FINAL TERM

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Section :- "B"

Subject :- INTRO. to structural dynamics & earthquake engineering.

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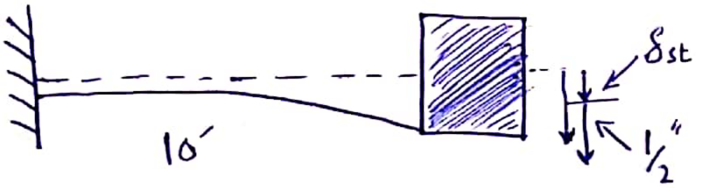
Q No. 1 :~

PROBLEM :~



GIVEN DATA :-

$E = 29000$ ksi

$I = 150\ in^4$

$\delta_{st} = 7695$ lb

SOLUTION :-

The genesal E.O.M for SDOF system;

$$Ku + c\ddot{u} + m\ddot{u} = p(t)$$

→ In our case system is undamped $(c=0)$ undergoing free vibration $(P(t)=0$.

→ Hence general EOM become $Ku + m\ddot{u} = 0$ — (1)

Now;

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000 \frac{k}{in^2} \times 150\ in^4}{(10 \times 12 in)^3}$$

$$K = 7.55\ k/in$$

$$\boxed{K = 90625\ lb/ft}$$

→ $$m = \frac{7695\ lb sec^2}{32.2 ft} \Rightarrow \boxed{m = 238.97\ slug}$$

⇒

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{238.97}} = 19.47 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.47} = 0.32 \text{ sec}$$

→ Substituting the corresponding value in eq (1)

$$Ku + m\ddot{u} = 0$$

$$90625\, u + 238.97\, \ddot{u} = 0$$

Where 'k' is in lb/ft and 'm' is in lb sec/ft²

General solution to the EOM for undamped free vibration is;

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$\because u(0) = \tfrac{1}{2}'' = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24} \text{ ft}$$

and

$$\dot{u}(0) = 0$$

$$\rightarrow u(t) = \left(\frac{1}{24}\right)\left(\cos(19.47)\right) + 0 = \left(\frac{1}{24}\right)\left(\cos(19.47)\right)$$

Equivalent static force at any time 't' is

$$f_{(s)}(t) = K.u(t) = \frac{90625 \times \cos(19.47)}{24}$$

$$f_{(s)}(t) = 3776 \cos(19.47)$$

⇒

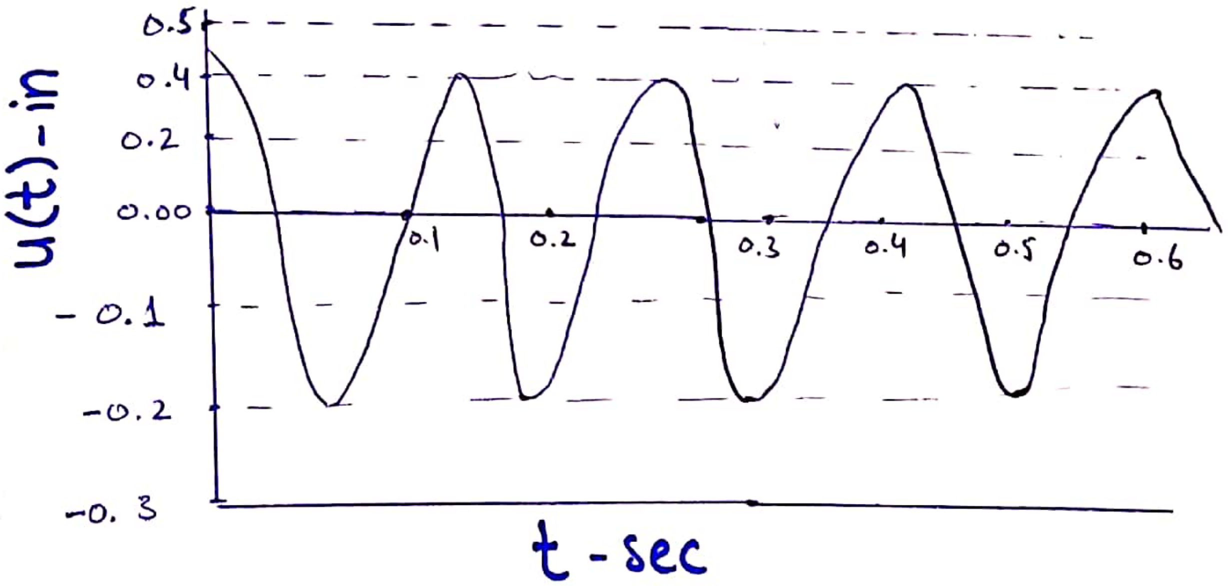
Amplitude of dynamic displacement, u_o for undamped free vibration;

