NAME :-

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ID # 7820

SECTION :- A

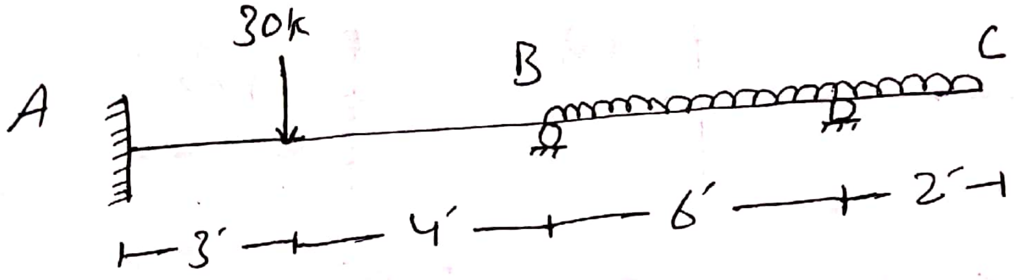
SUBJECT :-

Structural Analysis - II

INSTRUCTOR :-

ENGR. ADEED KHAN

PROBLEM# 1 :



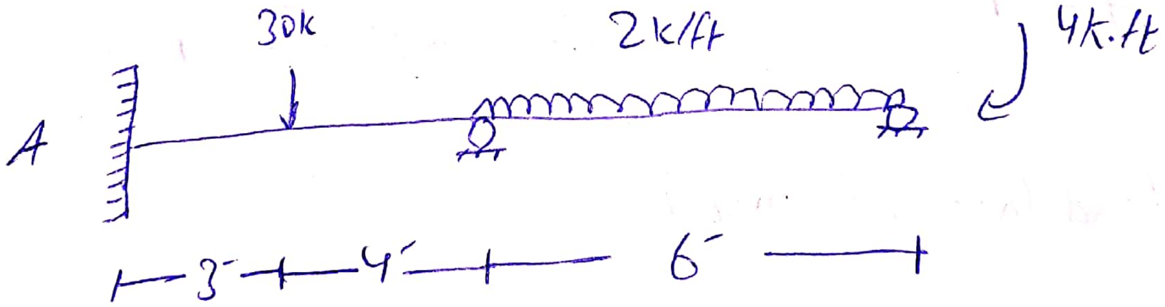
SOLUTION:

Step# 01:

Kinematic Indeterminacy

$K.I = 5^{\circ}$

So we have to reduce the extended portion.



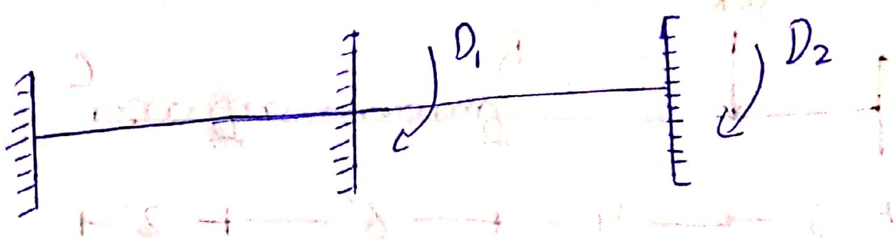
$\Rightarrow \frac{2(2)}{1} = 4 k.ft$

Now,

$K.I = 2^{\circ}$

Step#2:

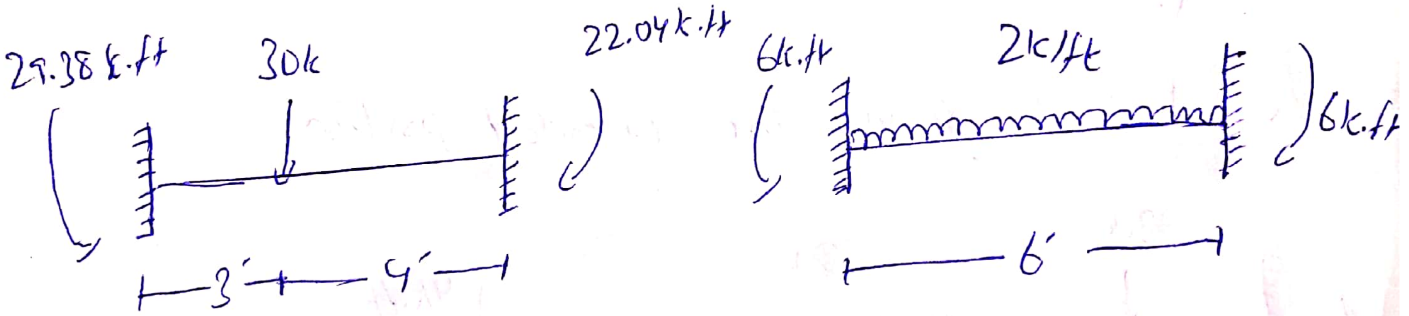
Determine unknown Joint Displacement.



