Linear Algebra

Summer Final Exam **Total: 50 Marks**

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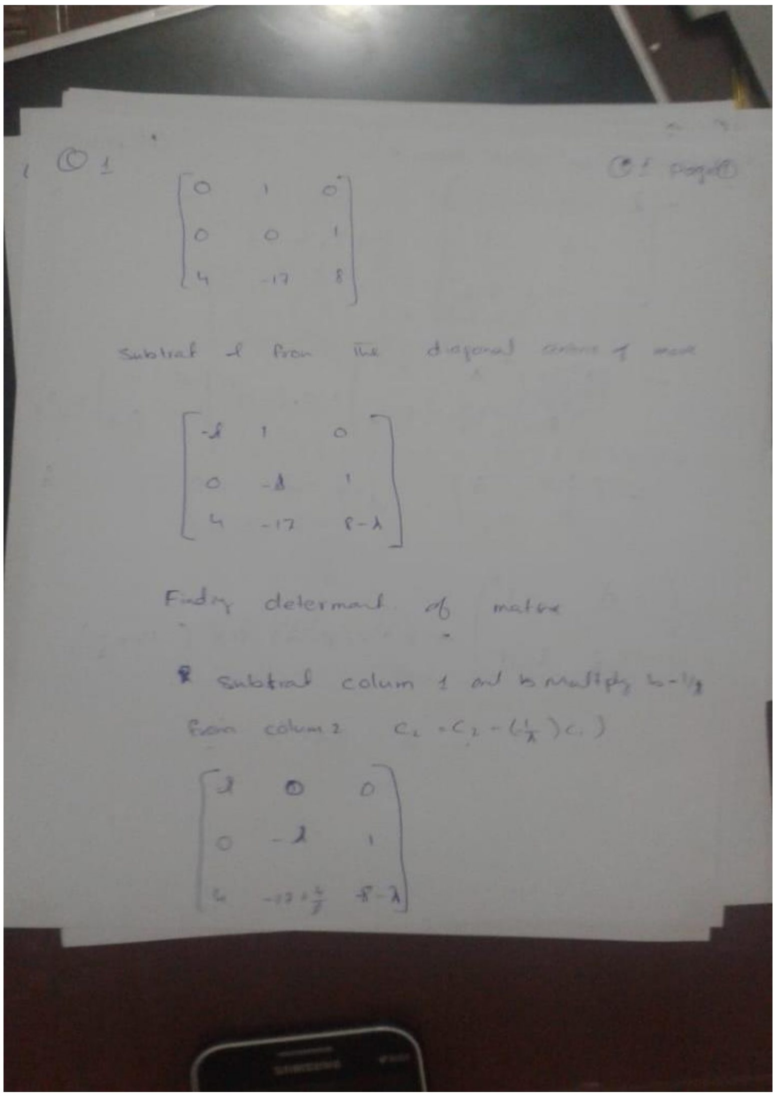
Question No: 1 **10 marks**

Find the eigenvalues of A

0 1 0



Q 1 Q1 page(1)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

Subtract λ from the diagonal entries of matrix

$$\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{bmatrix}$$

Finding determinant of matrix

Subtract column 1 and to multiply to $-1/\lambda$ from column 2 $C_2 = C_2 - (\frac{1}{\lambda})C_1$)

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17+\frac{4}{\lambda} & 8-\lambda \end{bmatrix}$$

