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Q2
i Find the Maclaurin's series for

$$Y(x) = x^2 + \sin x$$

Solution:-

$$F(x) = x^2 + \sin(x)$$

	Mac laurin's Series
$F(x) = x^2 + \sin(x)$	$f(0) = 0$
$f'(x) = 2x + \cos x$	$f'(0) = 1$
$f''(x) = 2 - \sin x$	$f''(0) = 2$
$f'''(x) = -\cos x$	$f'''(0) = -1$
$f^{iv}(x) = \sin x$	$f^{iv}(0) = 0$
$f^{v}(x) = \cos x$	$f^{v}(0) = 1$
$f^{vi}(x) = -\sin x$	$f^{vi}(0) = 0$
$f^{vii}(x) = -\cos x$	$f^{vii}(0) = -1$

Mac Laurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5} f^{v}(0) +$$

$$\frac{x^6}{x6!} f^{vi}(0) + \frac{x^7}{7!} f^{vii}(0)$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} \text{------}$$

Q3 ii $\quad y = x^3 (1+x)^9 e^{6x}$

Solution:-

natural log on b.S

$$\ln y = \ln \left[x^3 (1+x)^9 e^{6x} \right]$$

$$\ln y = \ln x^3 + \ln (1+x)^9 + \ln e^{6x}$$

Differentiate on b.S

$$\frac{1}{y}(y') = \frac{3}{x} + \frac{9}{x+1} + 6$$

$$y' = y \left[\frac{3}{x} + \frac{9}{x+1} + 6 \right]$$

$$y = x^3 (1+x)^9 e^{6x}$$

$$y' = \left[x^3 (1+x)^9 e^{6x} \right] \left[\frac{3}{x} + \frac{9}{x+1} + 6 \right]$$

Q3 i $y'' = 1$

$1 + xy = x^2 + y^2$

Using implicit differntiation w.r.t "x"

$0 + xy' + y \cdot 1 = 2x + 2yy'$

$xy' - 2yy' = 2x - y$

$(x-2y)y' = 2x - y$

$y' = \dfrac{2x-y}{x-2y}$ — ①

differentiate again using quotiont rule

$$y'' = \frac{(x-2y)(2-y') - (2x-y)(1-2y')}{(x-2y)^2}$$

$$y'' = \frac{(x-2y)\left(2 - \frac{2x-y}{x-2y}\right) - (2x-y)\left(1 - 2\frac{2x-y}{x-2y}\right)}{(x-2y)^2}$$

$$= \frac{\frac{\cancel{(x-2y)}(\cancel{2x} - 4y - \cancel{2x} + y)}{\cancel{(x-2y)}}}{}$$

$$= \frac{-3y - \frac{(2x-y)(-3x)}{(x-2y)}}{(x-2y)^2}$$

$$= \frac{-3y (2x-y) - (-6x^2+3xy)}{(x-2y)(x-2y)^2}$$

$$= \frac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)^2}$$

$$= \frac{-6xy + 6x^2 + 6y^2}{(x-2y)^2}$$

$$y'' = \frac{6(xy - x^2 - y^2)}{(x-2y)^2}$$

Q 1

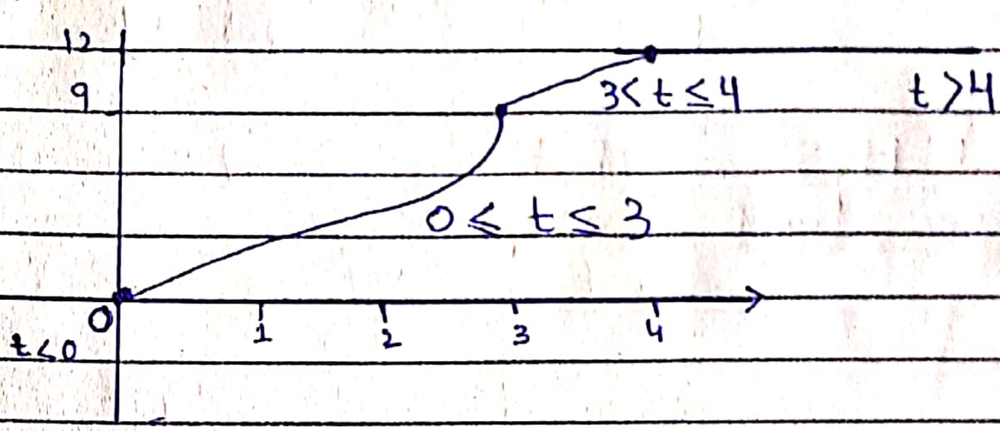
The functiong $g(t)$ is defined by

$g(t) = 0 \quad t < 0$
$t^2 \quad 0 \le t \le 3$
$2t+3 \quad 3 < t \le 4$
$12 \quad t > 4$

a. state any point of discontinuity

$g(t) = 0 \quad t < 0$
$t^2 \quad 0 \leq t \leq 3$
$2t+3 \quad 3 < t \leq 4$
$12 \quad t > 4$

graph :-



From the graph, we conclude that the function $g(x)$ is continuous at every point. There is no point of discontinuity.

Q1 b

Find if they exist

$\lim_{t \to 3} g$

$$g(t) = \begin{cases} t^2 & 0 \leq t \leq 3 \\ 2t+3 & 3 < t < 4 \end{cases}$$

L.H.S

$$\lim_{t \to 3} g(\cancel{t}) = \lim_{t \to 3} (t^2)$$

$$= (3)^2 = 9$$

R.H.S

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} (2t+3)$$

$= 2(3) + 3$

$= 9$

L.H.S = R.H.S

So limit exists