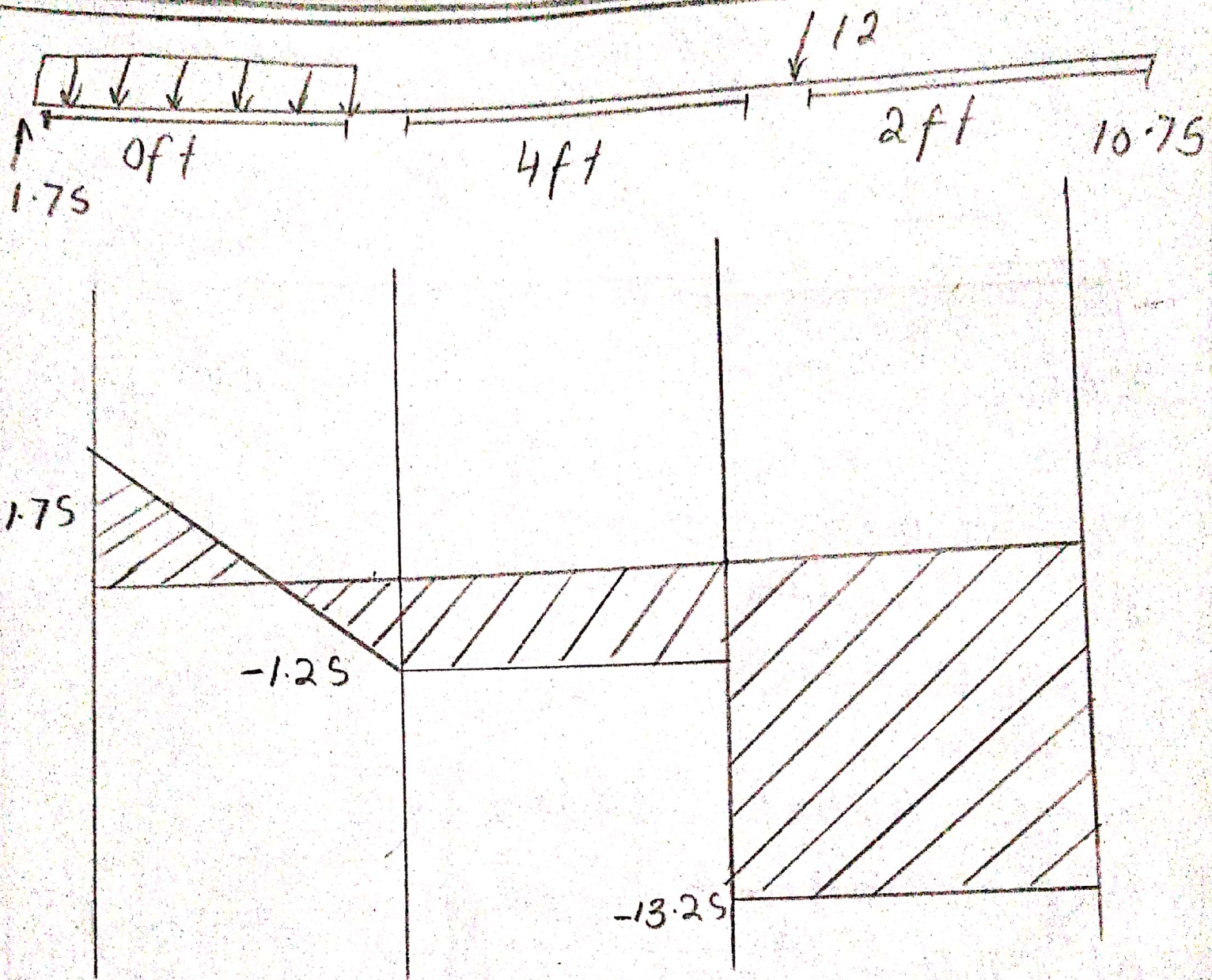
Now shear force at $3b/4$

$$\sum F_y = \uparrow \quad \downarrow$$

$$-1.75 + 3 + 12 + V_{ioft} = 0$$

$$V_{ioft} = 1.75 - 3 - 12$$

$$V_{ioft} = -13.25$$



Now moment at change point.
Find Zero shear point

$$\frac{1.75}{x} = \frac{1.25}{(6-x)}$$

$$1.75(6-x) = 1.25(x)$$

$$10.5 - 1.75x = 1.25x$$

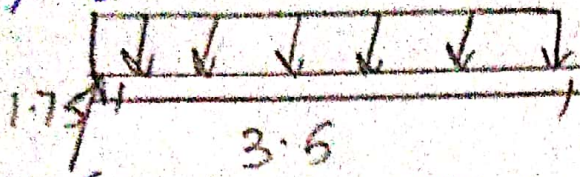
$$10.5 = 1.25x + 1.75x$$

$$\frac{10.5}{3} = \frac{3x}{3}$$

$$x = 3.3$$

As we know that moment is maximum where shear force is zero.

