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Subject:- Signals & Systems
Final exams.

Question 1 (a)

Sol:- Let,

$$x(T) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw)\, e^{jwt}\, dw$$

Now diff b.S w.r.t "t"

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(jw) \frac{d}{dt}\{e^{jwt}\} dw$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi} \left[\int_{-\infty}^{\infty} x(jw) \{e^{jwt} . jw\} dw \right.$$

$$\frac{dx}{dt} = (t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{Jw\, x(jw)\} e^{jwt}\, dw$$

$$F\left\{\frac{d}{dt} x(t)\right\} = jw \times jw$$

From the equation we observed that if f/n comes in differential time domain it is multiplied by jw in frequency domain

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(b)

Sol:- $X[z] = 2 - 4z^{-2} + 2z^{-3}$

$H[z] = 3 + z^{-1} + 2z^{-2}$

Now, $Y[z] = H(z) \cdot X(z)$

$= (3 + z^{-1} + 2z^{-1}) \cdot (2 - 4z^{-2} + 2z^{-3})$

$Y[z] = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

Now, Using delay property for finding $Y[n]$

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3] - 6\delta[n-4] + 4\delta[n-5]$

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Question 2

Sol:- As we know

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} f(x)\,dx + \frac{1}{2\pi}\int_{0}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -\pi/2\,dx + 1/2\pi\int_{0}^{\pi} \pi/2\,dx$$

$$a_0 = \frac{-\pi}{4\pi}\left[x\Big|_{-\pi}^{0}\right] + \frac{\pi}{4\pi}\left[x\Big|_{0}^{\pi}\right]$$

$$a_0 = \frac{-\pi}{4\pi}\left[0-(-\pi)\right] + \frac{\pi}{4\pi}\left[\pi-0\right]$$

$$a_0 = \frac{-\pi}{4\cancel{\pi}}(\cancel{\pi}) + \frac{\pi}{4\cancel{\pi}}(\cancel{\pi})$$

$$a_0 = -\frac{\pi}{4} + \frac{\pi}{4}$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\cos nx\,dx$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} \pi/2 \cos nx\,dx$$

$$a_n = \frac{-\pi}{2\pi}\left[\int_{-\pi}^{0} -\cos nx\,dx\right] + \left[\frac{\pi}{2\pi}\int_{0}^{\pi} \cos nx\,dx\right]$$

$$a_n = \frac{-\pi}{2\pi}\left[-\sin nx\Big|_{-\pi}^{0} + \frac{\pi}{2\pi}\left[\sin nx\Big|_{0}^{\pi}\right]\right]$$

$$a_n = \frac{-\pi}{2\pi}\left[-\left[\sin 0 - \sin(-\pi)\right] + \frac{\pi}{2\pi}\left[\sin\pi - \sin 0\right]\right]$$

$$a_n = \frac{-1}{2}\left[-(0-0) + \frac{1}{2}[0-0]\right]$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} -\pi/2 \sin nx\,dx + 1/\pi\int_{0}^{\pi} \pi/2 \sin nx\,dx$$

$$b_n = \frac{1}{\pi}\cdot -\frac{\pi}{2}\int_{-\pi}^{0} \sin nx\,dx + \frac{1}{\pi}\cdot\frac{\pi}{2}\int_{0}^{\pi} \sin nx\,dx$$

$$b_n = 1/\pi \times \pi/2\left[-\cos nx\Big|_{-\pi}^{0} + 1/\pi \times \pi/2\pi\left[\cos\pi - \cos 0\right]\right]$$

$$= \frac{\pi}{2\pi}\left(-1(1+1) + \pi/2\pi\left[-1-1\right]\right)$$

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$$b_n = \begin{cases} \text{"}0\text{"} & \text{if } n \text{ is even} \\ \text{"}\frac{2}{n\pi}\text{"} & \text{if } n \text{ is odd} \end{cases}$$

So,

$$f(x) = \frac{2}{1\pi} \sin x + \frac{2}{3\pi} \sin 3x + \frac{2}{5\pi} \sin 5x + \frac{2}{7\pi} \sin 7x \text{ ---------}$$

Question 3

Solve :- Given that

$$x(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z^2+3z-z-3}$$ Taking "2z" common

$$x(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$$ "Simplifying"

$$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

or we can write

$$\frac{2(z+1)}{z^2+2z-3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

putting

$$2(z+1) = A(z-1) + B(z+3) \Rightarrow \text{eq } (1)$$

Now putting $z = 1$ in eq (1)

$$2(1+1) = B(1+3$$

$$4 = 4B$$

$$\frac{\cancel{4}}{\cancel{4}} = \frac{\cancel{4}B}{\cancel{4}}$$

$$1 = B$$

$$B = 1$$

Now putting $z=-3$ in eq (1)

$$2(-3+1) = A(-3-1)$$

$$-4 = -4A$$

$$A = 1$$

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Now we will put A and B in eq (1)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{(z-1)}$$

$$x(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Inverse Z - Transform

$$X(z) = u[3] + 1(-1)^k$$

x ———— ✓ ———— x

Question 4

Sol:- $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$

$$G(s) = C[SI - A]^{-1} B + B$$

$$= [1 \;\; 2] \left(\left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= [1 \;\; 2] \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$= [1 \;\; 2] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$$

$$= [1 \;\; 2] \frac{1}{s(s+2)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= [1 \;\; 2] \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

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$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$G(s) = \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

[num, den] = ss2tf (A, B, C, D)

[A, B, C, D] = tf2ss [num, den]

x ———————— x

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(Question 5)

Sol:- Fourier transform of the given signal $x(t) = e^{-a(t)}$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a(t)}\, e^{-j\omega t}\, dt.$$

$\therefore$

$$x(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-j\omega)}\left[1-0\right] - \frac{1}{(a+j\omega)}\left[0-1\right]$$

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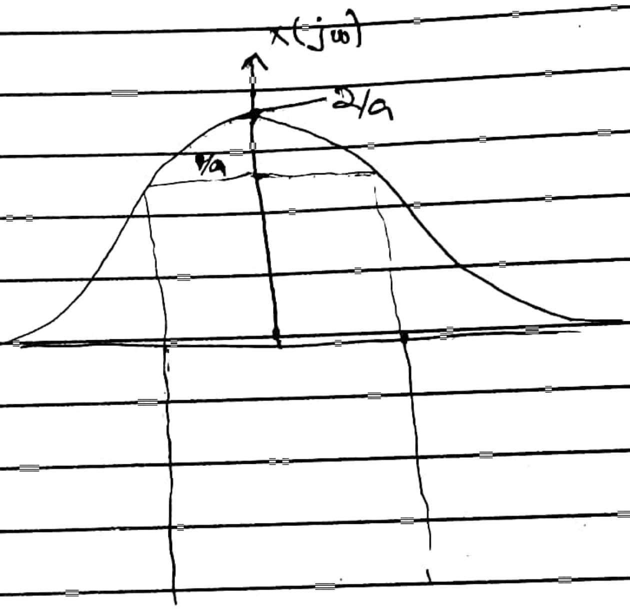
$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2-(j\omega)^2}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}$$



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