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step#3:

Compute (ADL) matrix



=) For Pointed load (not at mid)

=) For left end:

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k.ft}$$

=) For Right end:

$$= \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

$\Rightarrow$ For 4DL :

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k.ft}$$

$$ADL_1 = +22.04 - 6 = 16.04 \text{ k.ft}$$

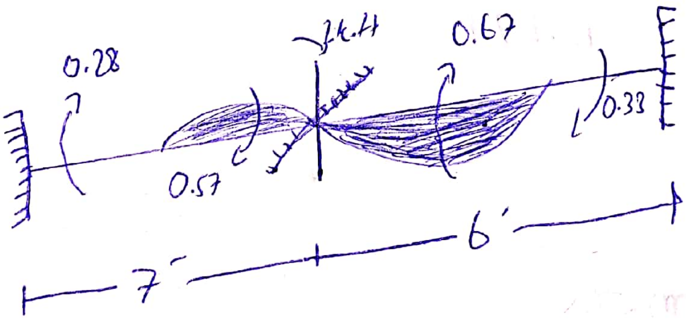
$$ADL_2 = 6 \text{ k.ft}$$

Step # 4 :

Compute (S) matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a) $D_1 = 1k$, $D_2 = 0$



$$\frac{4EI}{7} = 0.57$$

$$\frac{4EI}{6} = 0.67$$

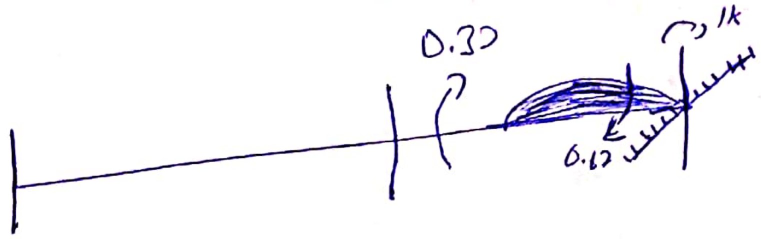
$$\frac{2EI}{6} = 0.33$$

$$\frac{2EI}{7} = 0.28$$

$$S_{11} = 0.57 + 0.67$$
$$= 1.24 EA$$

$S_{21} = 0.33EA$

b) $D_1 = D$, $D_2 = 1k$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$S_{12} = 0.33$

$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step # 5:

Compute (D) matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$

$= 0.8308 - 0.1089$

$|S| = 0.7219$

$$Adj\ A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now,

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0-16.04 \\ 4-6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

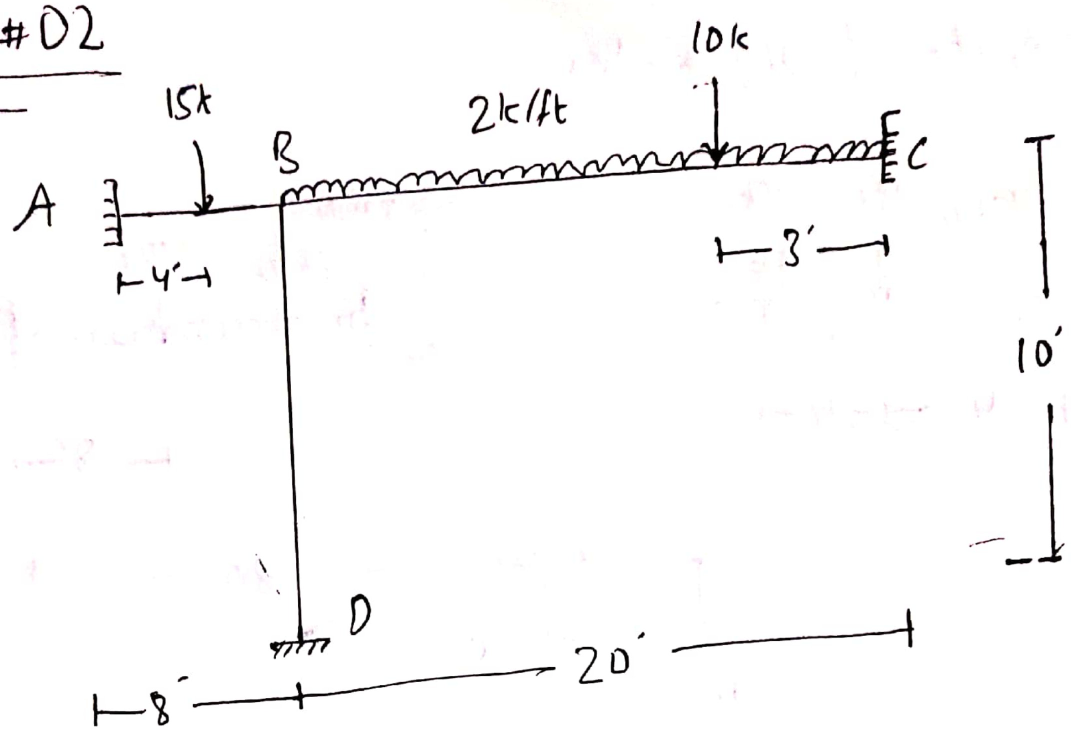
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.93 & -0.45 \\ -0.45 & 1.72 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.93(-16.04) + (-0.4)(-2) \\ -0.45(-16.04) + 1.72(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 14.91 + 0.8 \\ 7.21 - 3.44 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 15.71 \\ 3.77 \end{bmatrix}$$

