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SESSIONAL ASSINGNMENT

ADVANCED MECHANICS OF MATERIALS

1 Applications of Mohr's Circle to three dimensions:-

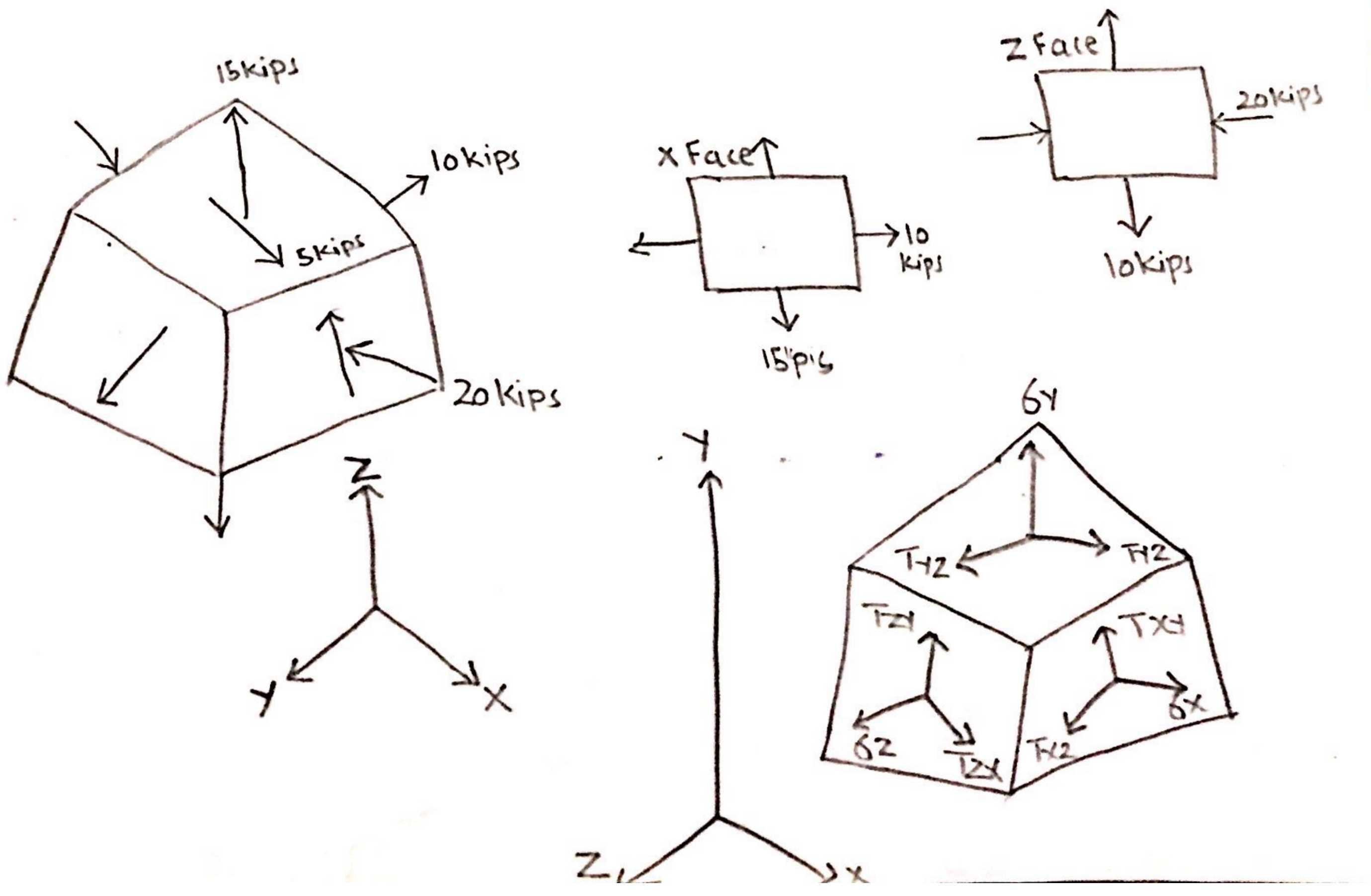
The mohr circle is then used to determine graphically the stress components acting on a rotated co-ordinate system, i:e, acting on a differently oriented plane passing through that point.