Take section at 3.5 from left support end find moment



$$\sum M_{3.5} = 0 \quad (+)$$

$$M_{3.5} - 1.75 \times 3.5 + 3 \left(\frac{3.5}{2} \right) = 0$$

$$M_{3.5} - 6.125 + 5.25 = 0$$

$$M_{3.5} = 6.125 - 5.25$$

$$M_{3.5} = 0.875$$

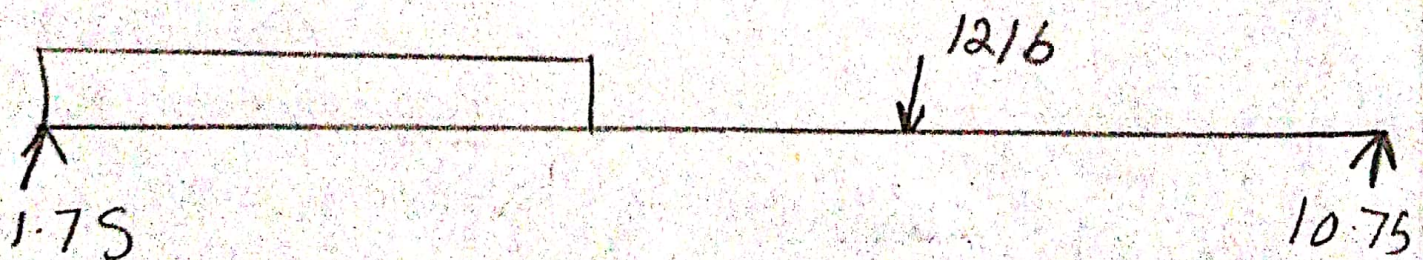
NOW:

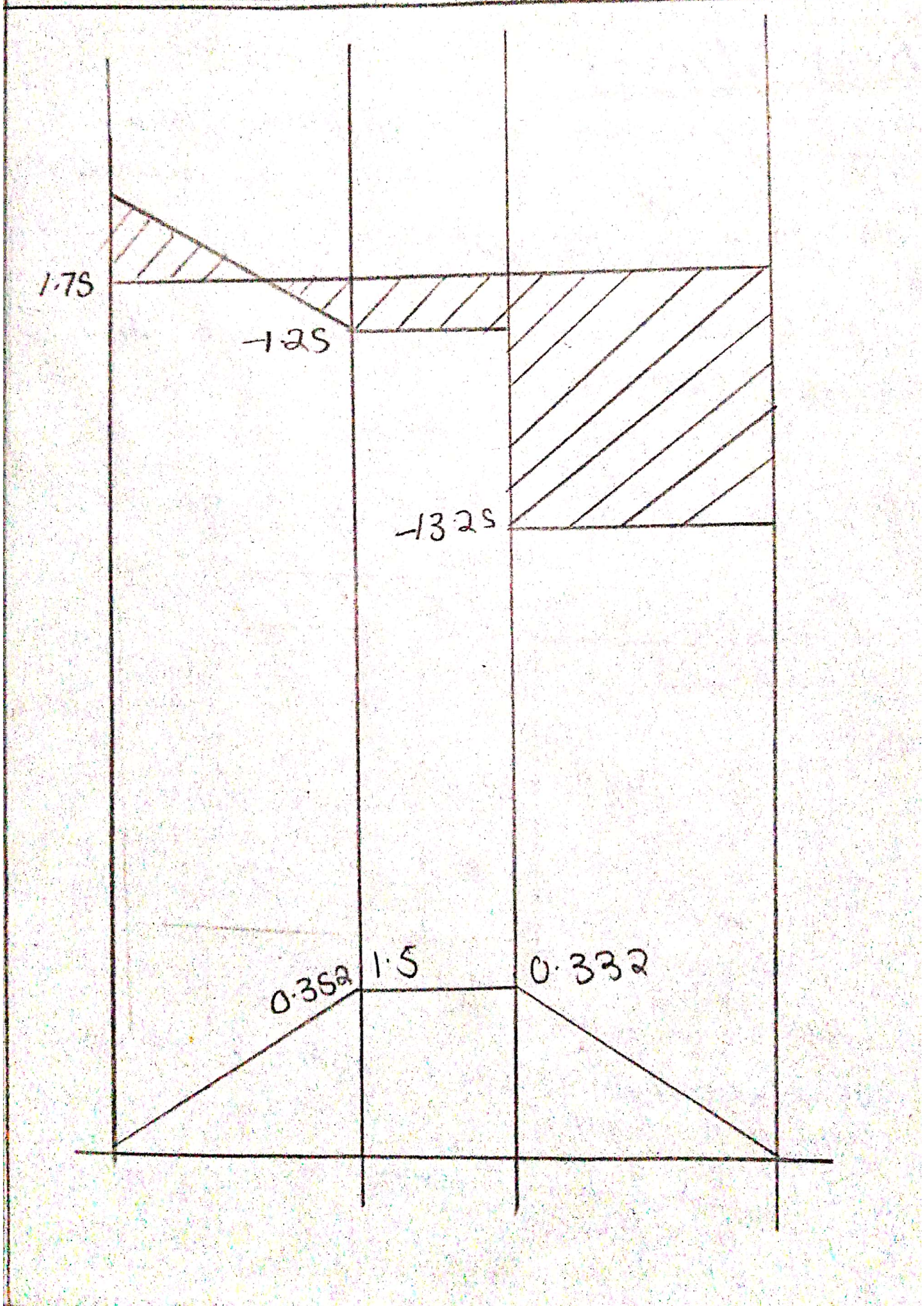
$$M_{of} = 1.75 \times 6 + 0.5 \times 6 \times 3 = 0$$

$$M_{of} = -10.5 + 9 = 0$$

$$M_{of} = 10.5 - 9$$

$$M_{of} = 1.5$$





Now; Shear Stress;

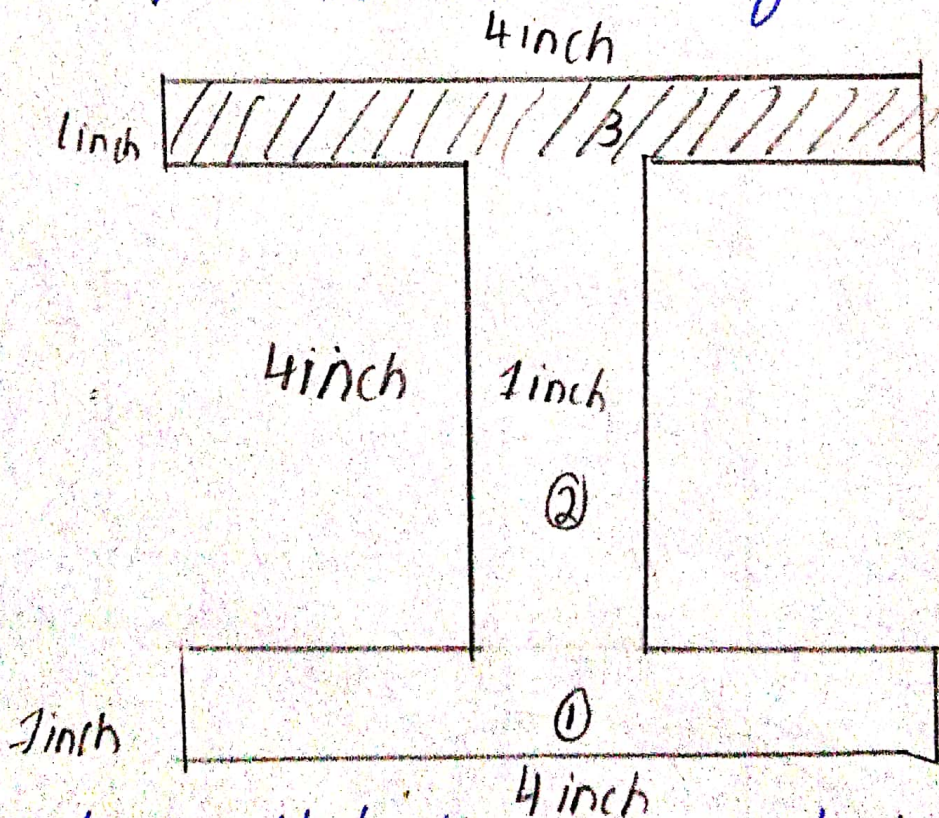
As per question the maximum shear stress $\tau = \frac{VQ}{It}$ occurs where the maximum shear force is 10.75 Ib.

