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Section: A

Subject: Structural Analysis(2)

Instructors: Engr. Sir Adeed Khan

Final Term Examination (Summer Semester 2020)

Dated:- 25-09-2020

Question#1

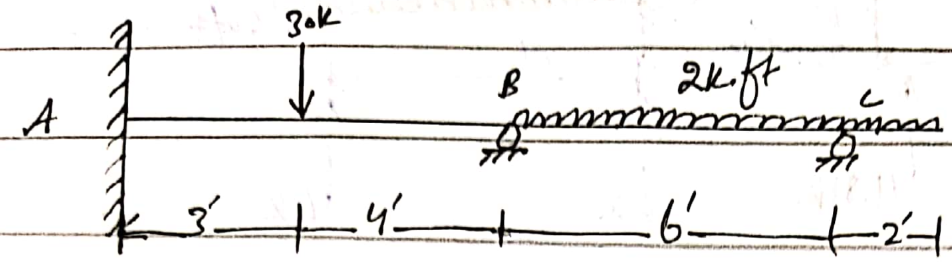


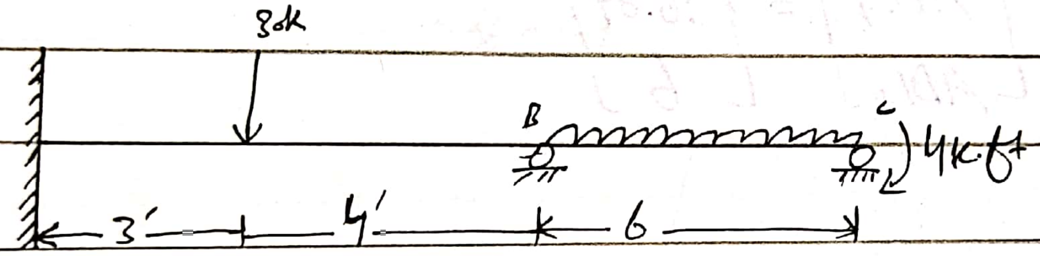
Figure.1

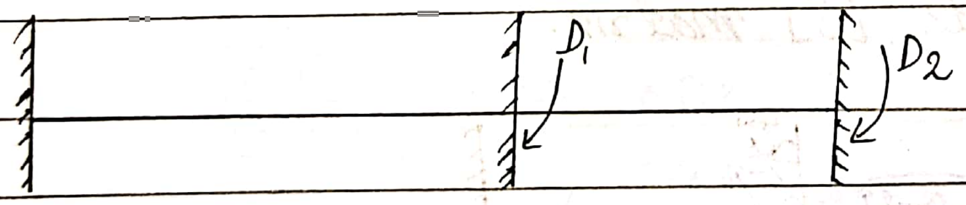
Sol:-

Step#1: K.I = 2° (Neglected Axial Effects)

Step #2:-

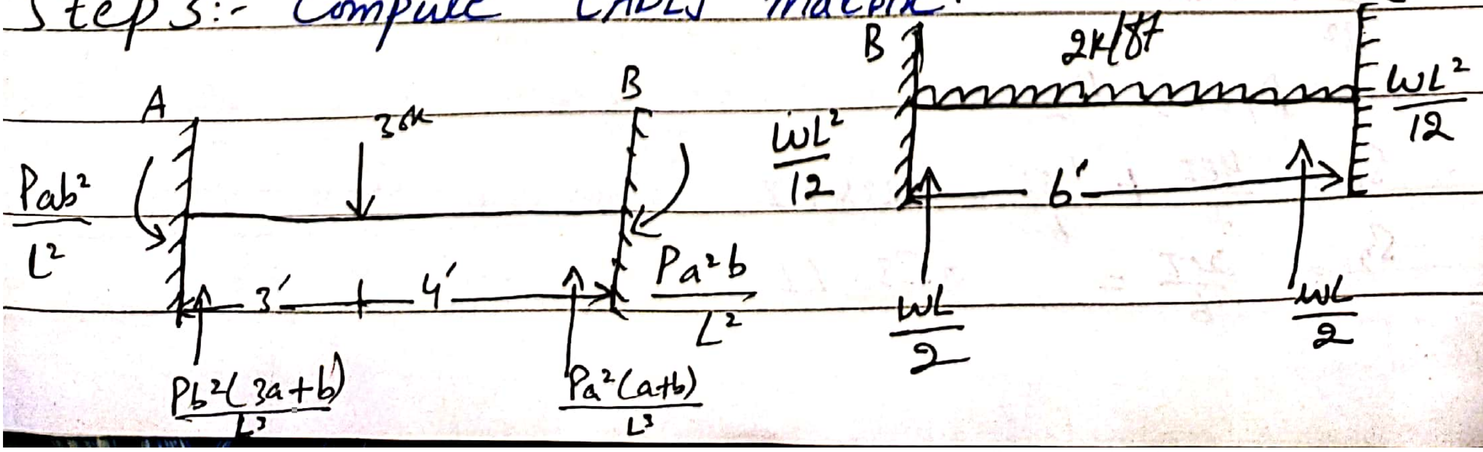
Select the unknown joint displacement.