The Mohr Circle can be applied to any symmetric 2x2 tensor matrix, including the strain and moment of inertia tensors.

The Mohr Circle construction enables the stresses acting in different directions at a point on a plane to be determined.

→ Plane problems (the intermediary principal stress σ_2 is perpendicular to the plane contarning σ_1 and σ_3) ax symmetric (3D revolution) problems ($\sigma_2 = \sigma_3$ and a meridian contains σ_1 and σ_3).

3D mohr's Circle Sketch



2] DIMENSIONAL ANALYSIS OF STRESS:-

A method of comparing the dimensions of the physical quantities occuring in a problem to find relationships between the quantities without having to solve the problem completely.

Solid elements are used for stress analysis of general three dimensional bodies that requires more precise analysis than is possible through two dimensional and or axismmetric analysis.

→ **Mathematically Form:-**

Stress $=$ Force $\times$ $[\text{Area}]^{-1}$

OR

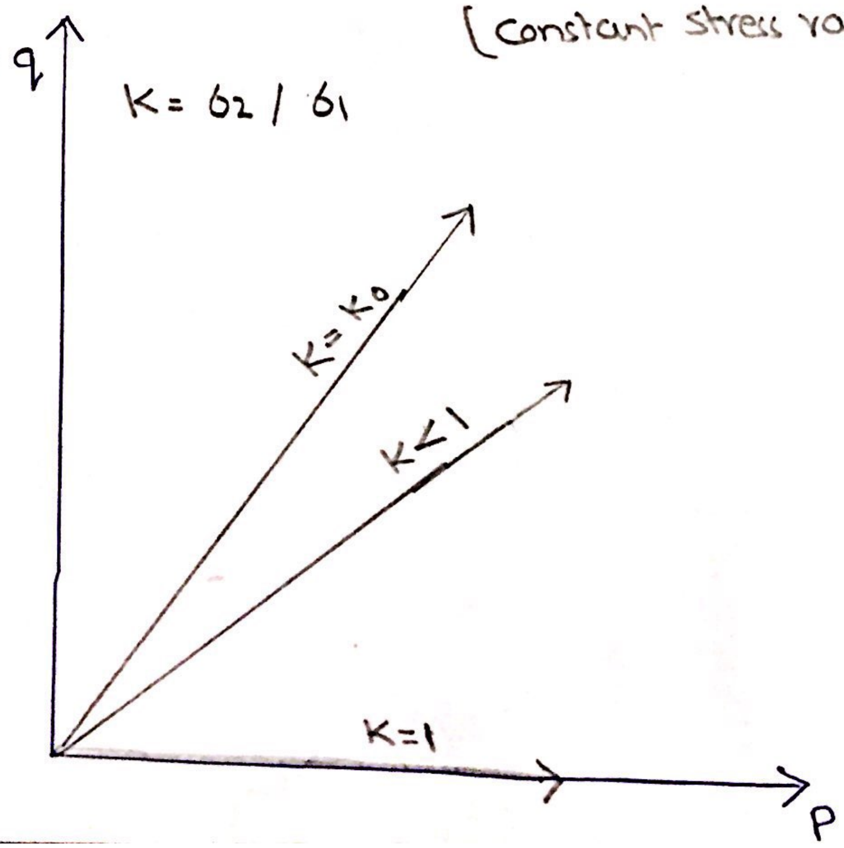
Stress $= [M^1 L^1 T^{-2}] \times [M^0 L^2 T^0] = [M^1 L^{-1} T^{-2}]$, Therefore stress is dimensionaly represented as $[M^1 L^{-1} T^{-2}]$.

→ **Purpose of dimensional analysis of stress:-**

Dimension analysis of stress is commonly used to determine the relationships between several variables i.e, to find the force as a function of other variables when exact functional relationships is unknown.