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Q1 page②

~~Row 1~~ . as select Row 1

①

$$\begin{bmatrix} \lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17\frac{4}{\lambda} & 8-\lambda \end{bmatrix} = (-\lambda).(-1)^{1+1}$$

$$\Rightarrow 8-\lambda \begin{pmatrix} \lambda & 1 \\ 17\frac{4}{\lambda} & 8-\lambda \end{pmatrix} + 0.(-1)^{1+2}\begin{vmatrix} 0 & 0 \\ 4 & 8-\lambda \end{vmatrix} + 0$$

$$(-1)^{1+3}\begin{vmatrix} 0 & -\lambda \\ 4 & -17\frac{4}{\lambda} \end{vmatrix} = -\lambda \begin{pmatrix} -\lambda & 1 \\ -17\frac{4}{\lambda} & 8-\lambda \end{pmatrix}$$

$$\begin{pmatrix} -\lambda & -1 \\ -17\frac{4}{\lambda} & 8-\lambda \end{pmatrix} = (-\lambda).(8-\lambda)-(1).\left(-17+\frac{4}{\lambda}\right)$$

$$= \lambda^2 - 8\lambda + 17 - \frac{4}{\lambda}$$

② P

①1 page③

$(-\lambda)\cdot\left(\lambda^2-8\lambda+17-\frac{4}{\lambda}\right) = -\lambda\left(\lambda^2-8\lambda+17\right)+4$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 4 & -17 & 8-\lambda \end{vmatrix} = \lambda\left(\lambda^2-8\lambda+17\right)+4$$

$\lambda\left(\lambda^2-8\lambda+17\right)+4 = \lambda^3+8\lambda^2 \mp 17\lambda - 4 = 0$

the root are

$\lambda_1 = 4$

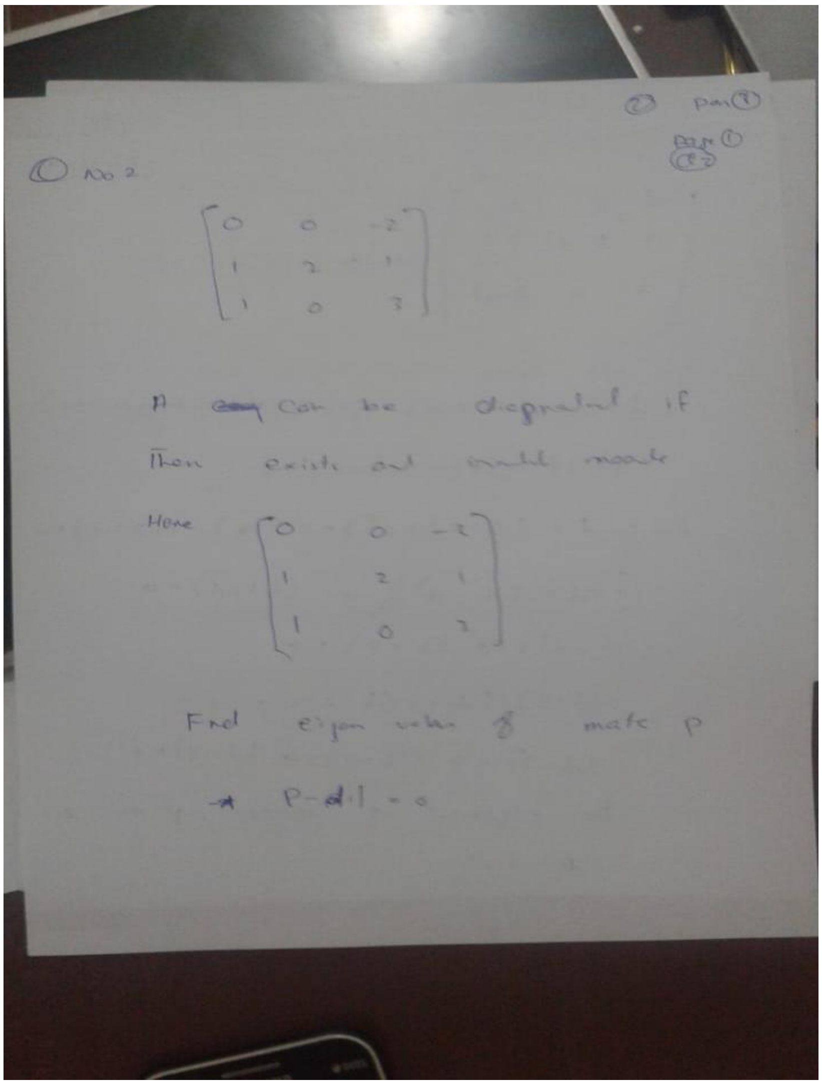
$\lambda_2 = 2-\sqrt{3}$

$\lambda_3 = \sqrt{3}+2$

Ans

Question No. 2 **10 marks**

Find a matrix P that diagonalizes the below matrix



Q No 2

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

A can be diagonalized if

There exists an invertible matrix

Here $\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Find eigen value of matrix P