So,

To find the shear stress we have the following formula

$$\tau = \frac{VQ}{It}$$

We first find the moment of inertia



As we know that to find centroid we have the following formula:

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$A_1 = 4 \times 1 = 4$$

$$A_2 = 4 \times 1 = 4$$

$$A_3 = 4 \times 1 = 4$$

$$\bar{y} = \frac{4 \times 0.5 + 4 \times 3 + 4 \times 5.5}{4 + 4 + 4}$$

$$\bar{y} = 3$$

Now moment of inertia

No A (in²) I_x (in)

$$d = (\bar{y} - y_1) \\ (\bar{y} - y_2) (\bar{y} - y_2)$$

$$\textcircled{1} \quad 4 \quad \frac{4 \times (1)^2}{12} = 0.333$$

$$\textcircled{2} \quad 4 \quad \frac{1 \times (4)^3}{12} = 5.333$$

$$\textcircled{3} \quad 4 \quad \frac{4 \times (1)^3}{12} = 0.333$$

(Now "d")

$$\textcircled{1} \quad d = (\bar{y} - y_2) = (3 - 0.5) = 2.5$$

$$\textcircled{2} \quad d = (\bar{y} - y_2) = (3 - 3) = 0$$

$$\textcircled{3} \quad d = (3 - 5.5) = -2.5$$

(Now d^2)

$$\textcircled{1} \quad 4 \times (2.5)^2 = 25$$

$$\textcircled{2} \quad 4 \times (0)^2 = 0$$

$$\textcircled{3} \quad 4 \times (-2.5)^2 = 25$$

Now

$$I_x = I_x + Ad^2$$

$$\textcircled{1} 0.333 + 25 = 25.333$$

$$\textcircled{2} 5.333 + 25 = 5.333$$

$$\textcircled{3} 0.333 + 25 = 25.333$$

Total

$$I = I_{x1} + I_{x2} + I_3$$

$$I = 25.333 + 5.333 + 25.333$$

$$I = 55.999 \text{ in}^4$$

Now shear stress

$$\tau = \frac{VQ}{Ib}$$

$$U_{\max} = 10.75$$

$$Q = JA$$

b = breadth of that fiber
Shear stress at point c located at centre of uniformly distributed load and inch below the top fiber