$$u_o = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

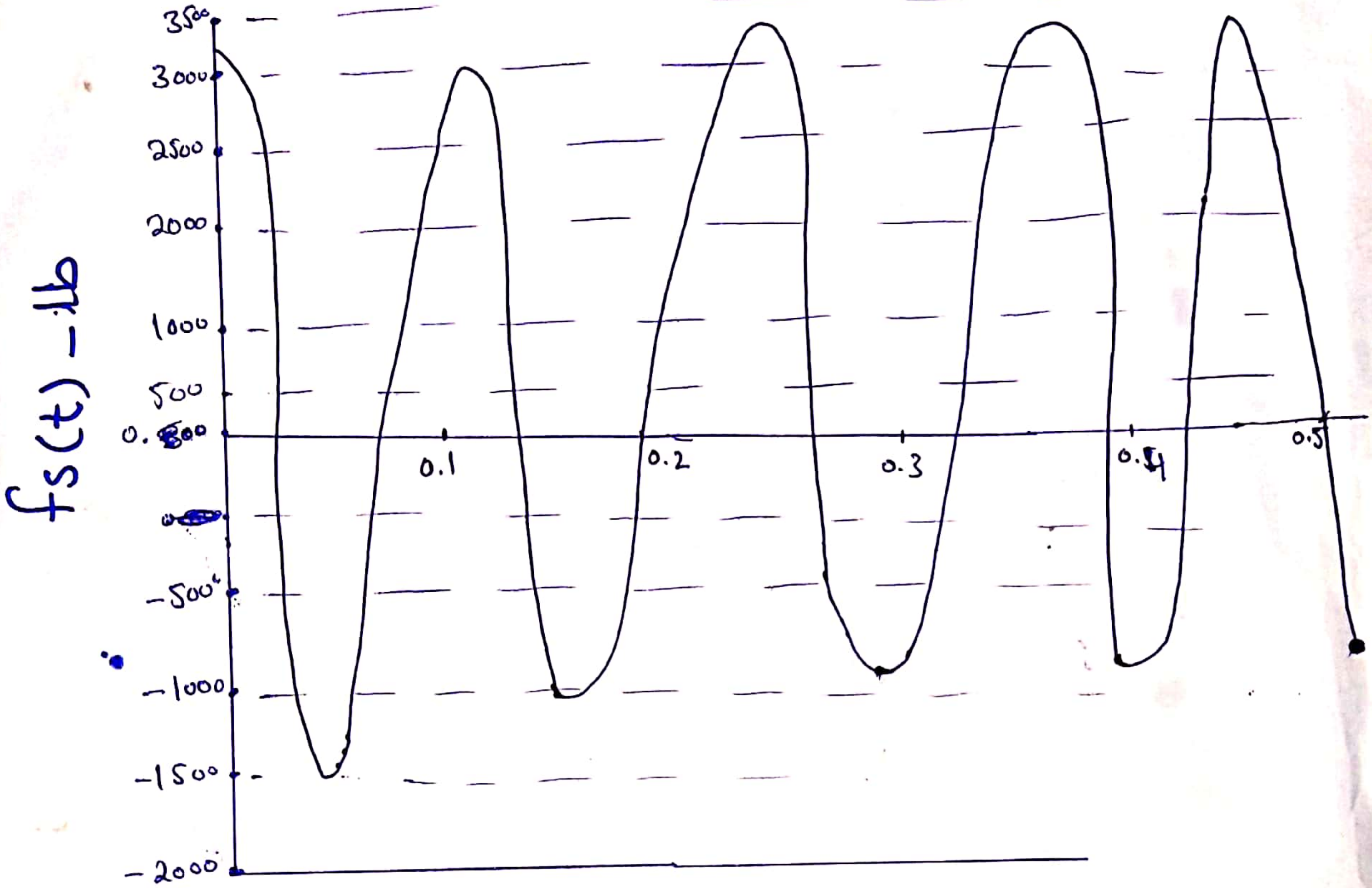
$$u_o = \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)} = \frac{1}{24}\ ft$$

$$\rightarrow \quad K u_o = 90625 \times \frac{1}{24} = \boxed{3776\ lb}$$

* Un damped Free vibration:



* Undamped Free vibration



QNO(2):-

PROBLEM:-

GIVEN DATA:-

$$E = 29000 \text{ ksi}$$

$$I = 150 \text{ in}^4$$

$$f_{st} = 7695 \text{ lb}$$

Take $\zeta = 2.5\%$

Solution:-

E.O.M for damped free vibration;

$$ku + c\dot{u} + m\ddot{u} = 0 \longrightarrow (1)$$

it is known from problem NO.1 that;

$$k = 90625 \text{ lb/ft} \quad \text{and}$$

$$m = 238.97 \text{ lb.sec}^2/\text{ft}$$

$$\rightarrow \quad c = \zeta \times 2 m \omega_n$$

$$c = 0.025 \times 2(238.97)(19.47)$$

$$c = 0.025 \times 9305.4918$$

$$\boxed{c = 232.637 \text{ lb.sec/ft}}$$

$\Rightarrow$

By subsitituting values of K, C and m in eq ①:

$$Ku + C\dot{u} + m\ddot{u} = 0$$

$$90625u + 232.637\,\dot{u} + 238.97\,\ddot{u} = 0$$

Solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[u(0)\zeta\omega_n\right].\sin(\omega_D t)\right]$$

$$\Rightarrow \omega_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{90625}{238.97}}$$

$$\omega_D = 19.47 \text{ rad/sec}$$

$$\rightarrow u(t) = e^{-0.025\times 19.47t}\left[\frac{1}{24}\times\cos(19.47t) + \frac{1}{19.47}\times\left[0+\frac{1}{24}\times 0.025\times 19.47\times\sin(19.47t)\right]\right]$$

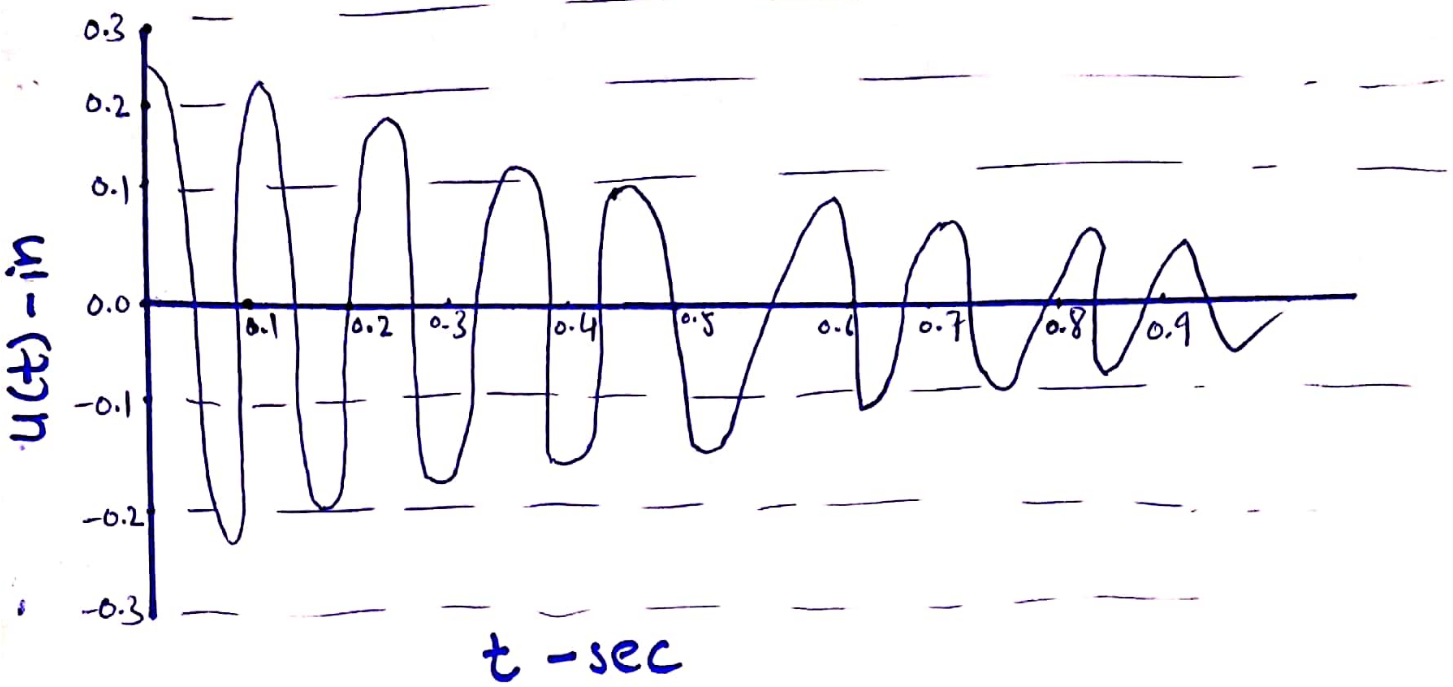
$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.47t) + 0.05136\times 0.02028\sin(19.47t)\right]$$

