

NAME :- Malik . m . Afnan

ID :- 7839

Section :- "B"

Department :- Civil Engineering

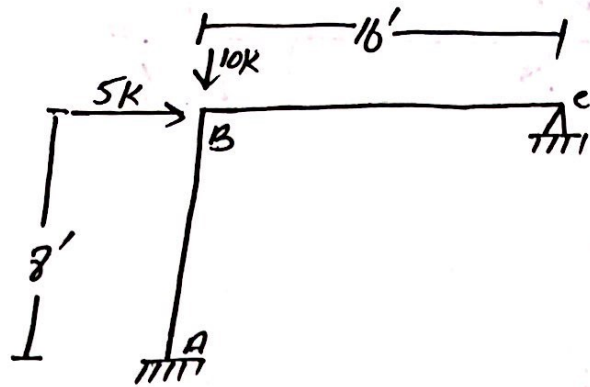
Subject :- Structure II

Date :- 21/08/2020



Q No 3

(1)



$$E = \text{Constant}$$

$$I = \bar{I}$$

$$I_B = 2I$$

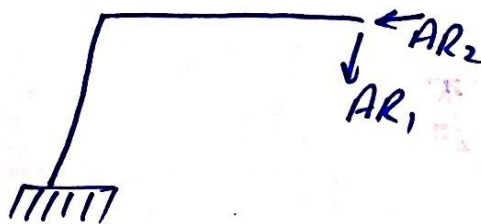
Solution:-

Total Statical indeterminacy

$$\Rightarrow R - 3 = 5 - 3 = 2^{\circ}$$

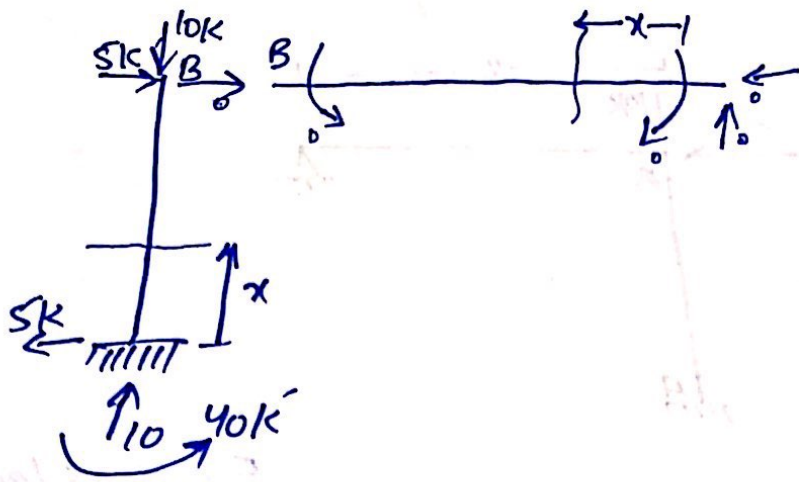
Step # 01:-

Identify Redundant Actions

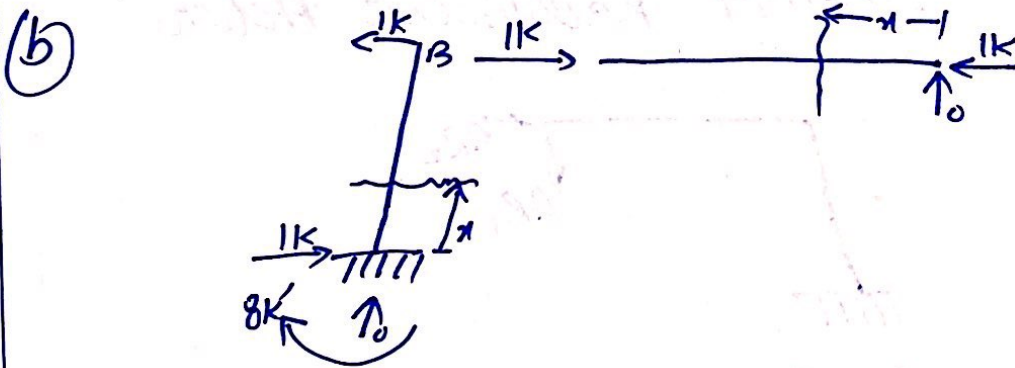
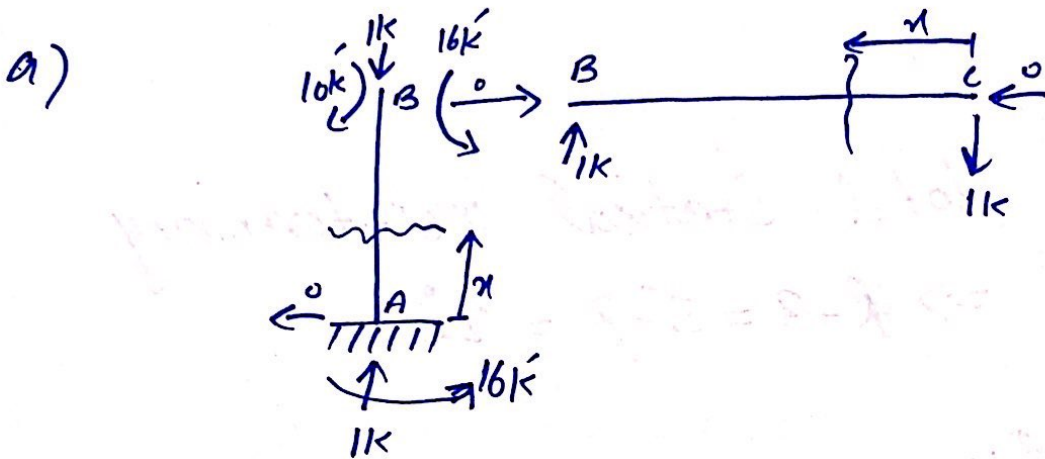


$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step # 2:- Compute value of [DRL]



Step #03:-  $[F]$  or  $[AMR]$



Member	AB	BC
Origin	A	C
Limit	0-8	0-16
$I$	$I$	$2I$
$M$	$5x-40$	0
$M_1$	-16	$x$
$M_2$	$8-x$	0

⇒ For finding the value of DRL:-

$$\begin{aligned}
 DRL_1 &= \int_0^8 \frac{M_{AB} \cdot M_1(AB)}{EI} dx + \int_0^{16} \frac{M_{BC} \cdot M_2(BC)}{EI} dx \\
 &= \int_0^8 \frac{(5x-40)(-16) dx}{EI} + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx
 \end{aligned}$$

$$\boxed{DRL_1 = \frac{2560}{EI}}$$

$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x) dx}{EI} + \int_0^{16} \frac{0 \cdot 0}{E(2I)} dx$$