3D Sketch of Analysis of Stress:-

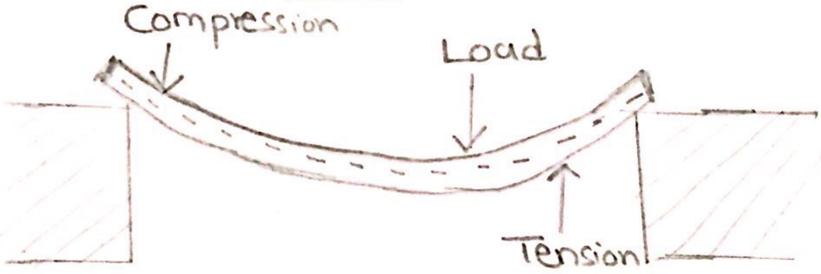
[Constant stress ratio loading]



3] SIMPLE BENDING:-

Bending will be called as simple bending when it occurs because of beam self-load and external load.

SKETCH:-

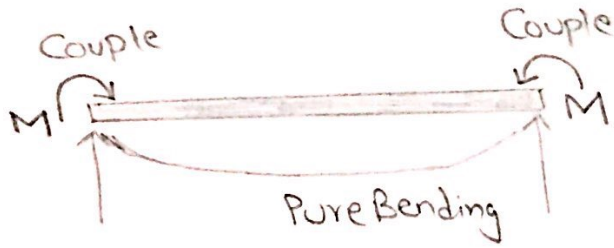


Simply Supported Beam.

* PURE BENDING:-

Bending will be called as pure bending when it occurs solely because of coupling on its end. In that case there is no chance of shear stress in the beam.

SKETCH:-



Pure Bending

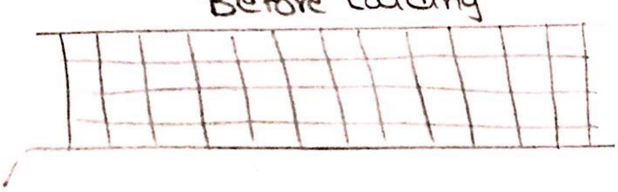
4] ASSUMPTIONS MADE IN THE THEORY OF PURE BENDING:-

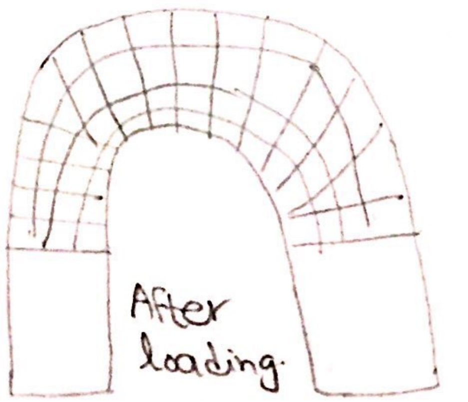
1) The material of the beam is homogeneous[1] and isotropic[2].
2) The value of Young's Modulus of elasticity is same in tension and compression.
3) The transverse sections which were plane before bending, remain plane after bending also.
4) The beam is initially straight and all longitudinal filaments bends into circular arcs with a common centre of curvature.
5) The radius of curvature is large as compared to the dimension of the cross section.
6) Each layer of the beam is free to expand or contract. Independently of the layer, above or below it.

Note:- [1] Homogeneous means the material is of same kind.
[2] Isotropic means that the elastic properties in all directions are equal.

→ Pure bending occurs only under a constant bending moment (M) since the Shear Force (V), which is equal to $\frac{dM}{dx} = V$, has to be equal to zero. In reality, a state of pure bending does not practically exist.