Problem # 02



Solution:-

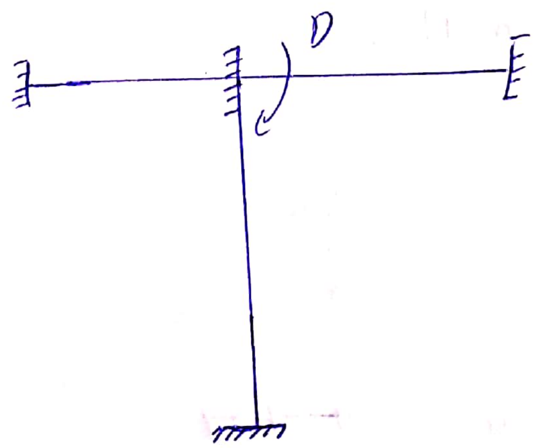
Step # 1:-

Kinematic Indeterminacy

K. I = 1°

Step # 2:-

Determine the unknown joint displacement

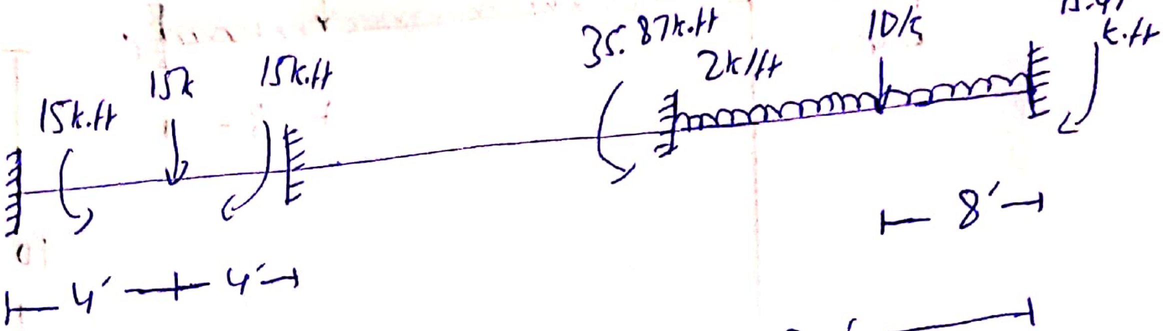


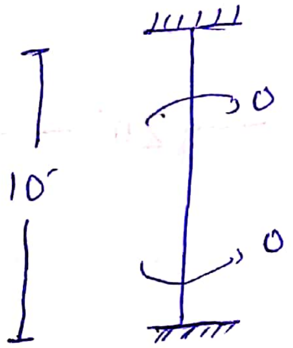
(D) = [?]