$$u(t) = e^{-0.48675}\left[0.041667\times\cos(19.47t) + 0.001041\sin(19.47t)\right]$$

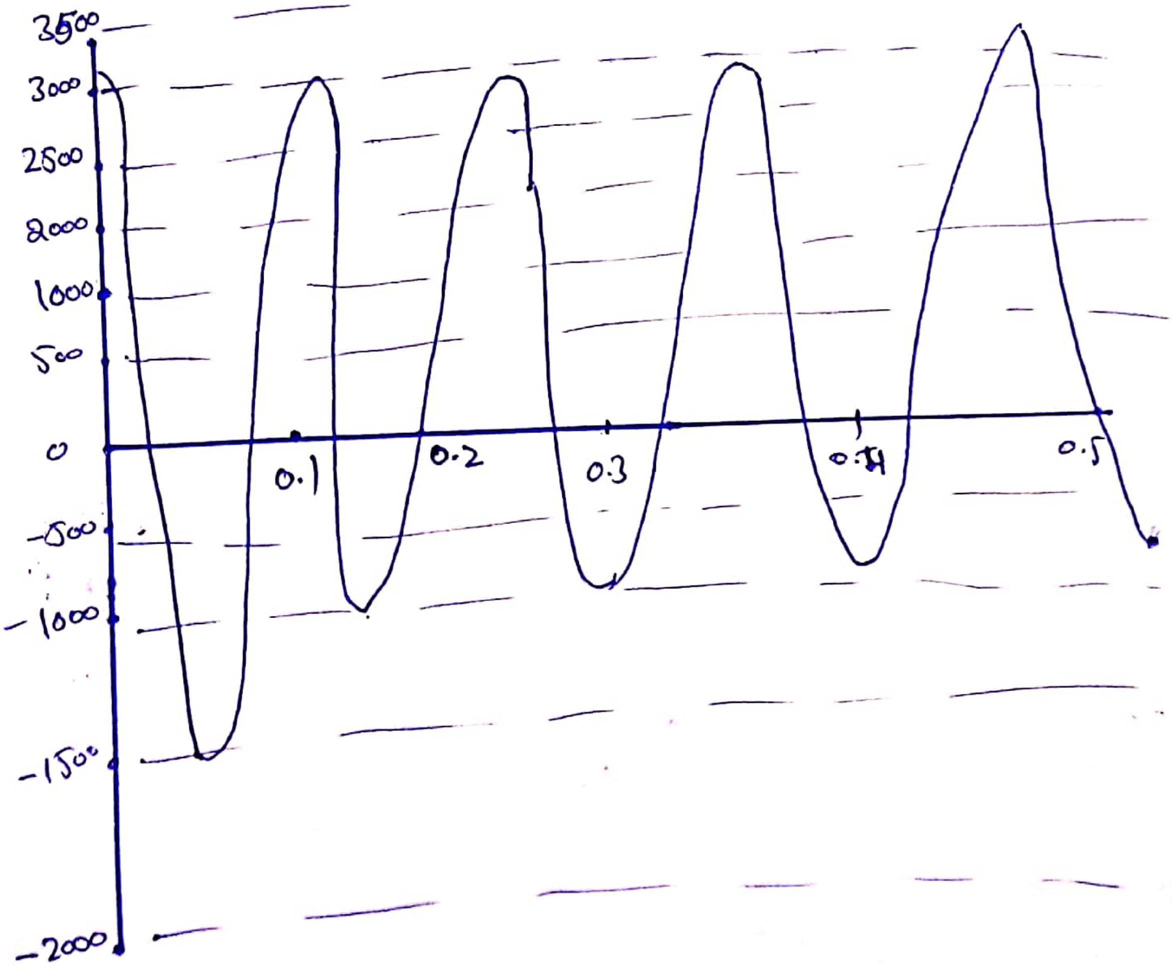
$$\Rightarrow f_s(t) = K.u(t) = 90625\times u(t)$$