$\rightarrow$ $|P - \lambda I| = 0$

(1) pan(1)

(1)2 pan(2)

$$\begin{vmatrix} -\lambda & 0 & -2 \\ 1 & (2-\lambda) & 1 \\ 1 & 0 & (3-\lambda) \end{vmatrix} = 0$$

$$(-\lambda)\big((2-\lambda)\times(3-\lambda)-(1\times 0)\big) - 0\big(1\times(3-\lambda)-1\times 1\big)$$

$$+(-2)\big((1\times 0)-(-2-\lambda)\times 1\big) = 0$$

$$(-\lambda)(6-5\lambda+\lambda^2) - 0(2-\lambda) - 2(2+\lambda) = 0$$

$$(-6\lambda+5\lambda^2-\lambda^3) - 0 - (4+2\lambda) = 0$$

$$= \quad -\lambda^3+5\lambda^2-8\lambda+4) = 0$$

$$(\lambda-1)(\lambda-2)(\lambda-2) = 0$$

$$(\lambda-1) = 0 \quad (\lambda-2) = 0 \quad (\lambda-2) = 0$$

The eigenval of matri P are given

$\lambda = 1, 2$

(2) par(9)

* 2 eigvaln 8o $\lambda = 2$

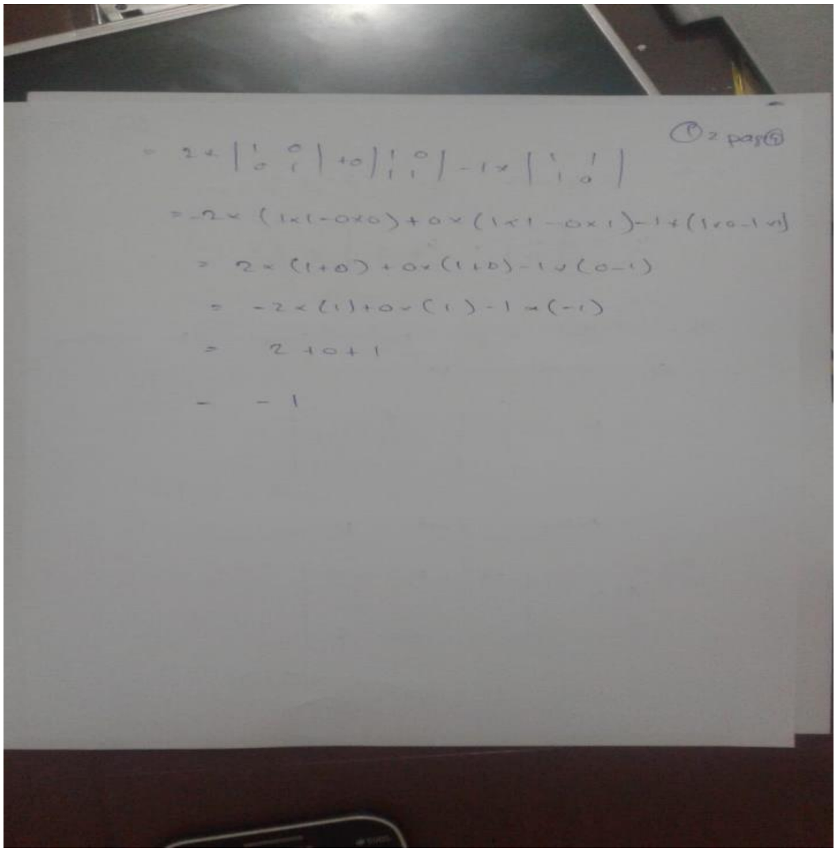
$$v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}.$$

