

Date: _____



NAME: MIAN DAUD JAN

CLASS: BS - SE, 4th Sem, Sec-A

ID: 14468

SUBJECT: Operation Research

TEACHER: Saif ulah

Class Timing: Monday (08:00 Am -
11:00 am)

QUESTION NO 1 :

There are total of 5 machines and five employments are to be relegated cost network is as per following.

Locate the best possible task

Machines

Jobs	A	B	C	D	E
1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	122	23	21
5	9	5	7	6	10

ANSWER NO 1 :

To find the best possible task,

First we need to do Row Reduction

Step 1 Row Reduction: Subtracting minimum value of row from each entry of row. The operations involved are,

$$R_1 - 3$$

$$R_2 - 1$$

$$R_3 - 7$$

$$R_4 - 16$$

$$R_5 - 5$$

	A	B	C	D	E	
1	3	9	0	8	12	$R_1 - 3$
2	3	1	6	0	9	$R_2 - 1$
3	1	4	3	0	4	$R_3 - 7$
4	0	3	106	7	5	$R_4 - 16$
5	4	0	2	1	5	$R_5 - 5$

Step 2: Column Reduction: Subtracting minimum value of column from each entry of column. The following operations are involved.

$$C_1 - 0$$

$$C_2 - 0$$

$$C_3 - 0$$

$$C_4 - 0$$

$$C_5 - 4$$

We get,

	A	B	C	D	E	
1	3	9	0	8	8	
2	3	1	6	0	5	
3	1	4	3	0	0	
4	0	3	106	7	1	
5	4	0	2	1	1	$C_5 - 4$

Step 3 | Zero Assignment:

Now we have to assign or cross out the zeros.

i) Starting with rows, assigning circle to ^{zero} bracket with in that row which have exactly one zero. cross all zeros in the respective assigned column.

ii) Starting with columns, assign circle to zero in that column which have exactly one zero and cross out all in respective assigned row.

Thus, we get matrix,

	A	B	C	D	E
Job					
1	3	9	0	8	8
2	3	1	6	0	5
3	1	4	3	8	0
4	0	3	106	7	1
5	4	0	2	1	1

Here, we have

Total assigned zeros = 5

→ optimal

⇒ 1 → C, 2 → D, 3 → E,

4 → A, 5 → B

Total time = 3 + 1 + 11 + 16 + 5 = 36

Darsi Notes

Date: _____

Page #04

Mian Daud
14468

QUESTION #02

Solve the following linear programming problem.

$$\min z = 2x_1 + 3x_2$$

$$\text{s.t } (1/2)x_1 + (1/4)x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1, x_2 \geq 0$$

Answer #02

Using the Big M method to solve linear programming problem. The given subject to constraints are,

$$(1/2)x_1 + (1/4)x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

Step 1: Converting the above inequalities into Equalities using the Slack and Artificial variables.

$$\text{If } \leq ; \text{ then } +S$$

$$\text{If } \geq ; \text{ then } -S + A$$

$$\text{If } = ; \text{ then } A$$

Thus we get,

Date: _____

Page # 05

Mian Daud
14468

$$(1) \quad \frac{1}{2}x_1 + \frac{1}{4}x_2 + S_1 = 4$$

$$(2) \quad x_1 + 3x_2 - S_2 + A_1 = 20$$

$$(3) \quad x_1 + x_2 + A_2 = 10$$

Step 2: Using objective function and set it equal to zero

i) Subtracting slack variable with co-efficient zero

ii) Adding Artificial variable with co-efficient (-M)

$$\rightarrow 0 = Z = 2x_1 + 3x_2 - 0S_1 - 0S_2 + MA_1 + MA_2$$

$$\rightarrow Z - 2x_1 - 3x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

Step 3: Creating the table

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	-2	-3	0	0	-M	-M	0
0	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
0	1	3	0	-1	-1	0	20
0	1	1	0	0	0	1	10

Darsi Notes

operation: $(R_3 + R_4)M + R_1 \rightarrow R_1$

$1, 2M-2, 4M-3, 0, 0, -M, 0, 30M$
 $\rightarrow R_1$

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	$2M-2$	$4M-3$	0	0	$-M$	0	$30M$
0	$1/2$	$1/4$	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

operation: $R_3/3, R_4 + 3R_3$

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	$2M-2$	$4M-3$	0	0	$-M$	0	$30M$
0	$1/2$	$1/4$	1	0	0	0	4
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	1	1	0	0	0	1	10

operation: $R_4 - R_3$

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	$2M-2$	$4M-3$	0	0	$-M$	0	$30M$
0	$1/2$	$1/4$	1	0	0	0	4
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3$

Date: 13th April, 2020

Page # 07

Mian Dawd
14463

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	$(2m-3)/3$	0	0	$(m-3)/3$	$(3-4m)/3$	0	$60+10m$ 3
0	$5/12$	0	1	$1/12$	$-1/12$	0	$7/3$
0	$1/3$	1	0	$-1/3$	$1/3$	0	$20/3$
0	$2/3$	0	0	$1/3$	$-1/3$	1	$10/3$

Finally, we obtain,

Z	x_1	x_2	S_1	S_2	A_1	A_2	R.H.S
1	0	0	0	$-1/2$	$(1-2m)/2$	$(3-2m)/2$	25
0	0	0	1	$-1/8$	$1/8$	$-5/8$	$1/4$
0	0	1	0	$-1/2$	$1/2$	$-1/2$	5
0	1	0	0	$1/2$	$-1/2$	$3/2$	5

In the optimal solution, we have

$$\{Z, S_1, x_1, x_2\} = \{25, 1/4, 5, 5\}.$$

As we don't have any of the artificial variables a_1 and a_2 in the solution, thus the solution is feasible.

Darsi Notes

QUESTION #03

Use Vogel's Approximation Method to obtain the Initial feasible Solution of,

Origin	Destination				Supply
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	

ANSWER #03

$$\text{Demand} = 60 + 40 + 30 + 110 = 240$$

$$\text{Supply} = 120 + 70 + 50 = 240$$

⇒ Demand = Supply

It is a balanced transportation problem thus no need of addition of dummy row and column.

Date: _____



	D ₁	D ₂	D ₃	D ₄	Supply				
S ₁	X	40	X	80	⁰ 36 120	13	(13)	-	-
S ₂	10	X	30	30	⁰ 16 70	2	2	2	(17)
S ₃	50	X	X	X	⁰ 50	5	5	5	17
Demand	⁰ 60	⁰ 40	⁰ 30	⁰ 110					
	4	(15)	8	3					
	4	-	8	3					
	8	-	(11)	8					
	8	-	-	8					

Total cost:

$$T.C = 40 \times 22 + 80 \times 4 + 10 \times 24 + 30 \times 9 + 30 \times 7 + 50 \times 32$$

$$T.C = 880 + 320 + 240 + 270 + 210 + 1600$$

$$T.C = 3520$$