Before Loading





5 CLASSICAL FLEXURE EQUATION:-

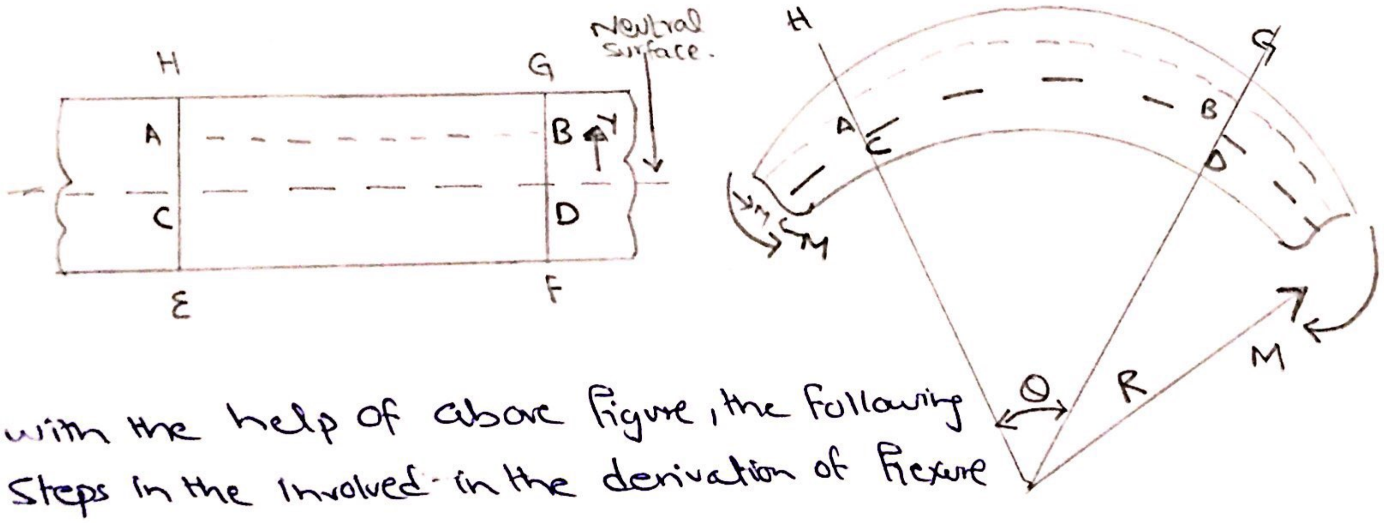
Bending theory is also known as flexure theory is defined as the axial deformation of the beam due to external load that is applied perpendicularly to a longitudinal axis which find applications in applied mechanics.

For a material, flexural strength is defined as the stress that is obtained from the yield just before the flexure test.

* FLEXURE EQUATION DERIVATION:-

Following are the assumptions made before the derivation of the bending equation.

→ The beam used is straight with a constant cross-section.
→ The beam used as of homogeneous material with a symmetrical longitudinal plane.
→ The plane of symmetry has all the resultant of applied loads
→ The primary cause of failure is buckling.
→ E remain same for tension and compression.
→ Cross section remains same before and after bending.



with the help of above figure, the following steps in the involved in the derivation of flexure equation.

Strain in fibre AB is the ratio of change in length to original length.

Strain in fibre AB $= \frac{A'B' \; AB}{AB}$ ∴ Strain $= \frac{A'B' \; C'D'}{C'D'}$

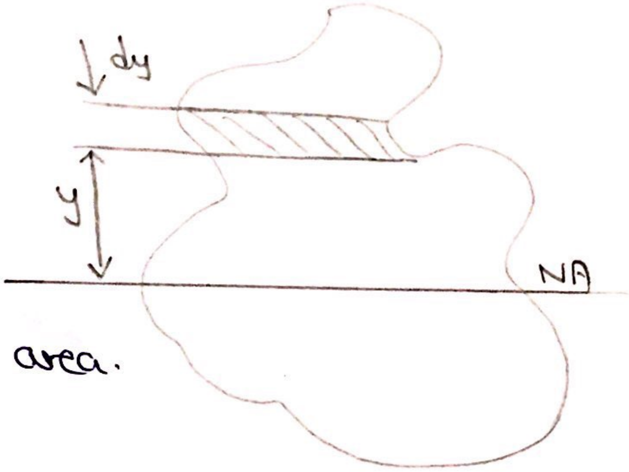
(as $AB = CD$ and $CD = C'D'$).

CD and C'D' are on the neutral axis and stressed is assumed to be zero, therefore strain also zero on the neutral axis.

$$= \frac{(R+y)Q - RQ}{RQ} = \frac{RQ + yQ - RQ}{RQ} = \frac{y}{R} \quad \frac{\sigma}{\varepsilon} = \frac{Y}{R}$$ where E is Young modulus of elasticity.