$$f_s(t) = e^{-0.48675}\left[3776\cos(19.47) + 94.34\sin(19.47t)\right]$$

* Damped free vibration :-



* Damped free vibration :-



Q NO. 3:-

PROBLEM:-

GIVEN DATA:-

→ Force = 60 kips

→ $U_1 = \frac{7695}{1000} = 7.695$ in

→ After ; $j = 7$ cycles

Completed = 3.57sec

→ $U_{J+1} = 2.286$ cm $= 0.9$ in

→ Ignore the vertical vibration:-

REQUIRED;

ⓐ Damping ratio.

ⓑ Natural period of undamped vibration.

ⓒ Stiffness of structures.

ⓓ Weight of tank.

ⓔ Damping Co-efficient.

ⓕ Number of Cycles to reduce the displacement amplitude to 0.5".

Solution:-

(a): Damping Ratio = $\zeta = ?$

As;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

By putting values

$$7 = \frac{1}{2\pi(\zeta)} \ln\left[\frac{7.695}{0.9}\right]$$

$$\zeta(7 \times 2 \times 3.14) = 2.145$$

$$\zeta(43.96) = 2.145$$

$$\zeta = \frac{2.145}{43.96}$$

$$= 0.0487 \times 100\%$$

$$\boxed{\zeta = 4.87\%}$$

(b): $T_n = ?$

As seven cycles are completed in "3.57" sec.
Thus time required to complete one cycle;

$$= \frac{7}{3.57} = 1.96 \text{ sec}$$

$$\boxed{T_D = 1.96 \text{ sec}}$$

Now;

$$\omega_D = \omega_n \sqrt{(1 - \zeta^2)}$$

$$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n} \left(\sqrt{(1-\zeta)^2} \right)$$

As;

$$T_D = T_n \Big/ \sqrt{(1-\zeta^2)}$$

$$\Rightarrow T_n = T_D (1-\zeta^2)$$

$$= 1.96 \left(\sqrt{1-(0.0487)^2} \right)$$

$$\boxed{T_n = 1.957 \text{ sec}}$$

⇒ Natural period of undamped vibration.

©: Stiffness of Structure, K = ?

As;

$$K = \frac{F \cos\theta}{2}$$

$$K = \frac{60 \cos(60°)}{2} \qquad \left(\begin{matrix} F = 60\text{kips} \\ \theta = 60° \end{matrix} \right)$$

$$K = 15 \text{ k/in}$$

$$\boxed{K = 18000 \text{ lb/ft}}$$

ⓓ: Weight of the tank, W = ?

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K.g}{W}}$$

$$\Rightarrow W_n^2 = \frac{k.g}{W} \Rightarrow (W = k.g/W_n^2)$$

By putting values of $W_n = 2\pi/T_n$

$$W_n = k.g/\left(\frac{4\pi^2}{T_n^2}\right) = k.g\,(T_n^2/4\pi^2)$$

$$W_n = 18000\text{lb/ft} \cdot 32.2\text{ ft/sec}^2 \left(\frac{(1.957)^2}{4(3.14)^2}\right)$$

$$W = 56284.75\text{lb} \Rightarrow \boxed{W = 56.284 \text{ Klb}}$$

(e): $c = ?$

It is know that $\zeta = \frac{c}{2m\omega_n}$

$$c = \zeta(2m\omega_n) = \zeta\left(2m\left(\frac{2\pi}{T_n}\right)\right)$$

By putting values

$$c = 0.0487\left(2\left(\frac{56284}{32.2}\right)\left(2(3.14)\right)\right) \Big/ 1.957$$

$$\boxed{c = 546.332 \text{ lb.sec/ft}}$$

(f): No. of cycles to reduse displacement amplitude from "6.872 in to 0.5in", $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2 \times 3.14 \times 0.0487} \ln\left(\frac{7.695}{0.9}\right)$$

$$= \frac{2.145}{2 \times 3.14 \times 0.0487} = 7.013$$

$$\boxed{J = 7 \text{ cycles}}$$

Ans