2: The eigen valor compos the colum of marlex P

$$P = \begin{bmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Now 7ind P

$$P = \begin{vmatrix} -2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$



(1) 2 page(5)

$$= 2 \times \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -2 \times (1 \times 1 - 0 \times 0) + 0 \times (1 \times 1 - 0 \times 1) - 1 \times (1 \times 0 - 1 \times 1)$$

$$= 2 \times (1+0) + 0 \times (1+0) - 1 \times (0-1)$$

$$= -2 \times (1) + 0 \times (1) - 1 \times (-1)$$

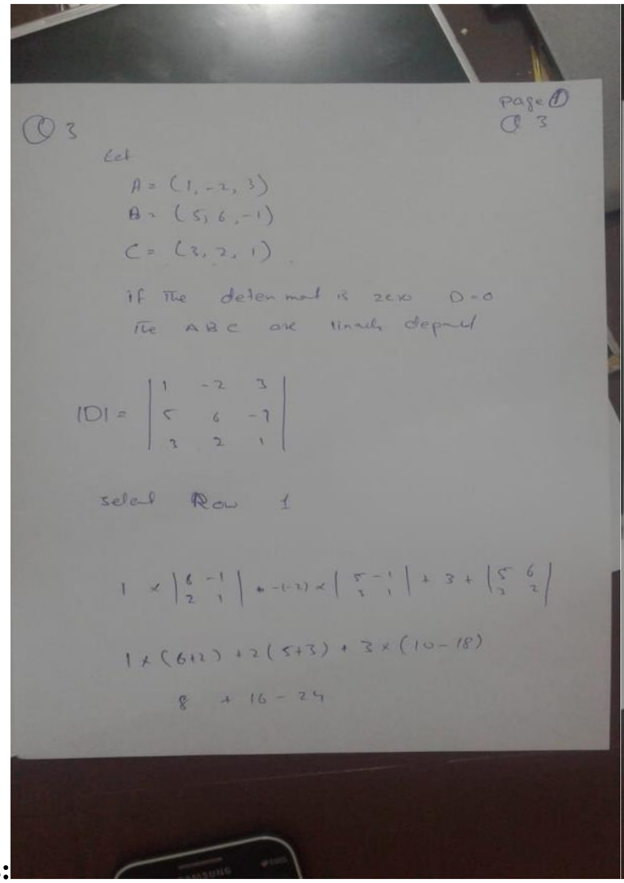
$$= 2 + 0 + 1$$

$$= -1$$

Question No. 3 **10 marks**

Determine whether the vectors form linear dependent or independent sets.

Ans:



Q 3

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Q 3

Let

$A = (1, -2, 3)$
$B = (5, 6, -1)$
$C = (3, 2, 1)$

if the determinant is zero $D = 0$
the A B C are linearly depend

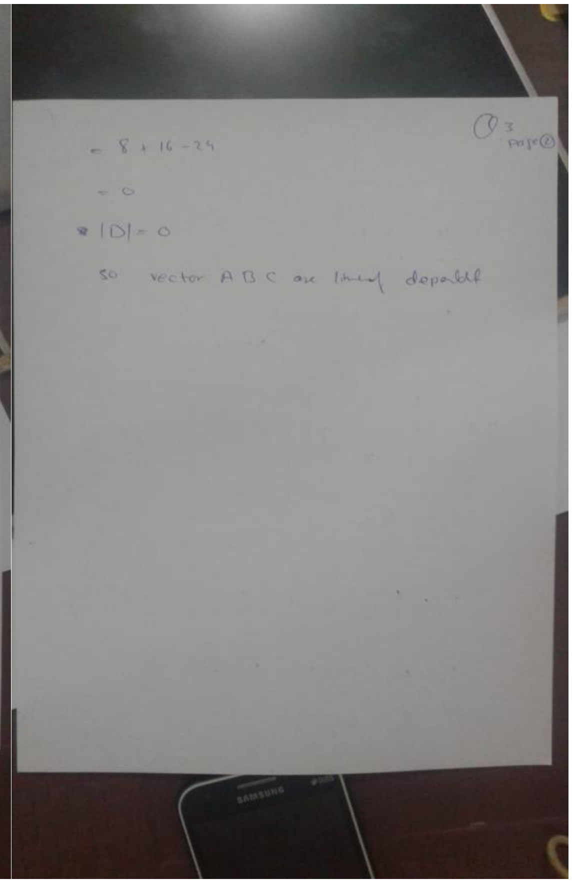
$$|D| = \begin{vmatrix} 1 & -2 & 3 \\ 5 & 6 & -1 \\ 3 & 2 & 1 \end{vmatrix}$$