$$DRL_2 = \frac{-853.33}{EI}$$

(4)

Compute Flexibility Matrix :-

$$F_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$\Rightarrow F_{11} = \int_0^8 \frac{m_1^2(AB)}{EI} dx + \int_0^{16} \frac{m_1^2(BC)}{EI} dx$$

$$= \int_0^8 \frac{(-16)^2}{EI} dx + \int_0^{16} \frac{x^2}{E(2I)} dx$$

$$F_{11} = \frac{2730.67}{EI}$$

$$F_{12} = F_{21} = \int_0^8 m_1(AB) \cdot m_2(AB) dx + \int_0^{16} m_1(BC) \cdot m_2(BC) dx$$
$$= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{2EI} dx$$

$$F_{12} = \frac{-512}{EI}$$

$$F_{22} = \int_0^8 (m_1)^2_{AB} dx + \int_0^{16} (m_1)^2_{BC} dx$$

$$= \int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{2EI} dx$$

$$F_{22} = 170.67$$

As we know

$$[DRS] = [DRL] + [AR] * [F]$$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$2) [AR] = [F]^{-1} * [DRS - DRL]$$

$$= \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} * \begin{bmatrix} 0 & -2560 \\ 0 & 253.33 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00005 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Q No 2

Ans

There are two main methods of Structural analysis using the matrix approach.

- i) Force Method.
- ii) Displacement Method.