(AD) = [0]

Step # 3:

Compute (ADL) matrix:





⇒ Point load at center

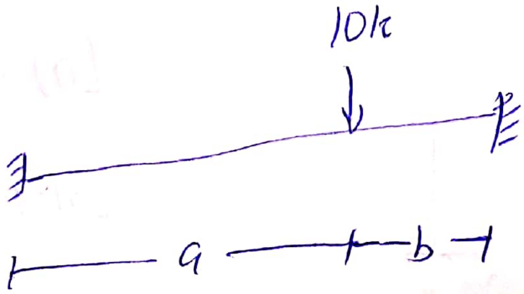
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip.ft}$$

⇒ Uniformly Distributed Load:

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

⇒ Point load (Not at mid):

Suppose



For Left End:

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

For Right End:

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

So Total Moment at left end:

$$19.2 + 66.67 = 85.87 \text{ k.ft}$$

Similarly at right end:

$$28.8 + 66.67 = 95.47 \text{ k.ft}$$

So $$(A_{DL}) = -85.87 + 15 = -70.87 \text{ k.ft}$$

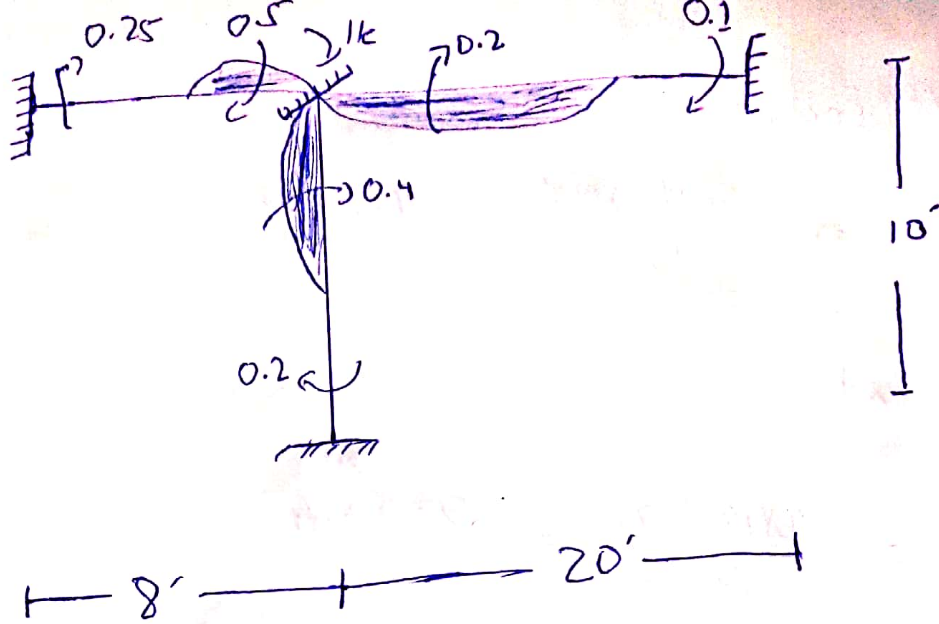
Step #4:

Determine $[S]$ Matrix

$$[S] = [S_{11}]$$

Now, $D = 1k$

P.T.O



$$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2)\,EI$$

$$= 1.1\,EI$$

$$[S] = 1.1\,EI$$

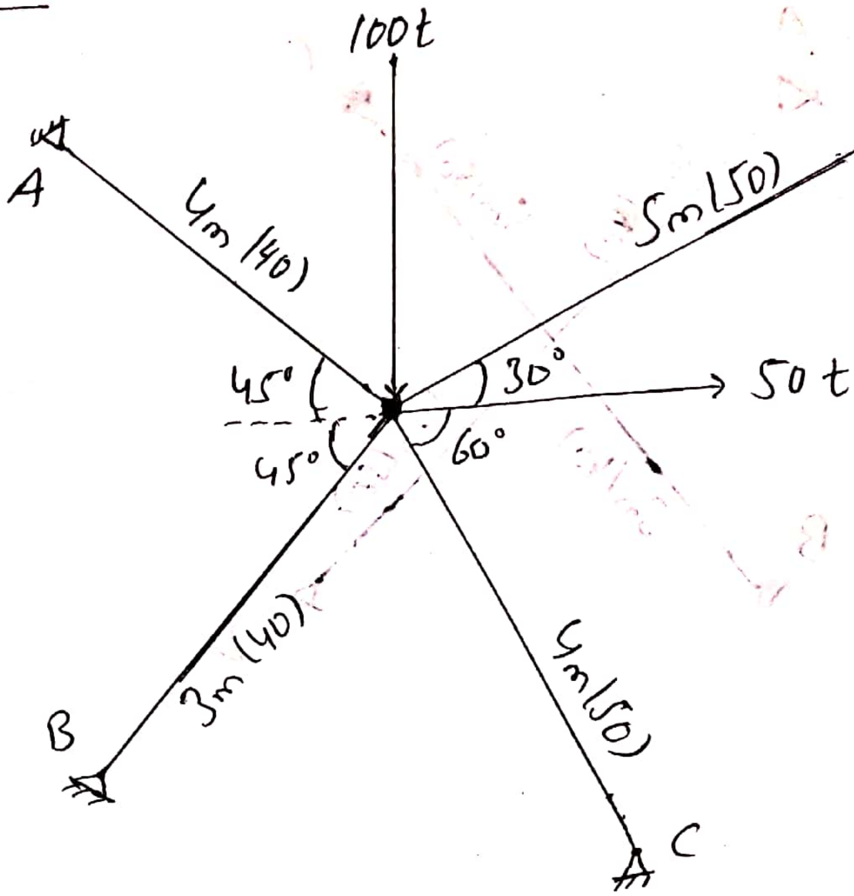
Step #5. Compute [D] matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$= \frac{1}{1.1} \times [0] - [-70.87]$$