OR

$$\frac{\sigma}{y} = \frac{\varepsilon}{R}$$



Cross sectional area.

$$\sigma = \frac{E}{R} y \ (\text{eq. } 1)$$

$F = \sigma \delta A = \frac{E}{R} y \delta A$ (Force acting on strip with area dA).

$Fy = \frac{\varepsilon}{R} y^2 \delta A$ (moment about neutral axis).

$M = \Sigma \frac{E}{R} y^2$ (total momentum for entire crossectional area).

$\delta A = \frac{E}{R} \Sigma y^2 \delta A$ $\Sigma y^2 \delta A$ is known as second moment of area and represented by I.

$$\therefore M = \frac{E}{R} I \ (\text{eq. } 2)$$

From eq 1 and eq 2 we get.

$$\boxed{\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}}$$

SECTION MODULUS:-

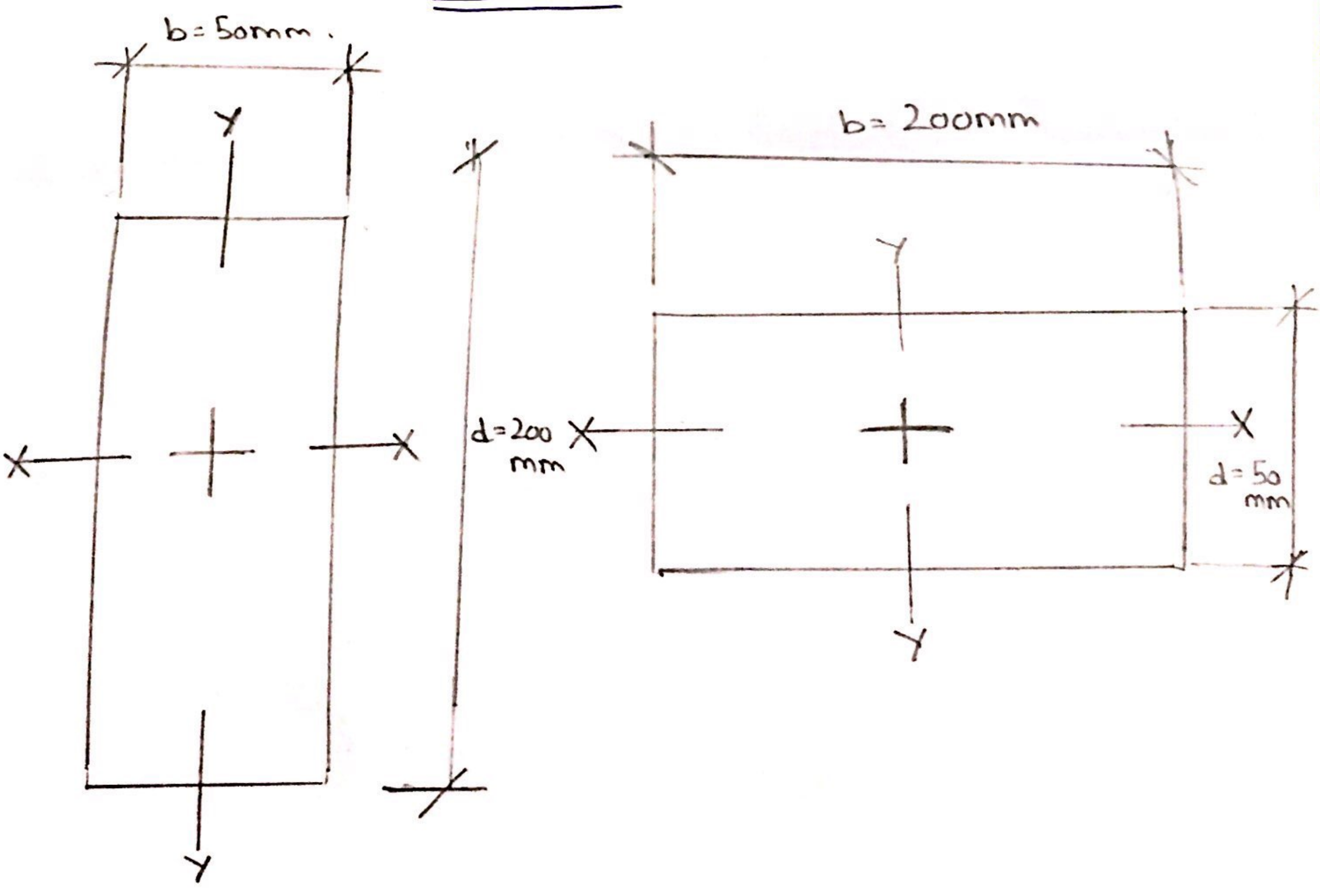
The section modulus of the cross-sectional shape is of Significant importance in designing beams. It is a direct measure of the strength of the beam. A beam that has a larger section modulus than another will be stronger. and capable of supporting greater loads.