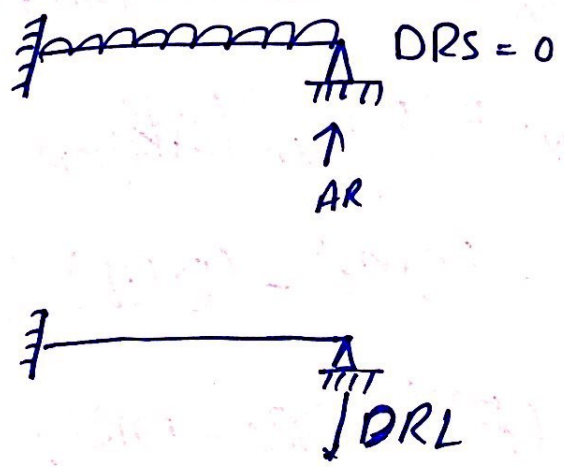
Force Method:-

This method is also known as Flexibility or Compatibility method. In this method the degree of static indeterminacy of the structure is determined and the redundants are identified. A Co-ordinate is assigned to each redundant. Thus  $AR_1, AR_2, \dots, AR_n$  are the redundants at co-ordinate  $1, 2, \dots, n$ . If all the the redundants are removed the resulting structure known as the

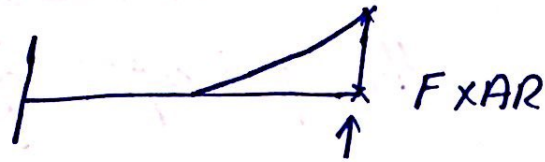


as the released strc, is statically determinate from the principle of superposition

The net displacement at any point in a statically determinate strc. is the sum of the displacement in the basic determinate strc due to applied loads and the redudants - This condition, know as compatibility condition may be expressed by the equations for "n" redudant actions.







$$DRS = DRL + F \times AR$$

$$DRS_1 = DRL_1 + F_{11}AR_1 + F_{12}AR_2 + \dots + F_{1n}AR_n$$

$$DRS_2 = DRL_2 + F_{21}AR_1 + F_{22}AR_2 + \dots + F_{2n}AR_n$$

$$DRS_n = DRL_n + F_{n1}AR_1 + F_{n2}AR_2 + \dots + F_{nn}AR_n$$

Writing these equation in matrix form

$$\begin{bmatrix} DRS_1 \\ DRS_2 \\ \dots \\ DRS_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} DRL_1 \\ DRL_2 \\ \dots \\ DRL_n \end{bmatrix}_{n \times 1} + \begin{bmatrix} F_{11} & F_{12} & \dots & F_{1n} \\ F_{21} & F_{22} & \dots & F_{2n} \\ \dots & \dots & \dots & \dots \\ F_{n1} & F_{n2} & \dots & F_{nn} \end{bmatrix} \begin{bmatrix} AR_1 \\ AR_2 \\ \dots \\ AR_n \end{bmatrix}$$

$$[DRS]_{n \times 1} = (DRL)_{n \times 1} + (F)_{n \times n} (AR)_{n \times 1}$$

$$(F)(AR) = (DRS) - (DRL)$$

$$AR = (F)^{-1} (DRS - DRL)$$

$n =$  Degree of indeterminacy

where,  $DRS =$  Support Settlement/Rotation

Corresponding to the redundant action

DRL = Displacement (Rotation/translation)

Corresponding to the redundant action in a released Strc. (Basic det str) due to applied loads.

AR = The redundant actions.

F = Flexibility Co-efficient i.e., displacement caused by unit action.

Differentiate between Force And Displacement Method

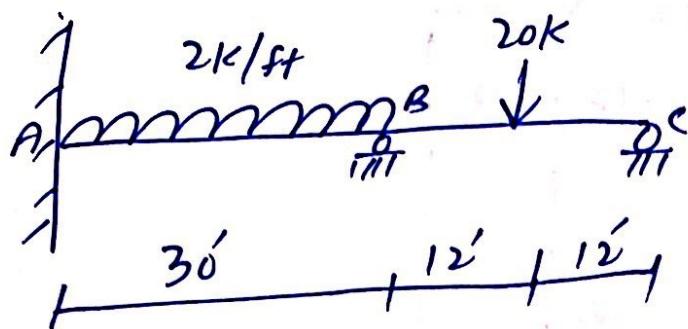
Force Method

- 1)  $D_s < D_k$
- 2) Force are redundant or unknowns
- 3) Starts with equilibrium of force
- 4) Number of redundants =  $D_s$
- 5) Forces found by compatibility of displacement

Displacement Method.

- $D_s > D_k$
- Displacement are redundants or unknowns.
- Starts with Compatible deformation.
- Number of redundants =  $D_k$
- Displacement found by equilibrium equation of forces.