$$\bar{y} = 2 + 0.5 = 2.5$$

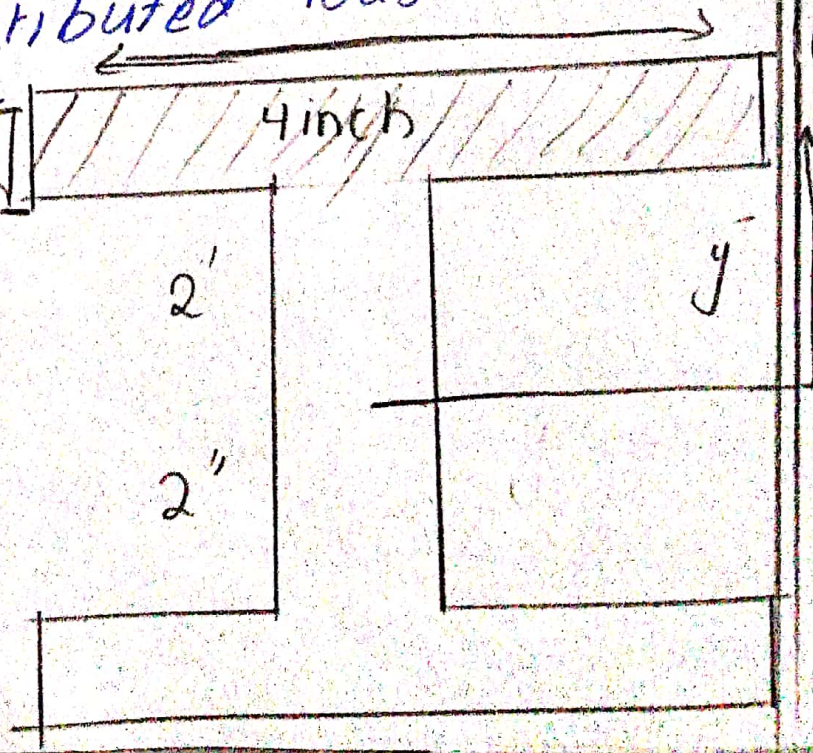
$$A = 1 \times 4 = 4$$

$$Q = 4 \times 2.5 = 10$$

As we know that

$$\tau = \frac{VQ}{Ib}$$

$$\tau = \frac{(10.75)(10)}{(55.996)(4)}$$



$$T = 0.479 \text{ Psi}$$

Now Flexural Stress Analysis

$$\sigma = \frac{My}{I}$$

where M is maximum moment in BMA

$$M = 1.5$$

$$\sigma = \frac{(1.5)(2)}{55.996}$$

$$\sigma = 0.0535$$

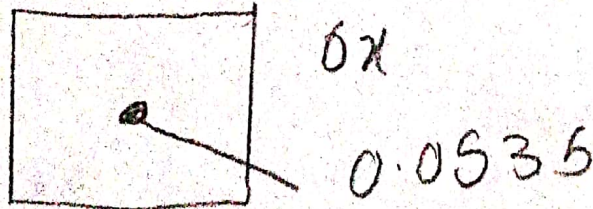
So, shear stress at point c is

$$t = 0.475 \text{ Psi}$$

Flexural stress at point c'

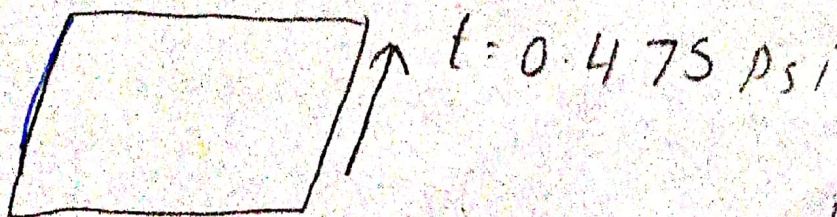
$$\sigma = 0.0535 \text{ Psi}$$

Now consider c is a plane element

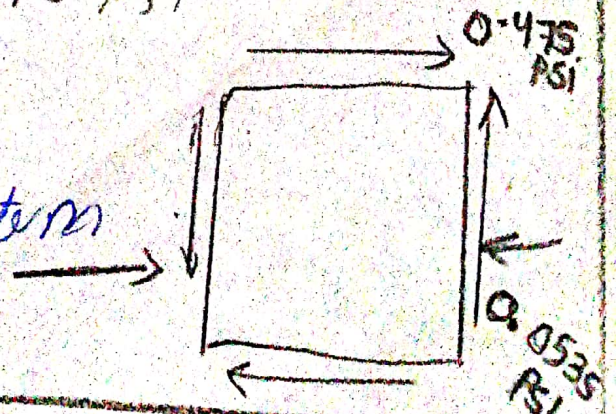


0.0535 psi is compressive Because point c lies in compression zone of Beam Cross

Now



combine stress on 2D element



Now we can find the stress state considers of point c at a degree of 20° clockwise orientation.