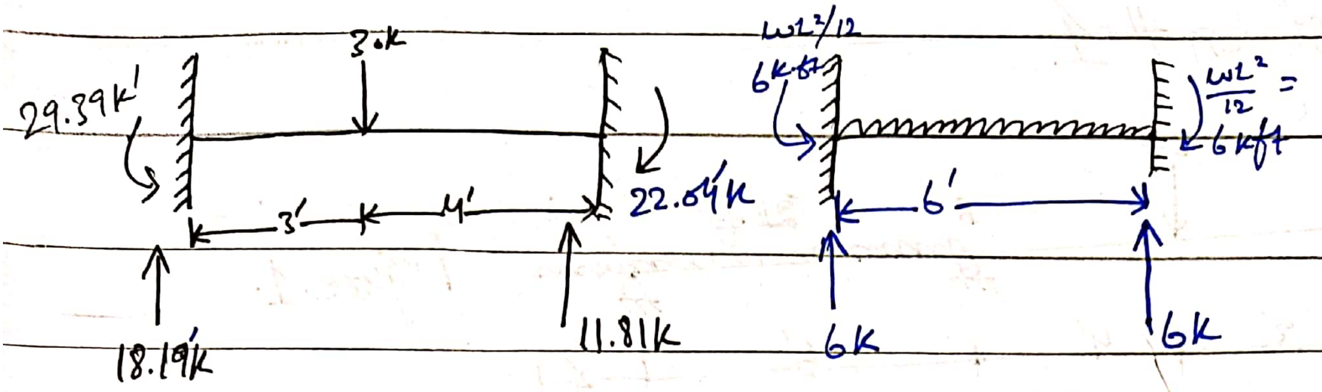




$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad [AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step 3:- Compute [ADL] matrix.





$ADL_1 = 22.04'k - 6'k$

$ADL_1 = 16.04'k$

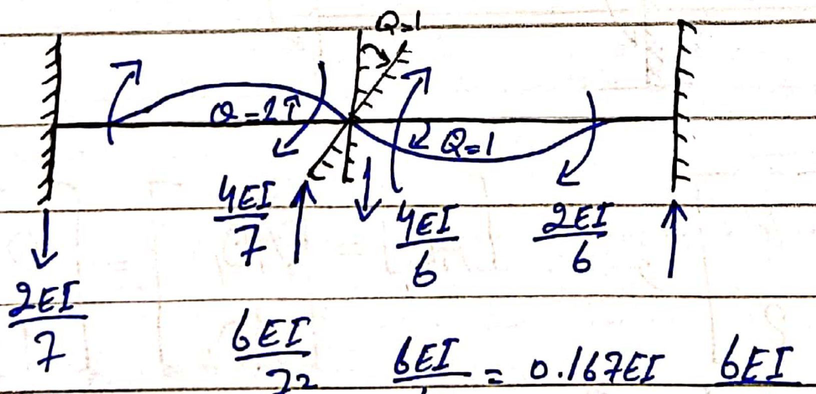
$ADL_2 = 6'k$

So, $[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$

Step#4:-

Compute [S] matrix.

