Course title:-
Electrical Network Analysis

Module:-
4th

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Date:-
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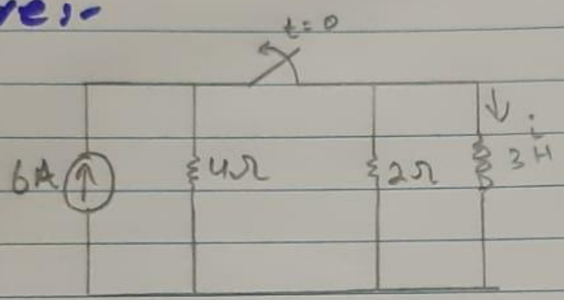
Question No 2:-

Determine the inductor current for both $t \geq 0$ & $t < 0$ for the current.

Figure:-



Solution:-

For $t < 0$:

The Switch is closed & inductor acts as a short circuit;

There fore inductor current

$\therefore \quad i = 6A$

P. 2

For $t > 0$:

The switch is open & the time constant

$$\tau = \frac{L}{R}$$

$$\tau = \frac{3}{2}$$

Now the indutor current

$$i(t) = e^{-\frac{t}{\tau}}$$

$$i(t) = 6e^{-\frac{t}{3/2}}$$

$$i(t) = 6e^{-2t/3}(t)\,A.$$

Q 3:-

A series RLC Circuit is described by

$$L\frac{d^2t}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5H$.

$R = 4\Omega$ & $L = 0.2F$,

P. 3

Let $i(0) = 1A$, $\frac{di(0)}{dt} = 0$

Solution:-

The step response of the branch voltage of the given RLC circuit is describe by;

$$L\frac{d^2i}{dt^2} + \frac{Rdi}{dt} + \frac{i}{C} = 10$$

Divide by L;

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{10}{L}$$

multiplying b $\frac{C}{C}$

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} \cancel{t} + \frac{i}{LC} = \frac{10C}{LC}$$

As $C = 0.2F$; thus;

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} \frac{i}{LC} = \frac{2}{LC}$$

P. 4

Substitue:

$$\frac{d^2t}{dt^2} + 8\frac{di}{dt} + 10i = 20 \quad (i)$$

The general form of source-free RLC is given by;

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{1}{LC} = \frac{I_s}{LC} \quad (2)$$

Comparing (1) & (2) we get.

$$\frac{R}{L} = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

From 3, α is given by;

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4\ rad/sec \rightarrow (6)$$

P. 5

The frequency ω_0 is given by;

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From 4,

$$\omega_0 = \sqrt{10} \text{ rad/sec} \rightarrow (7)$$

From (6) & (7)

$\because \alpha > \omega_0$

$\therefore$ The circuit is over damped

The roots of characteristic equation are given by;

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 + \sqrt{6} \text{ rad/s}$$

And $s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$$= -4 - \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 - \sqrt{6} \text{ rad/s}$$

From (5), the steady current is given by;

$$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A \rightarrow (8)$$

The current of over damped is given by;

$$i(t) = I_s + A_2 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \rightarrow (9)$$

Substitutes $t = 0$:

$$i(0) = I_s + A_1 + A_2$$

Thus,

$$A_1 + A_2 = -1 \rightarrow (10)$$

From 9, find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Substitute;-

$$1 = 2 + A_1 + A_2$$

Then,

$$A_1 + A_2 = -1 \rightarrow (10)$$

D. (7)

From 9, Find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Substitute:- $t = 0$:

$$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$$

Substitute values,

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \rightarrow (11)$$

Solving (10) & (11)

