Answer # 1

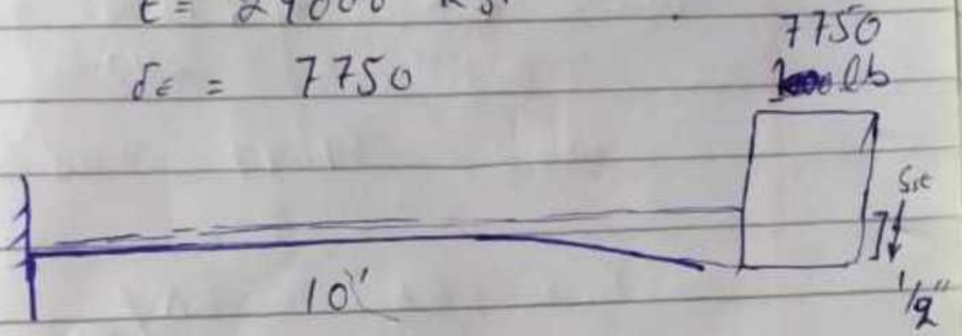
Given Data :-

Deflection = 1/2"

$I = 150 \; in^4$

$E = 29000$ ksi

$\delta_E = 7750$



Solution:- The genral E.O.M for SDOF system is.

$$ku + cu' + m\ddot{u} = P(t)$$

n our case system undamped (c=0) under going free vibration P(t)=0

$$k = 3EI/L^3$$

$$k = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$k = 7.55 \; k/in$$

n order to eliminate the chance of mistake during calculation it more appropriate

$$k = 7.55 \; k/in = 90625 \; lb/ft$$

$$m = \frac{7750 \text{ lb sec}^2}{32.2}$$

$$m = 240.68 \text{ slug}$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{96025}{240.68}}$$

$$\omega_n = 19.97 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} = 2\pi / 19.97$$

$$T_n = 0.31 \text{ sec}$$

Substitüig

$$90625 U + 240.68 \ddot{U} = 0$$

where k in lb/ft & m lb sec/ft^2
General solution to EOM for undamped free vibration

$$U(t) = U(0) \cos(\omega_n t) + U(0/\omega_n \sin(\omega_n t)$$

$$U(0) = \frac{1}{2}'' = 1/24 \text{ ft} \quad \& \quad \dot{U}(0) = 0$$

equilivent static force

$$F_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.97)}{48}$$

$$F_s(t) = 1888 \cos(19.97)$$

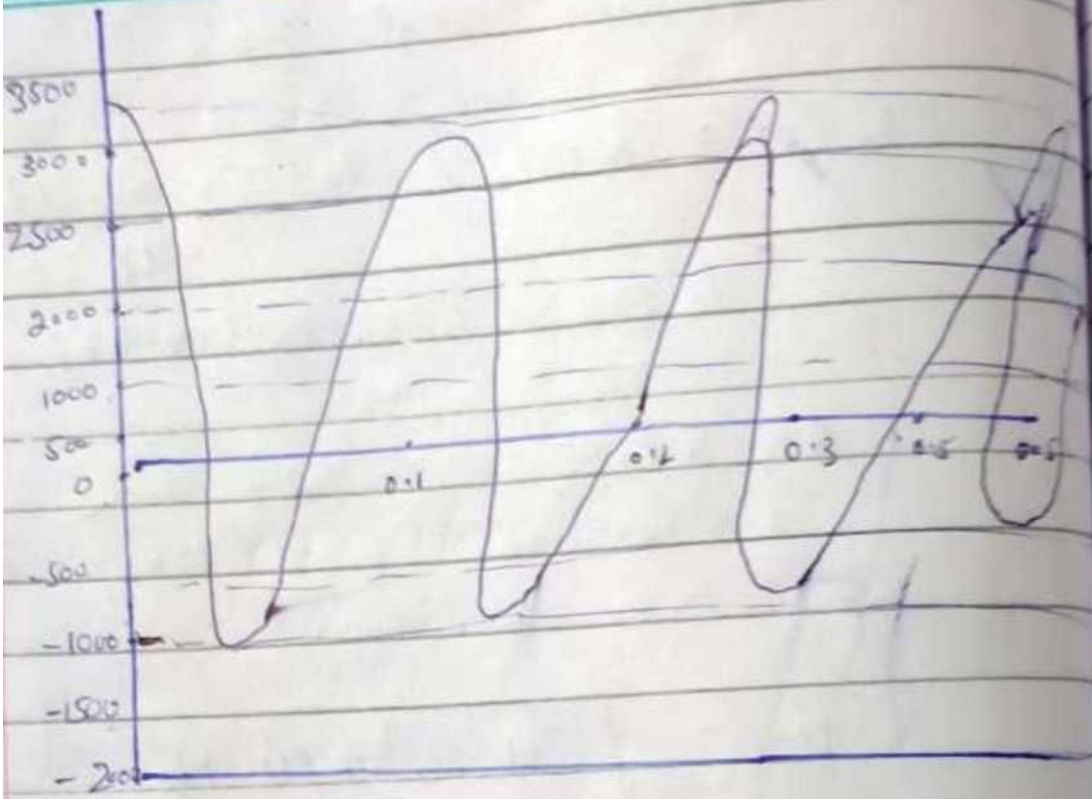
Amplitude of dynamic displacement, u_0 for undamped free vibration is -