(i)



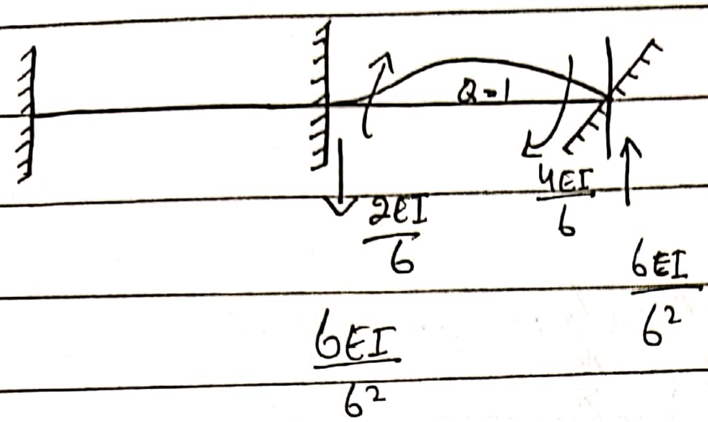
$\frac{6EI}{7^2} = 0.122EI$

when $D_1 = 1$, $D_2 = 0$

$S_{11} = \frac{4EI}{7} + \frac{4EI}{6} = 1.238\ EI$

$S_{21} = \frac{2EI}{6} = 0.333\ EI$

(ii) When $D_1 = 0,\ D_2 = 1$



$$S_{12} = \frac{2EI}{6} = 0.333\ EI$$

$$S_{22} = \frac{4EI}{6} = 0.667\ EI$$

Stiffness Matrix $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

$$[S] = \begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI$$

Step #5:-

Compute the values of D_1 & D_2

$[AD] = [ADL] + [S][D]$

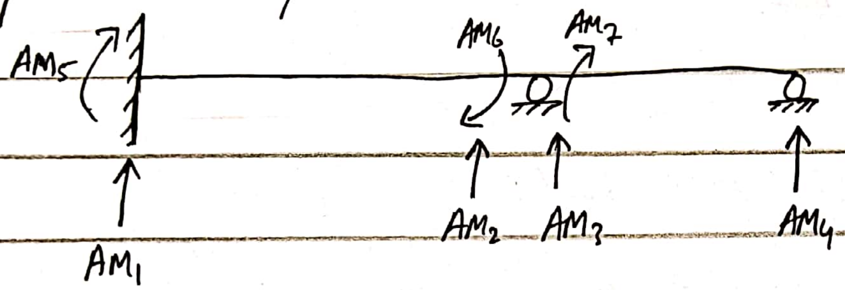
$[D] = [S]^{-1}[AD - ADL]$

$$[D] = \left(\begin{bmatrix} 1.238 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} EI \right)^{-1} \begin{bmatrix} 0 - (16.04) \\ 4 - (6) \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 0.933 & -0.466 \\ -0.466 & 1.732 \end{bmatrix}\begin{bmatrix} -26.04 \\ -2 \end{bmatrix}$$

$$D = \frac{1}{EI}\begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

Step 6: Compute Members & Actions.



$$[AM] = [AML] + [AMD][D]$$

[AML];

$$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 18.19\,K \\ 11.81k \\ 6k \\ 6k \\ -29.39'k \\ 22.64'k \\ -6'k \end{bmatrix}$$

[AMD];

$$[AMD] = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AM_{22} \\ AMD_{31} & AMD_{32} \\ AMD_{41} & AMD_{42} \\ AMD_{51} & AMD_{52} \\ AMD_{61} & AMD_{62} \\ AMD_{71} & AMD_{72} \end{bmatrix} = EI \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.833 \end{bmatrix}$$