Solve

Given stress state

$$\sigma_x = -0.0535$$

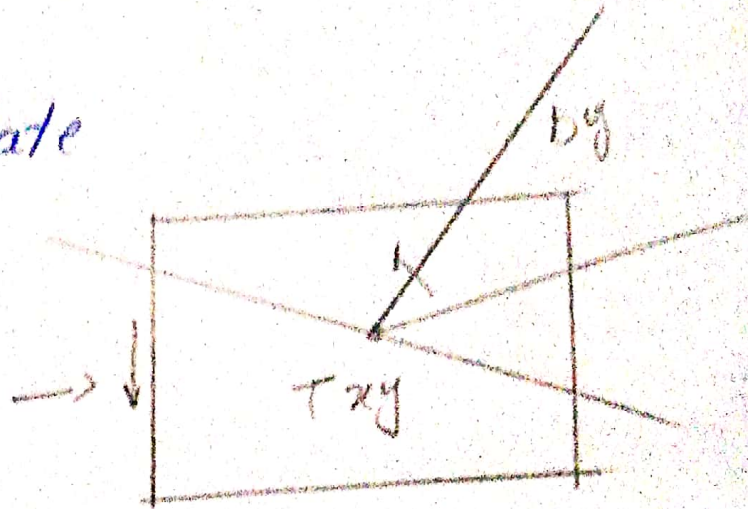
$$\sigma_y = 0$$

$$\tau_{xy} = 0.475$$

$$\sigma_{x'} = ?$$

$$\sigma_{y'} = ?$$

$$\tau_{x'y'} = ?$$



As we derive the following formulae equation for stress transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

For $\sigma_{x'}$

$$\sigma_{x'} = \frac{-0.0535}{2} + \frac{-0.0535}{2} \cos(2(-20))$$

$$+ (0.475) \sin(2(+20))$$

$$\sigma_{x'} = -0.02675 - 0.0204 \cdot 0.3053$$

$$\boxed{\sigma_{x'} = -0.3524 \text{ Psi}} \text{ compression}$$

For σ_y'

$$\sigma_y' = -\frac{0.0535}{2} - \frac{(-0.0535) \cos(2(-20))}{2}$$

$$- (0.475) \sin 2(-20)$$

$$\sigma_y' = -0.02675 - 0.0204 - 0.3053$$

$$\boxed{\sigma_y' = -0.3524} \text{ compressive}$$

For $\epsilon_{x'y'}$

$$\epsilon_{x'y'} = -\sigma_x - \sigma_y \sin 2\theta + \epsilon_{xy} \cos 2\theta$$

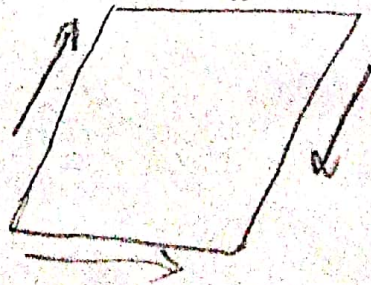
$$\epsilon_{x'y'} = -\frac{(-0.0535)}{2} - 0 \sin(2(-20))$$

$$+ 0.475 \cos(2(-20))$$

$$= -0.01719 + 0.3638$$

$$\epsilon_{x'y'} = 0.34661$$

So, the new stress state after 20° clockwise orientation is shown by $\sigma_y' = 0.3524$



$$\sigma_x' = 0.3524$$

$$\tau_{x'y'} = 0.34661$$

Find its Principle Stress:-

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-0.0535 + 0}{2} \pm \sqrt{\left(\frac{-0.0535 - 0}{2}\right)^2 + (0.479)^2}$$

$$\sigma_{1,2} = -0.0267 \pm \sqrt{7.155 + 0.2256}$$

$$\sigma_{1,2} = -0.0267 \pm \sqrt{7.3806}$$

$$\sigma_{1,2} = -0.0267 \pm 2.7166$$

$$\sigma_{1,2} = \sigma_1 = -0.0267 + 2.7166 = 2.689$$

$$\sigma_{1,2} = \sigma_2 = -0.0267 - 2.7166 = -2.7433$$

Max in Plane Shear Stress

$$\tau_{xy} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\tau_{xy} = \sqrt{0.00071 + 0.225}$$

$$\tau_{xy} = \sqrt{0.22571}$$

$$\tau_{xy} = 0.4750 \text{ Psi}$$

To Draw Mohr's Circle for the given Problem

Solve As we know that to draw the circle we need the coordinates of circle as well as radius. The coordinates of circle can be found by this

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$$\left(\frac{bx + by}{2}, 0 \right)$$

centre coordinates

$$(h, k) \left(\frac{-0.0535}{2}, 0 \right)$$

$$= (-0.02675, 0)$$

Radius of Mohr's circle is

$$r = \sqrt{\left(\frac{bx - by}{2} \right)^2 + \frac{1}{2}xy}$$

$$r = \sqrt{\left(\frac{-0.0535 - 0}{2} \right)^2 + (0.475)^2}$$

$$r = \sqrt{0.000715 + 0.2256}$$

$$r = \sqrt{0.226375}$$

$$r = 0.4757$$