$$[D] = [64.42]\,\frac{1}{EI}$$

PROBLEM # 03



SOLUTION :-

For A :-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow \quad P = 2.828 \text{ m}$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow \quad b = 2.828 \text{ m}$$

For B :-

$$\sin 45^\circ = P/3$$

$$\Rightarrow \quad P = 2.12 \text{ m}$$

$$\cos 45^\circ = b/h$$

$$\Rightarrow \quad b = 2.12 \text{ m}$$

For C :-

$$\sin 30^\circ = \frac{P}{5}$$

$$\Rightarrow \quad P = 2.5 \text{m}$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow \quad b = 4.33 \text{m}$$

Now,

$$EA_{(A)} = 2000 \times 40 = 80{,}000 \ t$$

$$EA_{(B)} = 2000 \times 40 = 80{,}000 \ t$$

$$EA_{(C)} = 2000 \times 50 = 10{,}000 \ t$$

$$EA_{(D)} = 2000 \times 50 = 10000 \ t$$

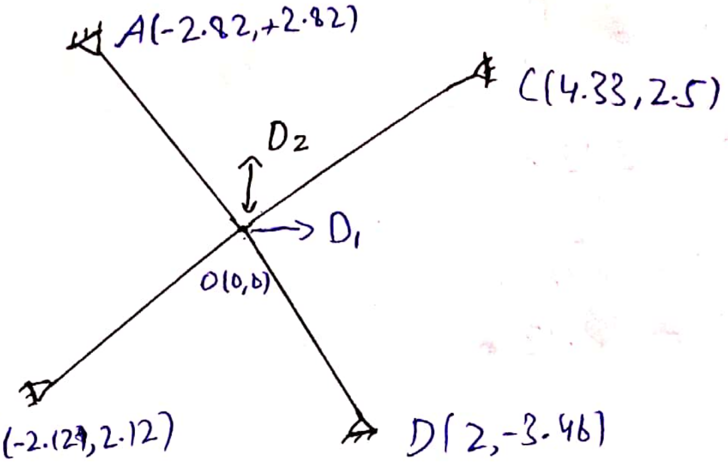
Step # 01:- K.I

$$K.I = 2j - r$$

$$= 2(5) - 8$$

$$= 2^\circ$$

Step # 02 :- Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} , \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 03 :- $[AMD]_{4\times 2}$ & $[S]_{2\times 2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(x_E - x_j)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80000}{(300)^2}(0+212) = 188.44$$

$$AMD_{31} = \frac{100000}{(500)^2}(0-433) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0-200) = -125$$

Now,

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3} (x_k - x_j)^2$$

$$= \frac{80{,}000}{400^3} \times (282)^2 + \frac{80{,}000}{(300)^3} \times (212)^2 + \frac{100{,}000}{(500)^3} \times (-433)^2$$

$$+ \frac{100{,}000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 82.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_E - x_j)(y_k - y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

iii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2}(y_k - y_j)$$

$$AMD_{12} = \frac{80{,}000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{(400)^2}(346) = 216.25$$

Now, $$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(y_k - y_j)^2$$

$$= \frac{80{,}000}{(400)^3}(-282)^2 + \frac{80{,}000}{(300)^3}(212)^2 + \frac{100{,}000}{(500)^3}(-250)^2$$

$$+ \frac{100{,}000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step # 04:

$$(D) = (S)^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step # 05: (AM)

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (-0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136\,t \\ -18.413\,t \\ 1.11\,t \\ -61.498\,t \end{bmatrix}$$