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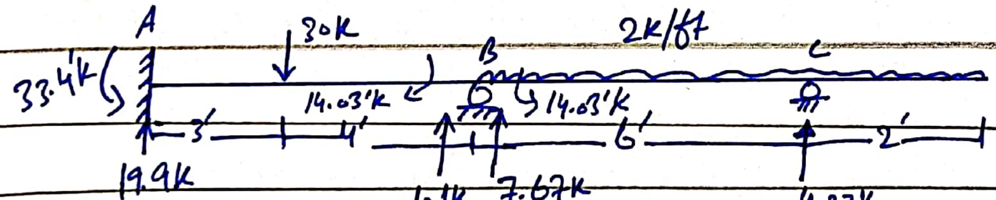
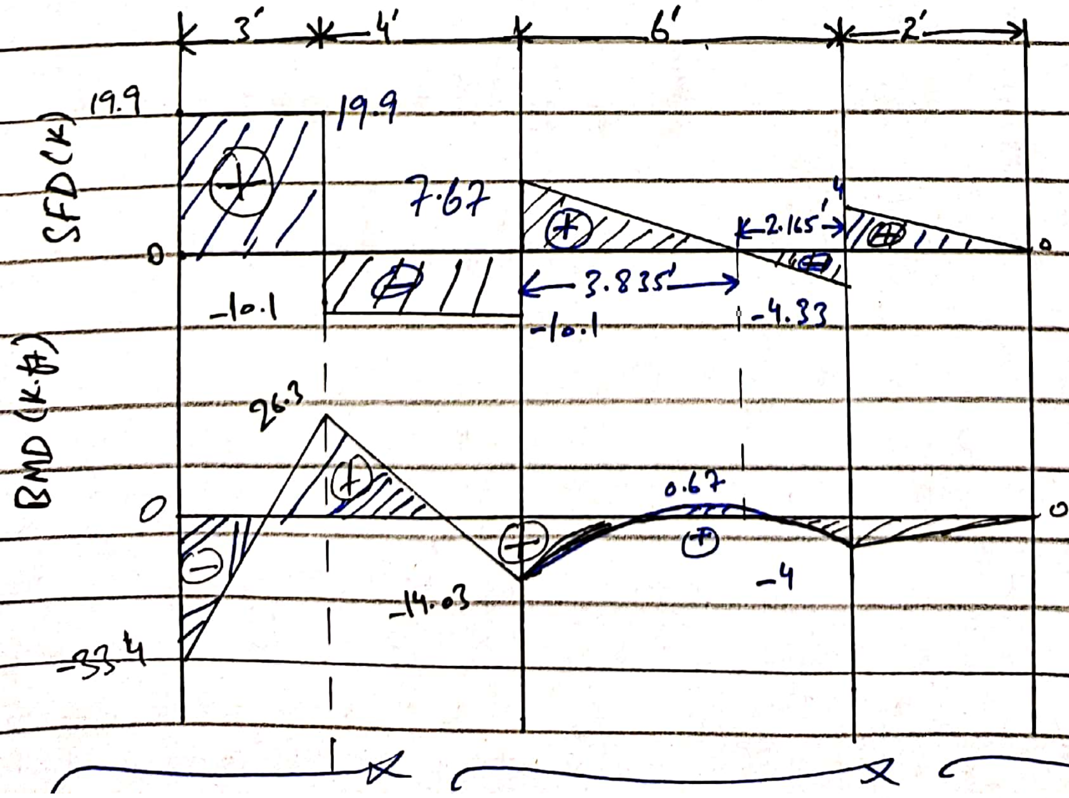
Now; $[AM] = [AML] + [AMD][D]$

$$[AM] = \begin{bmatrix} 18.19 \\ 11.81 \\ 6 \\ 6 \\ -29.39 \\ 22.64 \\ -6 \end{bmatrix} + \begin{bmatrix} -0.122 & 0 \\ 0.122 & 0 \\ -0.167 & -0.167 \\ 0.167 & 0.167 \\ 0.286 & 0 \\ 0.571 & 0 \\ 0.667 & 0.333 \end{bmatrix} EI \times \frac{1}{EI} \begin{bmatrix} -14.03 \\ 4.01 \end{bmatrix}$$

$$[AM] = \begin{bmatrix} 19.9k \\ 10.1k \\ 7.67k \\ 4.33'k \\ -33.4'k \\ 14.03'k \\ -14.03'k \end{bmatrix}$$