$$u_0 = \sqrt{(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)}$$

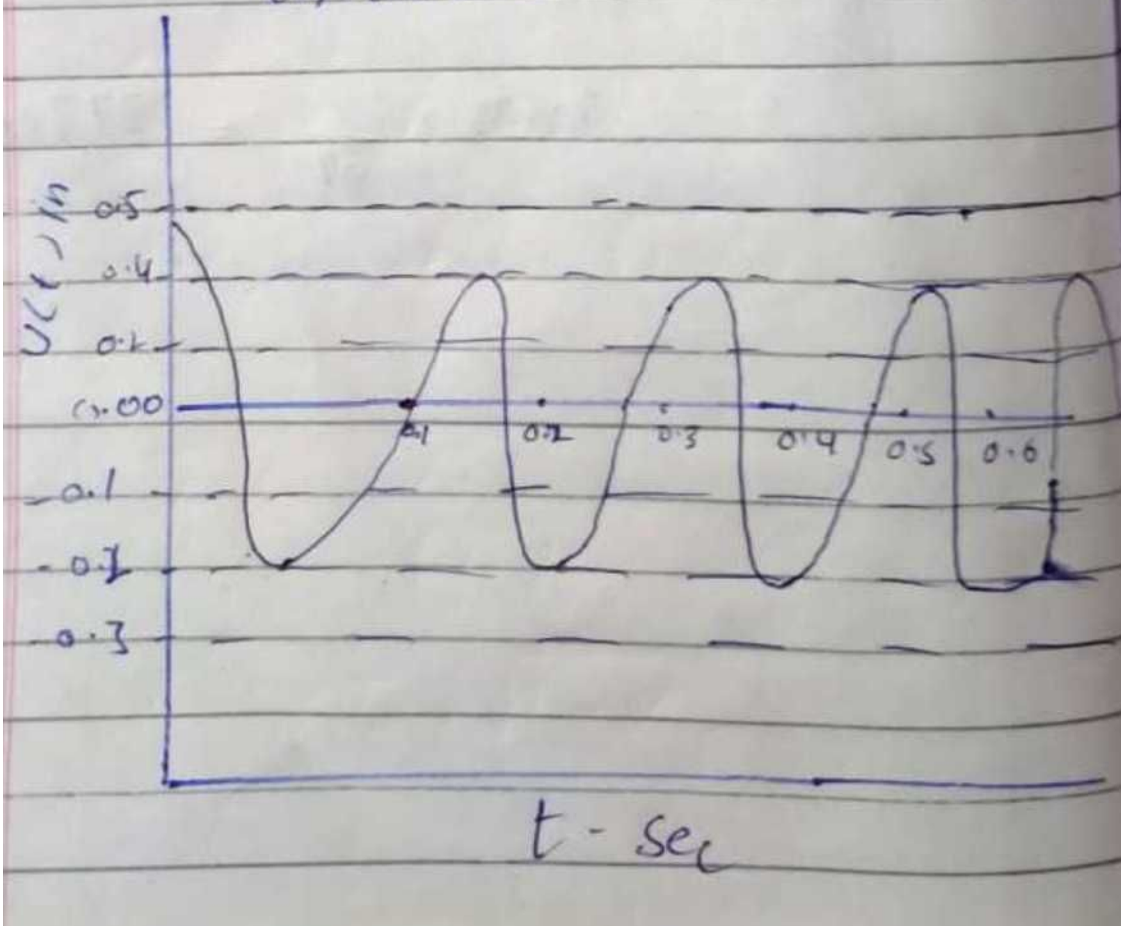
$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0} = \frac{1}{24} ft$$

