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Sub :- Differential Equat

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Part 2

Question NO 1:-

Given

$$\frac{\partial^2 w}{\partial t^2} = \frac{c^2 \partial^2 w}{\partial x^2}$$

Now

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct)+\cos(2x+2ct)\right]$$

$$= \frac{\partial}{\partial t}(\sin(x+ct)) + \frac{\partial}{\partial t}(\cos(2x+2ct))$$

$$\frac{dw}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[\frac{\partial c}{\partial t}\cos(x+ct) - 2c\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) \cancel{4c} \sin(2x+2ct)$$

Now.

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) - \cancel{2c}\cos(2x+2ct)\right]$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$\rightarrow (1)$.

$$= -c^2(x+ct) - 4c^2\cos(2x+2ct)$$

$$= c^2\left[-\sin(x+ct) - 4\cos(2x+2ct)\right]$$

$$= c^2 \cancel{\sin(x+ct)} - 4c^2 \cancel{\cos(2x+2ct)} =$$

$$-\cancel{\sin(x+ct)} - 4c^2\cancel{\cos(2x+2ct)}$$

$$0 = 0$$

Q NO 1
part B

(II) $w = \tan(2x + ct)$.

NOW

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}(c \sec^2(2x+ct))$$

$$= c^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$$

NOW

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct).$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct.$$

$\hookrightarrow (i)$.

$$\cancel{4c^2 \sec^2(2x+ct)\tan(2x+ct)}$$
$$\cancel{= 4c^2 \sec^2(2x+ct)}$$

$$0 = 0$$

Q NO 28: Given funct

$$f(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

we have to find the fourier co-efficients, a_0, a_n & b_n.

Now,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2} \rightarrow (1).$$

$$a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2} \rightarrow (4)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

2

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[2\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2} ; \text{ if } n \text{ is odd} \\ 0 ; \text{ if } n \text{ is even} \end{cases} \quad 2$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x \sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x \sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the require series is

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos(2x-1)}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}$$

$$\frac{\sin nx}{n}.$$

Question NO 3:-

solve the initial value problem.

$y'' - 4y' + 13y = 8\sin 3x$, $\rightarrow$ (1)

$y(0) = 1$ and $y'(0) = 2$.

$y'' - 4y' + 13y = 0 \rightarrow$ (2)

change (2) into auxiliary equation.

put $y = m$ in (2)

$m^2 - 4m + 13 = 0$

use quadratic formula

$a = 1$, $b = -4$, $c = 13$.

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

9

$$= \frac{4 \pm \sqrt{-36}\, i}{2}$$

$= 4 \pm 6i$

$= 2 \pm 3i$

$m_1 = 2 + 3i$

$m_2 = 2 - 3i$

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \to$ (A)

$y_p = A \cos 3x + B \sin 3x \to$ (*)

Difference w.r.t x.

$y_p' = -3A \sin 3x + 3B \cos 3x$

Again Diff w.r.t x.

$y_p'' = -9A \cos 3x - 9B \sin 3x$

put in (1)

$\Rightarrow (-9A \cos 3x - 9B \sin 3x) - 4(-3A \sin 3x + 3B \cos 3x)$

$+13(A \cos 3x + B \sin 3x)$ $8 \sin 3x$

③

$\Rightarrow -9A\cos3x - 12B\cos3x + 13A\cos3x$

$- 9B\sin3x + 12A\sin3x + 13B\sin3x - 8\sin 3x$

$\Rightarrow (-9A - 12B + 13A)\cos3x + (-9B + 12A + 13B)\sin3x = 8\sin 3x$

comparing co-efficient

$\sin3x \Rightarrow \quad 4B + 12A = 8$

$\cos3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow 4A = 12B$

$\Rightarrow A = 3B \rightarrow (b)$

Put (b) in ⓐ

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$B = 1/5 \rightarrow$ ⓒ

put C in B -

$A = 3B$

$A = 3(1/5)$

$A = 3/5 \rightarrow$ ⓓ

5
$\cancel{45}$
$/81$

(4)

Put C and d in (*)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$$

The General solution is

$$y = y_c + y_p.$$

$$y = e^{2x}(C_1\cos 3x + C_2\sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (C)$$

we have to find the values of C_1 & C_2 for this

put $x=0$ & $y=1$ in (C)

$$1 = e^{2(0)}(C_1\cos 3(0) + C_2\sin 3(0)) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0).$$

$$1 = (C_1(1) + C_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = C_1 + \frac{3}{5}$$

$y = 2/5 \rightarrow (**)$ ⑤

Diff C w.r.to x^2

$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + C_2(2e^{2x}\sin 3x$

$+ 3e^{2x}\cos 3x) - \frac{6}{5}\sin x + \frac{3}{5}\cos x$

$\rightarrow (D)$

Put

$y' = 2, \ x = 0$ in (D)

$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$

$+ C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$

$- \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x)$

Put $y' = 2, \ x = 0$

$2 = C_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0))$

$+ C_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\sin 3(0))$

$- \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0) \rightarrow (B)$

Put $y' = 2, \ x = 0$ in (D)

(b)

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$$
$$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$
$$-\frac{6}{5}\sin 3x + \frac{3}{15}\cos 3x)$$

put $y=2$, $x=0$

$$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0))$$
$$+ c_2(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0))$$
$$-\frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0).$$

$$2 = c_1(2) + c_2(3) - 0, + \frac{3}{5}$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

put $c_1 = \frac{2}{5}$

$$2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3c_2$$

$$3c_2 = 2 - \frac{7}{5}$$

$$3c_2 = \frac{3}{5} \Rightarrow c_2 = \frac{3}{15} \rightarrow (***)$$

put (**) & (***) in $\rightarrow$ C

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right)$$
$$+ \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x.$$

(1) $y = \frac{2}{5} e^{2x} \cos 3x + \frac{3}{15} e^{2x} \sin 3x + \frac{3}{5} \cos 3x$
$+ \frac{1}{5} \sin 3x$

Required General solution.

Q4
1

Question NO 4:

Solution:

It is already in symbolic form.

$(D^2 - DD')z = \cos x \cos 2y \rightarrow (a)$

Put A.E $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i.e $D = m, D' = 1.$

$\Rightarrow m^2 - m = 0$

$m = 0, 1.$

Therefore

$C.F = f_1(y) + f_2(y+x)$

From eq(a).

$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$

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(2)

$$= \frac{1}{2} \frac{1}{D^2 - DD'} 2\cos x \cos 2y$$

As

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$CF = f_1(y-x) + x f_2(y-x)$$

$$PI = \frac{1}{D^2 + 2DD' + D'^2} [2(y$$

By General method

$$m_2 - 1 \; ; \; y - x = c$$

$$= \frac{1}{D+D'} [(x + \sin(-c)] \, dx$$

$$= \frac{1}{D+D'} [2c x - (\sin c) x]$$

Replacing ~~y-x~~ by c by $y-x$

$$y - x = c$$

③

$$= \frac{1}{D+D'}[2cx - (\sin c)x]$$

$$= \frac{1}{D+D'}[2x(y-x) - x\sin(y-x)].$$

Again

put $y-x=c$

$$= \int (2xc - x\sin c)\,dx \Rightarrow$$

$$cx^2 - \frac{x^2}{2}\sin c$$

replacing

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y)$$

$$z = C.F + P.I = f_1(y-x) + x f_2(y-x) + x^2y - x^3 + \frac{1}{2}x^2\sin(x-y).$$