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Iqra National University, Peshawar
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Mid – Term Examinationspring2020
Date:

Course Code:	MTH 102			**Course Title:**	Calculus and analytic geometry		
Prerequisite:				**Instructor:**	HIMAYATULLAH		
Module:	3	**Program:**	BEE	**Total Marks:**	30	**Time Allowed:**	90 min

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	. **Identify** $\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y=\left(x+\frac{1}{x}\right)(x-\frac{1}{x}+1)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s=160t-16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y=x^4-2x^2+2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

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Name: Sajid Ahmad Module: 1st Semester

I D: U671 Program: BE(E)

Subject: Calculas & Analytic Geometry Instructor: Sir Himayatullah

Q1: (a)

Identify $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Solg→

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0}$$

$$= \frac{0}{0} \Rightarrow \%\ \text{form}$$

So then

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Xing & ÷ing by $\sqrt{2+h} + \sqrt{2}$

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$\lim_{h \to 0} \frac{2 + h - 2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})}$$

$$\lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

Apply limit

$$= \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \boxed{\frac{1}{2\sqrt{2}}}$$ Ans.

Q1 (b) Find the first order derivatives of the function $y = (x + \frac{1}{x})(x - \frac{1}{x} + 1)$

Solg:-

$$y = (x + \frac{1}{x})(x - \frac{1}{x} + 1)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x + \frac{1}{x})(x - \frac{1}{x} + 1)$$

Product Rule

$= (x + x^{-1}) \frac{d}{dx}(x - x^{-1} + 1) + (x - x^{-1} + 1) \frac{d}{dx}(x + x^{-1})$

$= (x + x^{-1})(1 + x^{-2}) + (x - x^{-1} + 1)(1 - x^{-2})$

$= \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right)\left(1 - \frac{1}{x^2}\right)$

$= x + \cancel{x \frac{1}{x^2}} + \cancel{\frac{1}{x}} + \frac{1}{x^3} + x - \cancel{x \frac{1}{x^2}} - \cancel{\frac{1}{x}} + \frac{1}{x^3} + 1$

$= 2x + 1 - \frac{1}{x^2} + \frac{1}{x^3}$ Ans.

$- \frac{1}{x^2}$

(a) A dynamite blast blows up a heavey rock with launch velocity of 160 m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec.

Sol:-

$s = 160t - 16t^2$ ft

(i) Velocity is:

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$= \frac{d}{dt}160t - \frac{d}{dt}16t^2$$

$$v = 160 - 32t$$

Maximum height

$$v = 0$$

So

$$160 - 32t = 0$$

$$\frac{160}{32} = \frac{32t}{32}$$

$$\boxed{t = 5 \text{ sec}}$$

$$S_{max} = 5(5) = 160(5) - 16(5)^2$$

$$S_{max} = 400 ft$$

(ii) Given that:

$$S = 256 ft$$

then

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

(iii) Find the acceleration of the rock at time 5 sec.

Sol:→

Since

$$V = 160 - 32t$$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$

$$a = 0 - 32 \text{ m/s}^2$$

$$\boxed{a = -32 \text{ m/s}^2}$$

Q 3 (4) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where?

Sol:→

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx}x^4 - \frac{d}{dx}2x^2 + \frac{d}{dx}2$$

(5)

$$\frac{16}{16}(t^2 - 10t + 16) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 0$$

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t-8) - 2(t-8) = 0$$

$$(t-8)(t-2) = 0$$

$$t = 8, \quad t = 2,$$

$$t_1 = 8\text{sec}, \quad t_2 = 2\text{sec}$$

Since

$$V = 160 - 32t$$

$t_1 = 2s$

$$V_{(2)} = 160 - 32(2)$$

$$= 160 - 64$$

$V_{(2)} = 96 m/sec \Rightarrow$ upward velocity

$t_2 = 8s$

$$V_{(8)} = 160 - 32(8)$$

$$= 160 - 256$$

$= -96 m/s \Rightarrow$ downward velocity.

$$= 4x^3 \frac{dx}{dx} - 2x\,2x \frac{dx}{dx} + 0$$

$$= 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the Tangent is horizontal then

$$\frac{dy}{dx} = 0$$

So

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0, \quad x^2 - 4 = 0$$

$$\frac{x^4}{4} = \frac{0}{4}, \quad x^2 - 1 = 0$$

$$x = 0, \quad x^2 - 1$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = 0, \quad x = \pm 1$$

So

$$x = 0, 1, -1$$

Their corresponding point in.

$y = x^4 - 2x^2 + 2$

for $x = 0$

$y = x^4 - 2x^2 + 2$

$= (0)^4 - 2(0)^2 + 2$

$= 0 - 0 + 2$

$\boxed{y = 2}$

* for $x = 1$

$y = x^4 - 2x^2 + 2$

$y = (1)^4 - 2(1)^2 + 2$

$y = 1 - 2 + 2$

$\boxed{y = 1}$

$x = -1$

$y = x^4 - 2x^2 + 2$

$= (-1)^2 - 2(-1)^2 + 2$

$= 1 - 2 + 2$

$\boxed{y = 1}$

Hence, $(0, 2)$, $(1, 1)$, $(-1, 1)$ point