$$ku = 90625 \times \frac{1}{24} = 3776 lb$$

Un dampe Free vibration



Un-damped Free Vib



Answer # 2

Sol:

Given Data.

$E = 29000$ ksi

$I = 150 \ in^4$

$f_{st} = 7750$ lb

$\xi = 2.5\%$

Sol:

E.q^n = for free vib

$ku + cu + m\ddot{u} = 0$

$K = 90625$ lb/ft

~~$m = 238$~~

$m = 240.68$ lb

$c = \xi = 2m\omega_n$

$= 0.025 \times 2(240.68)(20.04)$

$c = 241.1$ lb sec/ft

by Substituting

$ku + c\dot{u} + m\ddot{u} = 0$

$90625 + (240.68) + (240.68)$

Sol: in the EOM for damped free vib.

$$v(t) = e^{-\zeta \omega_n t}\left[v(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left(v(0) + \zeta\omega_n\right)\sin(\omega_D t)\right]$$

$$\omega_D = \sqrt{\frac{K}{m}} = \sqrt{\frac{906254}{240.68}}$$

$$\omega_D = 19.40 \ \text{rad/sec}$$

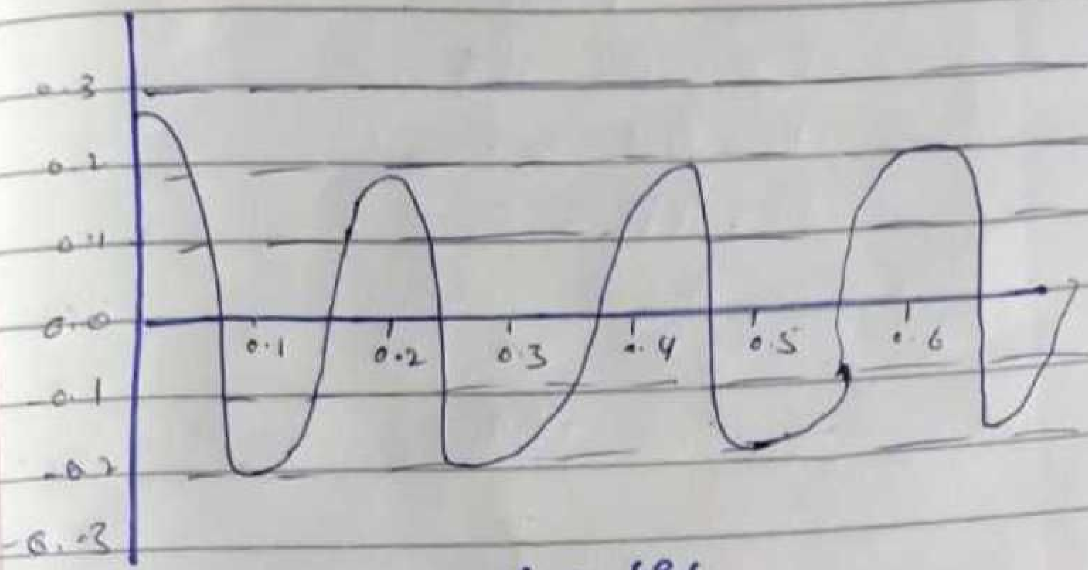
$$U(t) = e^{-0.025 \times 19.40}\left[\frac{1}{24} \times \cos(19.40) + (0.05136) \times 0 + \frac{1}{24} \times 0.025 \sin(19.40)\right]$$