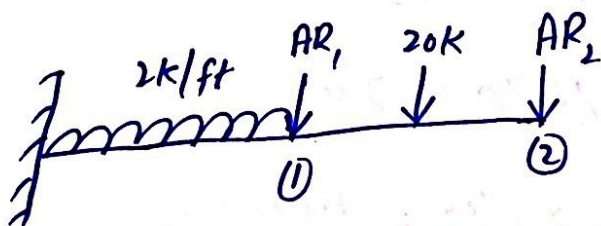
$EI = \text{Constant}$

Solution:-

Structural Indeterminacy = 2

Step # 01:-

Select Redundant Actions

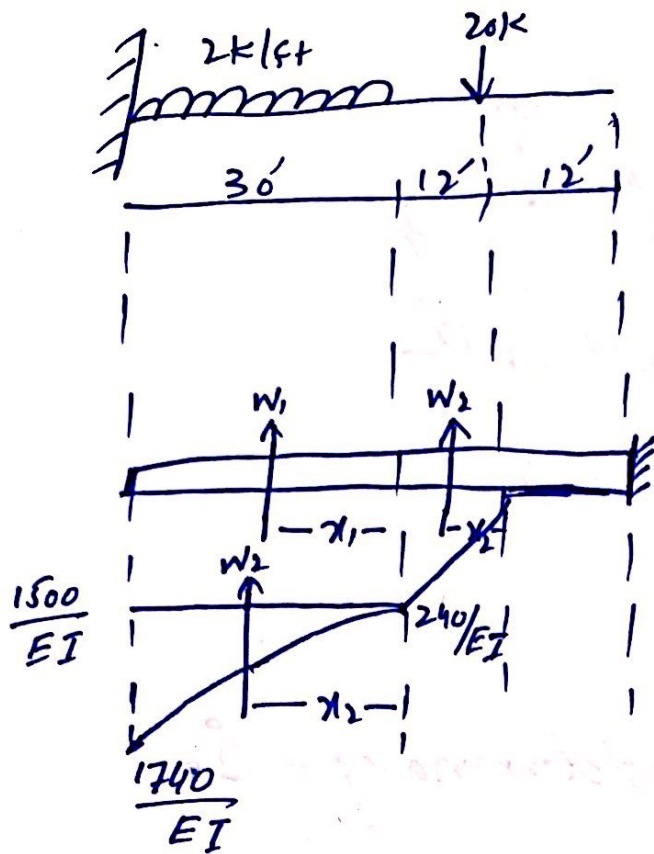


$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL] + [F] \times [AR]$$

Step # 2:- Compute the value of  $[DRL]$





$$20 \times 12 = 240$$

$$20 \times (12 \times 30 + 2 \times 30 \times 15) = 1740$$

$$W_1 = 1500 \times 30 = 45000$$

$$W_2 = \frac{1}{3} \times 30 \times 240 = 2400$$

$$W_3 = \frac{1}{2} \times 12 \times 240 = 1440$$

$$x_1 = \frac{b}{2} = \frac{30}{2} = 15'$$

$$x_2 = \frac{3}{n+2} \times L = \frac{3}{2+2} \times 30 = 22.5'$$

$$x_3 = \frac{2}{3} \times L = \frac{2}{3} \times 12 = 8'$$

Now Finding DRL:-

$$\begin{aligned} DRL_2 &= w_1 \times (x_1 + 24) + w_2 \times (x_2 + 24) + w_3 \times (x_3 + 12) \\ &= 45000(15 + 24) + 2400(22.5 + 24) + 1440(8 + 12) \\ &= 1755000 + 111600 + 28800 \end{aligned}$$

$$DRL_2 = 1895400/EI$$

$$\begin{aligned} DRL_1 &= w_1(x_1) + w_2(x_2) \\ &= 45000(15) + 2400(22.5) \\ &= 675000 + 54000 \end{aligned}$$