→ **Unit:-**

Its unit is mm^3.

Section modulus depends only on the cross section shape of the beam. Shapes like rectangular, square and circle etc.

SKETCH:-



BEAM A

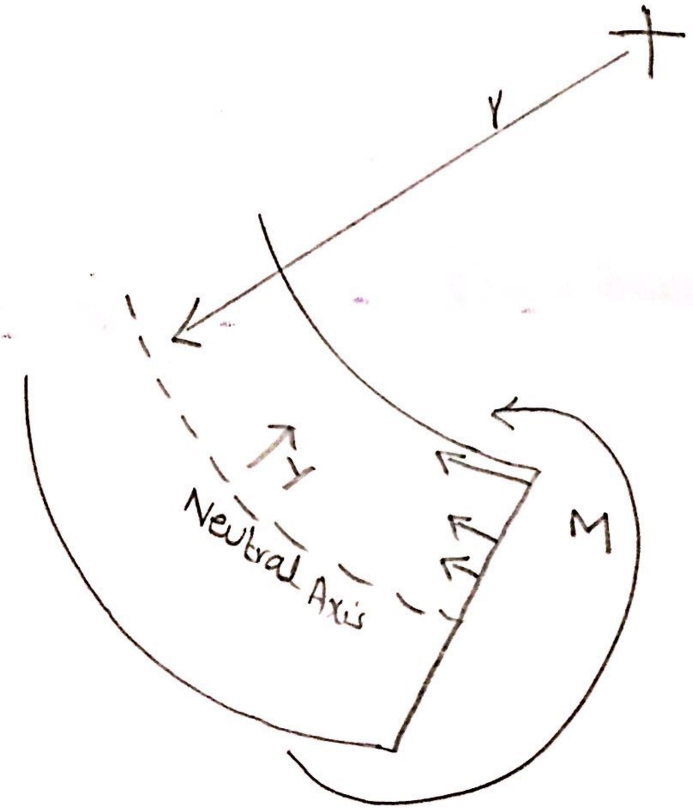
BEAM B

7 APPLICATION OF BENDING EQUATION IN AN OBJECT:

Bending equation is also known as flexure equation or equation for theory of simple bending. At the NA, bending stress or bending strain is zero. The first moment of area of a beam section about neutral axis is also zero.

$$E/R = M/I = F/y.$$

This equation gives the bending normal stress and also commonly called the flexure formula.



8] Moment of Resistance

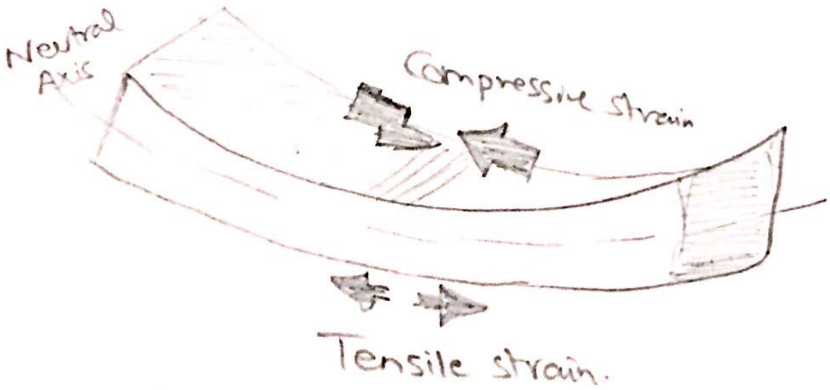
The algebric sum of moments of the internal forces (compressive and tensile forces developed in the cross section due to bending). about the neutral axis of the section is called the moment of resistance of the section.