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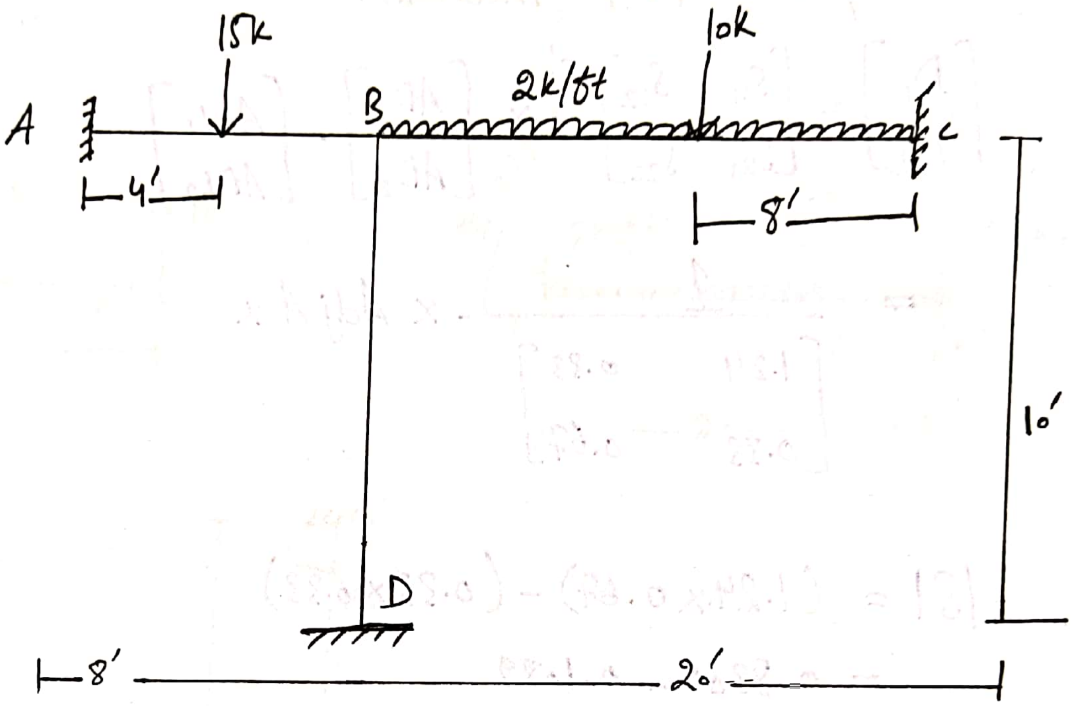
Question # 02

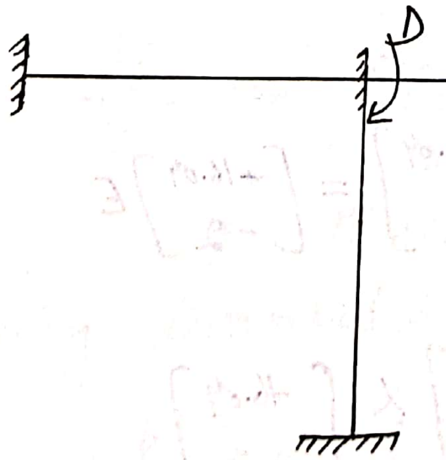


Sol # 1 Step # 1:- Determine Kinematic Indeterminancy

$K.I = 1^{\circ}$

Step # 2:-

Determine Unknown Joint displacement.



$[D] = [?]$

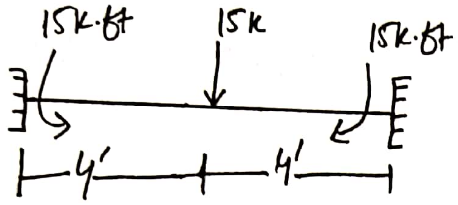
$[AD] = [0]$

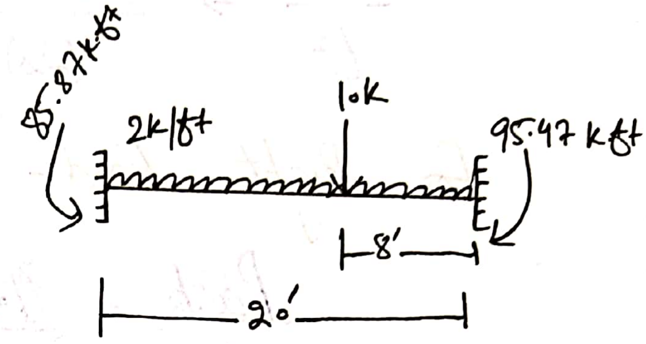
Step # 3:-

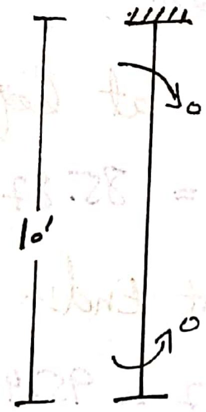
Compute [ADL] Matrix.

