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Department # BEE
Semester # 12th
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Question no: 1 a Part.

Solution:

The equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2.2$$

Hence

$$y^h(n) = c_1 2^n + c_2 n 2^n$$

The particular solution is

$$y^p(n) = k(-1)^n u(n)$$

Substituting this solution in to difference equation obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For

$$n = 2,\ k(1+4+4) = 2$$

$k = 2/9$ The total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from the initial conditions.

we obtain. $y(0) = 1,\ y(1) = 2$ then

$$c_1 + 2/9 = 1$$

$$\Rightarrow c_1 = 7/9$$

$$2c_1 + 8c_2 - 2/9 - 2$$

$$\Rightarrow c_2 = \frac{1}{3}$$

Question no 1

B Part.

Solution:

The characteristics equation is

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = \frac{1}{2}, \frac{1}{3}$ Hence

$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$

with $x(n) = \delta(n)$, we have

$y(0) = 2$

$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$

$\Rightarrow y(1) = 1.4$

Hence

$C_1 + C_2 = 2$ and

$\frac{1}{2}C_1 + \frac{1}{5}C_2 = 1.4$

$= 7/5$

$C_1 + \frac{2}{5} C_2 = 14/5$

The equation yield

$C_1 = 10/2, C_2 = \frac{-4}{3}$

$h(n) = [\frac{10}{3}(\frac{1}{2})^n \cdot 4/3(\frac{1}{5})^n] u(n)$

x —— x —— x —— x

Question no 2.
A Part.

Sol:

$$x(z) - \frac{1}{1-2z-1} (1.z^{-1})^2$$

$$x(z) = \frac{1}{(1-2z-1)(1.z-1)^2}$$

$$= \frac{A}{(1-2z-1)} + \frac{B}{(1-z-1)} + \frac{Cz-1}{(1-z^{-1})^2}$$

$A = 4, B = -3, c = -1$

Hence, $x(n) = [4(2)^n - 3 - n]u(n)$

X X X

Question no 2
B Part:

Sol:

we have

$x(n) = \frac{1}{2\pi j} \oint \cdot \frac{z^n - 1}{1 - az^{-1}} \, d \cdot z = \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$

where "C" is a circle at radius greather than $|a|$ we shall evaluate this integral using $f(z) = z^n$ we distingiush two cases

if $n \geq 0$, $f(z)$ has only zeros and no pole inside "c" The only pole "c" is $z = a$ Hence

if $n < 0$ $f(z) - z^n$ has n^{th} order poles at $z = 0$ which is also inside "C" Thus these are condributions from both poles

For $n = -1$ we have

$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a} \Big|_{z=0} + \frac{1}{z^2} \Big|_{z=a} = 0$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$ thus

$x(n) = a^n u(n)$

x x x x

Question no = 3

a Part

Sol:

At $\omega = 0$ we have

$$H(0) - \frac{b_0}{(1-P)^2} = 1$$

Hence

$$b_0 = (1-P)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^3}{1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{1-P\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^4}{[(1-P/\sqrt{2})^2 + P^2/2]^2} = \frac{1}{2}$$

as equivalentry

$$\sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

The valve of $P = 0.32$ satifies this equation Consequently the system function for the disired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can be applied for the design of bandpass fiters.

x — x — x — x — x —

Question no 3

B Part:

Sol:

Clearly the filters must have poles at

$Pb_2 = re^{\prime} s\mathcal{U}k_{12}$

and zeros at $z = 1$ and $z = -1$

consequently the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr}$$

$$\Rightarrow G \frac{z_2 - 1}{z^2 + r^2}$$

The gain factors is determined by evaluating the frequency respons $H(z_0)$

$$H(\pi/2) = \frac{G2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating. $H(\omega)$ at $\omega = 4\pi/9$ Then

$$|H(\frac{4\pi}{9})|^2 = (\frac{1-r^2}{4})^2 \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

The value of $r^2 = 0.7$ satisfies this equation there fore the system function desired filter is

$$H(Z) = 6.15 \frac{1-Z^{+8}}{1+0.7Z^{2}}$$

x — x — x — x

Question no 4 .

A Part.

Sol:

A finite duration squence of length L is given as

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{allies wise} \end{cases}$$

Solution:

The fouster tronform of this sequences

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega}$$

$$\sum_{n=0}^{L-1} e^{jm} = \frac{1-e^{-j\omega L}}{1-e^{j\omega}}$$

$$\frac{\sin(\omega L/2)}{\sin(\omega L/2} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated $L=10$ The N-Point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequency ωk $2\pi k/N$

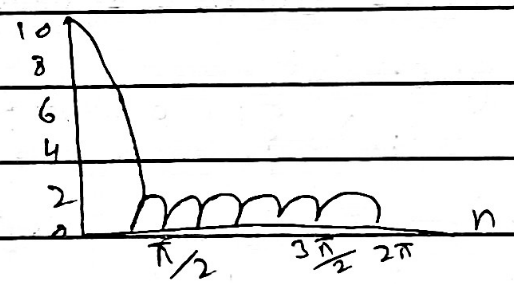
$k = 0, 1 \ldots\ldots\ldots N-1$ Hence

$$X(k) = \frac{1-e^{-j2\pi k L/N}}{1-e^{-j2\pi k/N}} \qquad k = 0, 1, \ldots N-1$$

$$= \frac{\sin(\pi k L/N)}{\sin(\pi k/N)} e^{j\pi k(1-1)/N}$$

if N is selected such that $N=2$ then DFT

$$X(k) = \begin{cases} L, & k=0 \\ 0, & k = 1, 2 \ldots L-1 \end{cases}$$



Thus there is only one non zero value in DFT this apparent from observation of $X(\omega)$ since $X(\omega)=0$

at the frequences $\omega k = 2\pi k/L$

~~k≠~~ $k \neq 0$

x ——— x ——— x

Question 4

B Part.

$x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$

Sol:

Each sequence consits of four zero points for the purposes of illustracting the operation involved in circular convalution it is describable to graph each sequance as paint on a circle Thus the sequances $x_1(n)$ and $x_2(n)$ are graphed Now $x_3(m)$ is obtained by circularly can volving $x_1(n)$ and $x_2(n)$

$$x = (0) = \sum_{x=+}^{3} x_1(n) x_2((1-n))N$$

$x^2(1-n)$ is simply the squence $x_2(n)$ Folded and graphed in a circle

$$\boxed{x_3(0) = 14}$$

For $m = 1$ we have

$$x_2(1) = \sum_{x=0}^{3} x_1(n) x_2((1-n))_4$$

it is easily verified that $x((1-n))_4$ is simply the squence $x_2((1-n))_4$ rotated by one unit in time as illustrated

for $m=2$

we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n))_4$$

Now $x_2(2-n))_4$ is the folded squence.