select Row 1

$$1 \times \begin{vmatrix} 6 & -1 \\ 2 & 1 \end{vmatrix} + -(-2) \times \begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + 3 \times \begin{vmatrix} 5 & 6 \\ 3 & 2 \end{vmatrix}$$

$$1 \times (6+2) + 2(5+3) + 3 \times (10-18)$$

$$8 + 16 - 24$$



Q 3
page ②

$= 8 + 16 - 24$

$= 0$

$\therefore |D| = 0$

so vector A B C are linearly dependent

Question No. 4 **20 marks**

What are the four main things we need to define for a vector space? Which of the following is a vector space over R? For those that are not vector spaces, modify one part of the definition to make it into a vector space.

a. V = { 2 x 2 matrices with entries in R}, usual matrix addition, and

$$k.\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix} for\ k \in R$$

b. V = {Polynomials with complex coefficients of degrees ≤3}, with usual addition and scalar multiplication of polynomials.

Q 4

Q4 page ①

Part a

$V = \{2\times 2$ matrix with entries in $\mathbb{R}\}$,

usual matrix addition and

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & a \end{pmatrix} \text{ for } k \in \mathbb{R}$$

The set $P_n(x)$ of all the polynomial over $\mathbb{R}$ in variable x of degree $\le$ ... from a vector space over $\mathbb{R}$

if

$f_n(x) = a_0 + a_1 x + a_2 x_2 \cdots + a x^n$ and

$g_n(x) = b_0 + b_1 x + b_2 x^2 \cdots b_n x^n$ ...

Then

$f_n(x + g_n(x)) = (a+b) + (a+b)x \ldots (a_n$

x^n is polynomial of $p_n(x)$

The associative addition property.

is ... from the addition

associative property of R

Page ② Q14

The zero polynomial $Z_0(x) = 0$ of degree zero act as the additive identity of $P_n(x)$ and

$$Z_n(x) = a_0 + (a_1)x + \dots + (a_n)x^n$$ is

the additive inverse of $Z_n(x)$

Commutative property follows the commutative property of R - Hence $P(x)$ is additive abelian group

The scalar multiply of $a \in R$ by

$$Z_n(x) = a a_0 (a a_1)x + (a a_2)x_2 \dots x^n \in P_n(x)$$

it observe property 3.2.3 b(i - iv) of Scalar multiplication which can easily be

$P_n(x)$ from a vector space on

R

part b

(3)

Let F be a Field A nonemp

set V together with two binary operations

① {C} From The respective stand 8

a vector space over the Field F a

F?

(a) V From an addition (usc) abstract

gap

(b) The scalar multiplication (·) as a

Function From FV into V observe

The following properties

(i) ∀ a ∈ F, x, y ∈ V, a(x+y) = ax + ay

(ii) ∀ a, B ∈ F, (a+B) x = ax + Bx ∀ x ∈ V

(iii) ∀ a, B ∈ F ∀ x ∈ V : a(Bx) = (aB)x

(iv) 1 ∈ F The identity of F, 1·x = x ∀ x ∈ V