Step # 3 :-

Compute [ADL] Matrix







⇒ Point load at Center:-

$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8}$$

$$= 15 \text{ kip.ft}$$

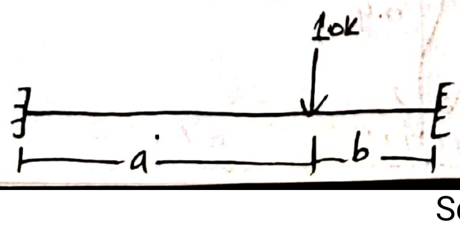
⇒ Uniformly Distributed load:-

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12}$$

$$= 66.67 \text{ k.ft}$$

⇒ Point load (Not at mid):-

Suppose :-



For left End:-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} \Rightarrow 19.2 \text{ k.ft}$$

For Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

So total moment at left end:-

$$\Rightarrow 19.2 + 66.67 = 85.87 \text{ k.ft.}$$

Similarly at Right End:-

$$\Rightarrow 28.8 + 66.67 = 95.47 \text{ k.ft.}$$

So $[ADL] = -85.87 + 15 = -70.87$ k.ft.

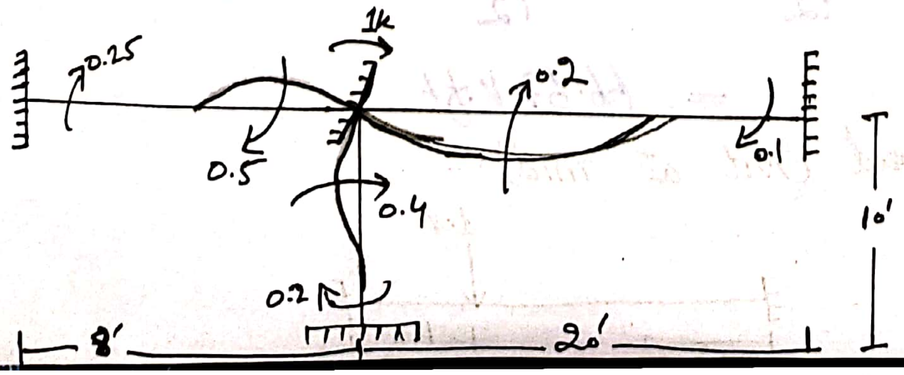
Step #4:-

Determine $[S]$ Matrix

$$[S] = [S_{11}]$$

Now:-

$D = 1k$



$$\Rightarrow \frac{4EI}{8} = 0.5$$

$$\Rightarrow \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \quad , \quad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \quad , \quad \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2)\,EI$$

$$\boxed{[S] = 1.1\ EI}$$

<u>Step #5</u>

Compute [D] Matrix

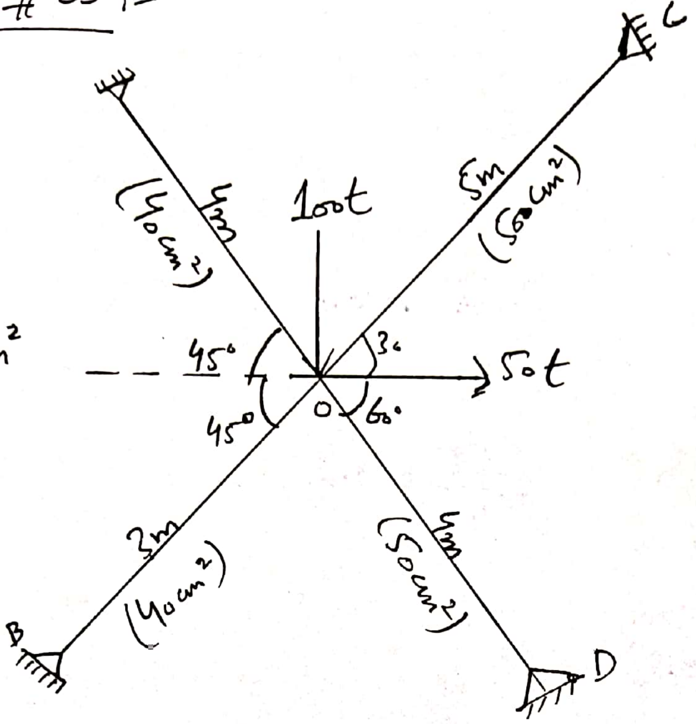
$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$\boxed{[D] = [64.42]\ ^{1}/_{EI}}$$