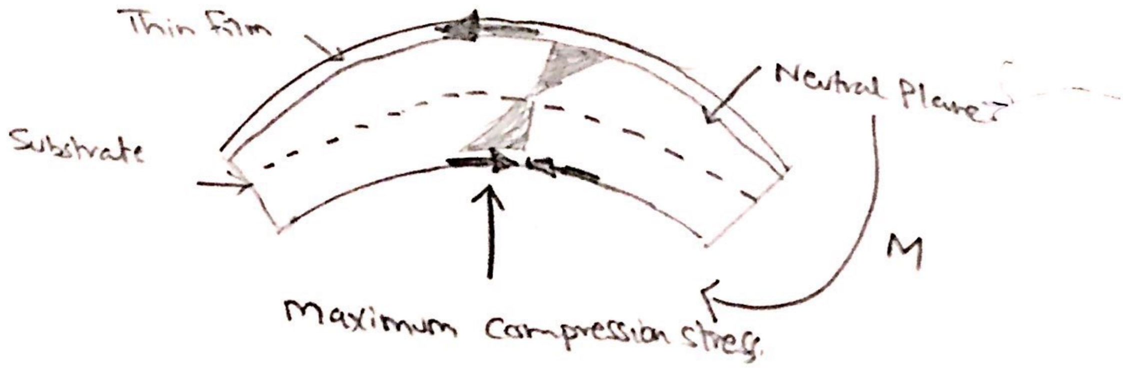
For equilibrium condition, the moment of resistance of a section will be equal to the applied bending moment at the section.

The moment of resistance of a section corresponding to the maximum permissible stresses in the material is called the limiting moment of resistance of the section. This indicates the maximum bending moment that could be resisted by the section without the stresses exceeding the permissible value.

* Denotation:-

Moment of resistance denotes the resistance offered by the beam to the external moment applied. In other words It's the the capacity of a beam to withstand the applied moment without failure.





9] Design of columns under centric load:

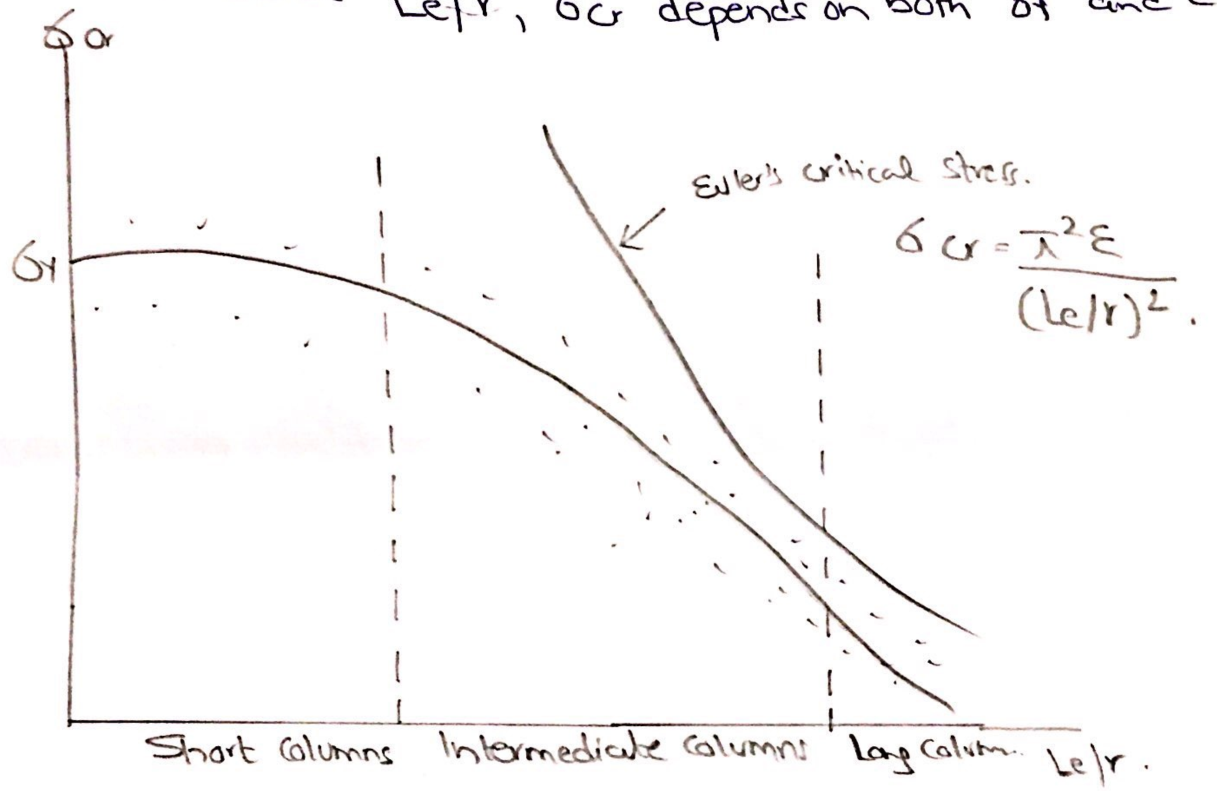
Previous analysis assumed stresses below the proportional limit and initially straight, homogeneous columns.