$$DRL_1 = 729000$$

So

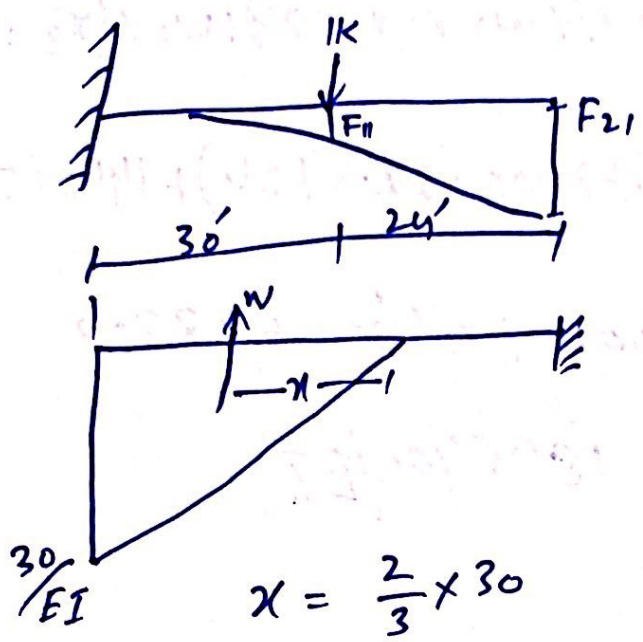
$$DRL = \frac{1}{EI} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

Step #3:-

Flexibility Matrix

$$[F]_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

(9) Applying unit load on AR<sub>1</sub>



$$x = \frac{2}{3} \times 30$$

$$x = 20'$$

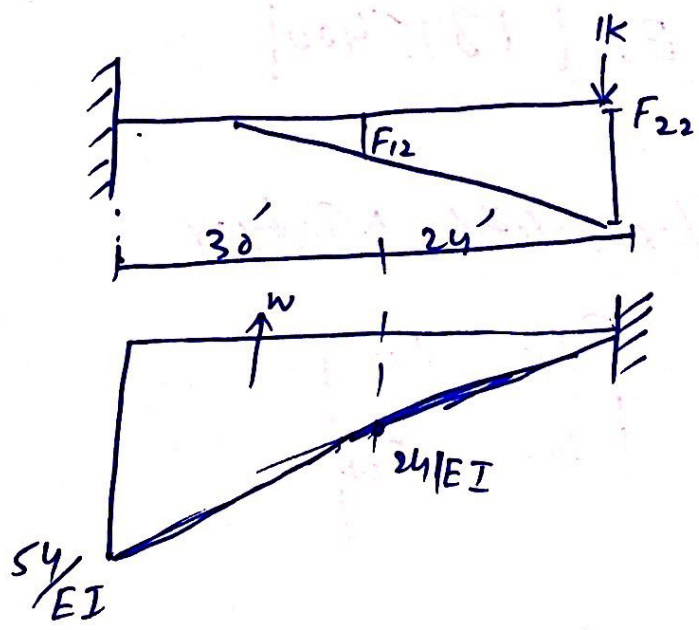
$$W = \frac{1}{2} \left( \frac{30}{EI} \times 30 \right)$$

$$W = 450/EI$$

So,  $F_{11} = \frac{450}{EI} (20) = 9000/EI$

$$F_{21} = \frac{450}{EI} (20+24) = 19800/EI$$

Now Apply unit load on AR<sub>2</sub> :-



$$W = \left( \frac{54+24}{2EI} \right) \times 30$$

$$W = 1170/EI$$



Now the distance .

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$$x = \frac{L}{3} \left[ \frac{b + 2(a)}{a+b} \right]$$

$$= \frac{30}{3} \left[ \frac{24 + 2(54)}{54 + 24} \right] = 16.92'$$

$$\Rightarrow F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92 + 24) = \frac{47876.4}{EI}$$

Hence

$$F_{2 \times 2} = \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix} \frac{1}{EI}$$

Step #4:- Compute the value of AR

$$[DRS] = [DRL] + [F] \times [AR]$$

$$[AR] = [DRS - DRL] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{|F|} \times \text{Adj} F$$

$$= \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{vmatrix}} \times \text{Adj} \begin{bmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{bmatrix}$$

$$\begin{aligned} |F| &= (9000 \times 47876.4 - 19796.4 \times 19800) \\ &= (430887600 - 391968720) \end{aligned}$$

$$|F| = 38918880$$

$$\Rightarrow \text{Adj } A = \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 0 - 729000 \\ 0 - 1895400 \end{bmatrix} \frac{1}{EI} \times \frac{1}{38918880} \begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}$$

$$= \begin{bmatrix} -729000 \\ -1895400 \end{bmatrix} \frac{1}{EI} \times \frac{\begin{bmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{bmatrix}}{38918880}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} 66.193 \\ -67.505 \end{bmatrix}$$