Problem # 03 :-



$E = 2000\ t/cm^2$

Sol :- For A:

$$\sin 45° = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828m$$

$$\cos 45° = \frac{b}{h}$$

$$\Rightarrow b = 2.82\ m$$

For B:-

$$\sin 45° = \frac{P}{3}$$

$$\Rightarrow P = 2.12m$$

$$\cos 45° = \frac{b}{h}$$

$$\Rightarrow b = 2.12m$$

For C:-

$$\sin 30° = \frac{P}{h=5}$$

$$\Rightarrow P \Rightarrow 2.5m$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33m$$

Now $EA_{(A)} = 2000 \times 40 = 80{,}000\,t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\,t$

$EA_{(C)} = 2000 \times 50 = 100{,}000\,t$

$EA_{(D)} = 2000 \times 50 = 100{,}000\,t$

Step #01:-

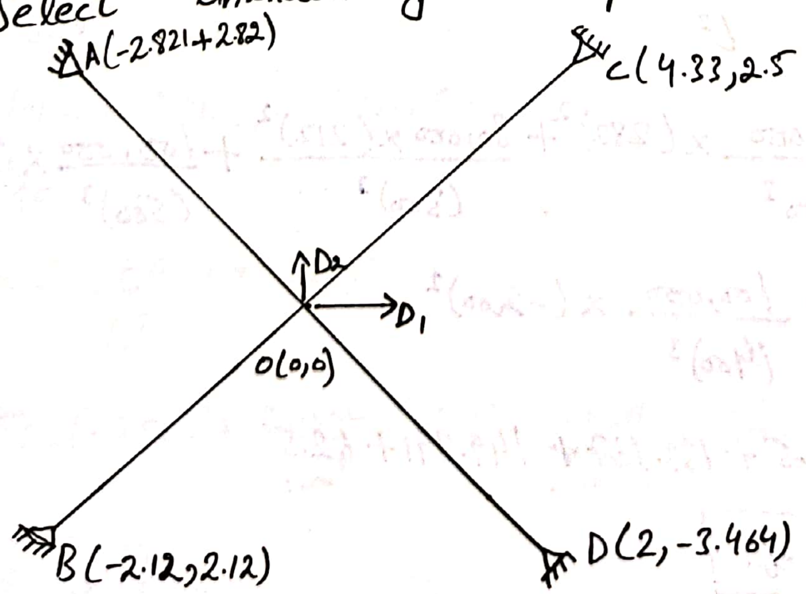
We have to find kinematic Indeterminancy

$K.I = 2j - r$

$= 2(5) - 8 = 2^\circ.$

Step #02:-

Select Unknown Joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step#03:- $[AMD]_{4\times2}$ & $[S]_{2\times2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

Now $$S_{11} = \sum^{m} \frac{EA}{L^3}(X_k - X_j)^2$$

$$= \frac{80,000}{400^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2 + \frac{100,000}{(500)^3} \times (-433)^2$$

$$+ \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.167 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_k - x_j)(y_k - y_j)$$

$$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212)$$

$$+ \frac{100,000}{(500)^3} \times (-433)(0-250) + \frac{100,000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2} (y_k - y_j)$$

$$AMD_{12} = \frac{80,000}{400^2} (-282) = -141$$

$$AMD_{22} = \frac{80,000}{300^2} (212) = 188.44$$

$$AMD_{32} = \frac{100,000}{500^2} (-250) = -100$$

$$AMD_{42} = \frac{100,000}{400^2} (346) = 216.25$$

Now $S_{22} = \sum^{m} \frac{EA}{L^3} (y_k - y_j)^2$

$$S_{22} = \frac{80,000}{400^3} (-282)^2 + \frac{80,000}{300^3} (212)^2 + \frac{100,000}{500^3} (-250)^2 + \frac{100,000}{400^3} (346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step # 04

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step # 05 :- [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 20.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$