$$A_1 = -1.316$$
$$A_2 = 0.316$$

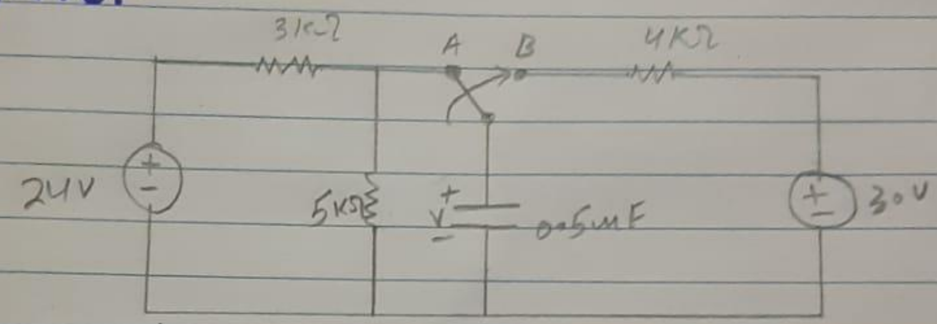
Substitute:- (9)

$$i(t) = 2 - 1.316\, e^{(-4+\sqrt{6})t} + 0.316\, e^{(-4-\sqrt{6})t} \text{ A}$$

Question No 1:-

The switch in Figure has been in position A for a long time. At $t=0$ for $t>0$ & calculate its value at $t = 2s$ & $8s$.

Figure:-



Solution:-

For $t<0$:

The switch is at Position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across 5kΩ resistor. Hence the voltage across the capacitor just before $t=0$ is obtained by voltage division as;

P. 9

$$v(0^-) = \frac{5}{5+3}(24) = 15V$$

As the capacitor can't change instantaneously.

$$V(0) = V(0^+) = 15V$$

For $t>0$; The switch is in position B. The $R_{th} = 4k\Omega$

Time constant is ;

$$\tau = R_{th}C = 4\times 10^3 \times 0.5\times 10^{-3}$$
$$\tau = 2s$$

Since the capacitor acts like an open circuit to dc at steady state;

$$V(\infty) = 30V.$$

Thus;

$$v(t) = V(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15-30)e^{-t/2}$$
$$= (30 - 15e^{-0.5t})V$$

At $t = 2$:

$$V(2) = 30 - 15e^{-\frac{2}{2}}$$
$$= 30 - 15e^{-1}$$
$$V(2) = 24.48\,V$$

At $t = 8$:

$$V(8) = 30 - 15e^{-\frac{8}{2}}$$
$$= 30 - 15e^{-4}$$

$$V(8) = 29.72.$$

Question 4:.

A series RLC circuit has
$R = 100\,\Omega$
$L = 240\,H$
$C = 10\,mF$.

If the input voltage is $v(t) = 10\cos 2t$, find the current flowing through the ciruit.

Q. 11

Solution:-

The input voltage,

$v(t) = 10 \cos 2t \ V$

Amplitude $= V_m = 10V$

Angular frequency $= 2$ rad/s

Phase angle, $\phi = 0°$

So,

Phasor for the voltage $v(t)$:

$V(t) = 10\angle 0° V.$

Now for inductive reactance.