→ Experimental data demonstrate.

→ For large L_e/r, σ_{cr} follows Euler's formula and depends upon E but not σ_Y.

→ For small L_e/r, σ_{cr} is determined by the yield strength σ_Y and not E

→ For intermediate L_e/r, σ_{cr} depends on both σ_Y and E.

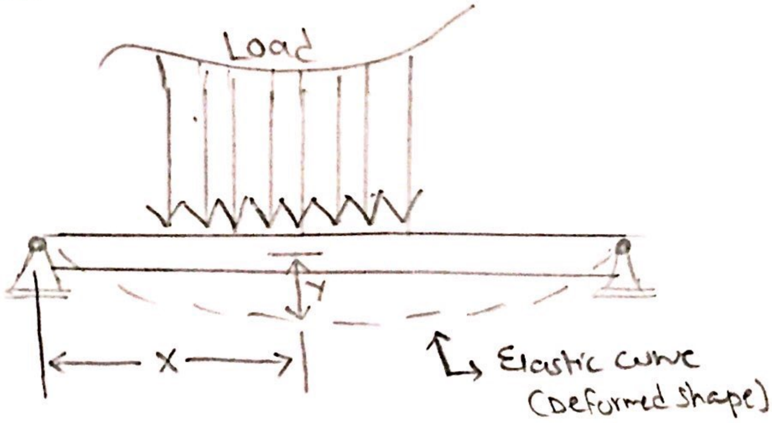


★ Centric Load:

A load which passes through the centroid of the cross section of a structural member and acts normal to the cross-section

DEFLECTION OF BEAM BY CASTIGILANO'S THEOREM:

In 1879, Castigliano published two theorems. Castigiliano's 2nd theorem. The first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the direction of its line of action.

→ 

⇒ method of Determining Beam Deflections:

By Castigilo's method theorem we find at deflection of beam.

$$U = W_i$$

Where "U" denote strain energy. "Wi" work done by internal forces.

$$\delta = \frac{\partial U}{\partial P} \quad \text{or} \quad \theta = \frac{\partial U}{\partial \bar{M}}$$

wher δ is the deflection at point of application force, P is direction. Q is rotation at point. $\bar{M}$ direction of $\bar{m}$. U is the strain energy.

$$U = \int_0^L \frac{M^2}{2EI} dz \,..$$

$$\delta = \int_0^L \left(\frac{\partial M}{\partial P}\right) \frac{M}{EI} dz \quad \& \quad Q = \int_0^L \left(\frac{\partial M}{\partial \bar{M}}\right) \frac{M}{EI} dx.$$