$X_L = \omega L$

~~Note for inductor~~

So $\omega = 2$ rad/s, $L = 240H$

$X_L = (2)(240)$

$X_L = 480\Omega$

Now For capacitive Reactance

$$X_C = \frac{1}{\omega C}$$

$\omega = 2$ rad/s , $C = 10mF$

$$\frac{1}{2(10 \times 10^{-3})}$$

$$\frac{1}{2 \times 10^{-2}}$$

$$\frac{1 \times 10^{2}}{2}$$

$$X_C = 50\,\Omega$$

Now For impedence

$$Z = R + jX_L - jX_C$$

$R = 100\,\Omega$, $X_L = 480\,\Omega$, $X_C = 50\,\Omega$

putting in eq

$$Z = (100 + 480 - 50)\,\Omega$$

$$Z = (100 + j430)\,\Omega$$

P. 13

Represent "Z" in phasor form

$$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$$

$$= \sqrt{10{,}000 + 184{,}900} \angle \tan^{-1}(4.3)$$

$$= \sqrt{194{,}900} \angle \tan^{-1}(4.3)$$

$$Z = 441.47 \angle 76.90^\circ \Omega$$

Now for current flowing in the circuit

$$i = \frac{v(t)}{Z}$$

$$i = \frac{v(t)}{Z}$$

$v(t) = 10\angle 0^\circ$, $Z = 441.47 \angle 76.90^\circ \Omega$

putting in eq

$$i = \frac{10\angle 0^\circ V}{441.47 \angle 76.90^\circ \Omega}$$

P. 14

$$i = \frac{10}{441.47} \angle [0 - 76.90°] A$$

$$= 22.6 \times 10^{-3} \angle -76.90° A$$

$$= 22.6 \angle -76.90° mA$$

So,

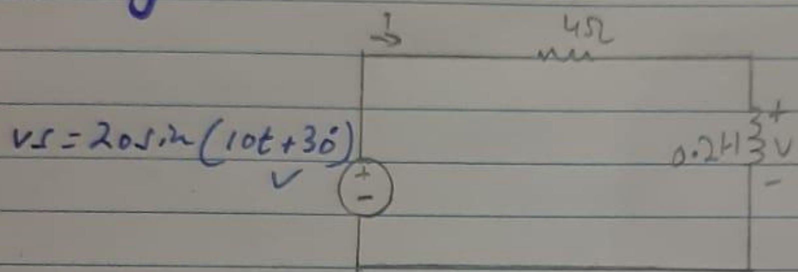
The general expression for "i"

$$i = 22.6 \cos(2t - 76.90°) mA.$$

Question 5:-

Find $v(t)$ & $i(t)$ in the circuit in Figure

Figure:-



Solution:-

For $i(t)$

From the voltage source

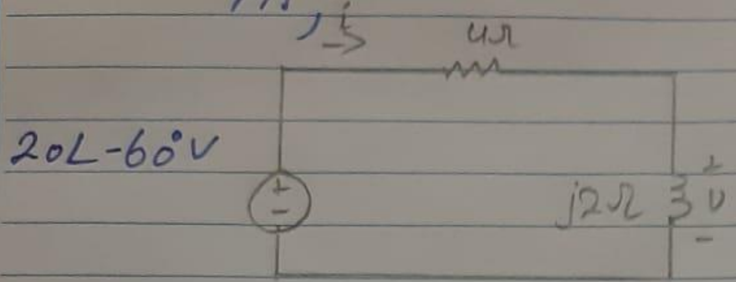
$$V_s = 20\sin(10t + 30^\circ)\,v$$

$$V_s = 20\cos(10t + 30^\circ - 90^\circ)\,v$$
$$V_s = 20\cos(10t - 60^\circ)\,v$$
$$V_s = 20\angle -60^\circ\,V$$

$$\omega = 10\ \text{rad/sec}$$
$$X_L = j\omega L$$
$$0.2H = j \times 10 \times 0.2 \Rightarrow 0.2H = j2\Omega$$

P. 16

Given circuit can be represented As;



From the above Circuit,

$$Z = 4 + j2\,\Omega$$

Hence the current is,

$$i = \frac{20\angle -60^\circ}{4 + j2}$$

$$i = \frac{20\angle -60^\circ}{\sqrt{4^2+2}\ \angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$i = \frac{20\angle -60^\circ}{4.472\angle 26.57^\circ}$$

$$i\ 4.472\angle -86.57^\circ$$

Converting this into time domain.

$i(t) = 4.472 \cos(10t - 86.57°)$

$i(t) = 4.472 \sin(10t - 86.57° + 90°)$

$i(t) = 4.472 \sin(10t + \cancel{8}3.43°)A$

For $v(t)$:

From the circuit voltage across the inductor is ;

$v = j2 \times i$

$v = j2 \times (4.472 \angle 86.57°)$

Converting polar form to rectangular form we gets

$v = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j\,0.53512.$

Convert retangular form to polar form.

P. 18

$v = \left(\sqrt{(8.926)^2 + (0.53512)^2}\right) \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$

$v = 8.944 \angle 3.40$

Converting this in time domain.

$v(t) = 8.944 \cos(10t + 3.4^\circ)$

$v(t) = 8.944 \sin(10 + 3.4^\circ + 90^\circ)$

$v(t) = 8.944 \sin(10t + 93.4^\circ)$