$$U(t) = e^{-0.48675}\left[0.041667 \times \cos(19.40) + 0.001042 \sin(19.40)\right]$$

$$F_{st} = k\,U(t) = 90625 \times U(t)$$

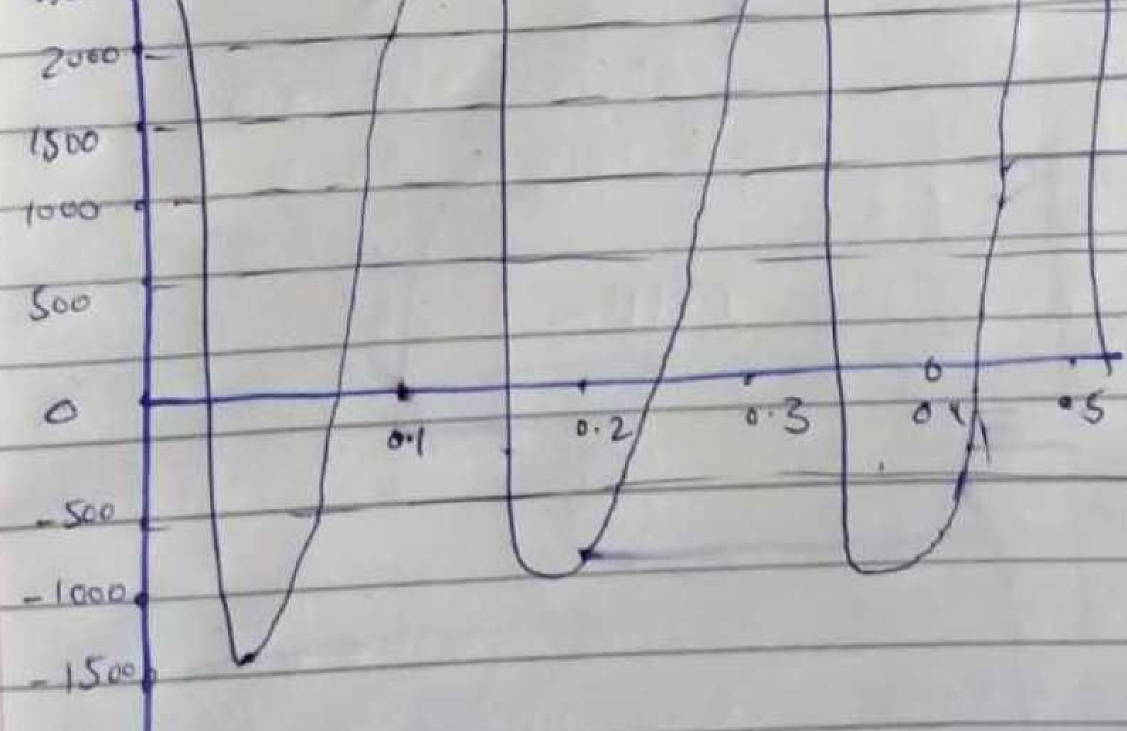
$$F_s(t) = e^{-0.48675}\left[3776 \cos(19.47) + 94.34 \sin(19.40)\right]$$

Damped Free vib



t = sec

Damped Free vib



(Answer # 3)

Given Data :-

Force = 60 kip

Horizontal displace = $D/1000$ $= \frac{7750}{1000} = 7.75 in$

Cycle = 7

time = 3.57 sec

amplitude displacement = 2.286 cm = 0.9"

(1) Damping ratios = ?

$\xi = ?$

$j = 7$, $u_{j+1} = u_8 = 2.286$.

$u_1 = 7.75 in$

$$\Rightarrow 7 = \frac{1}{2\pi\xi} \ln \left(\frac{u_1}{u_{j+1}} \right)$$

$$\Rightarrow 7 = \frac{1}{2\pi\xi} \ln \left(\frac{7.75}{0.9} \right)$$

$$7 = \frac{1}{2\pi\xi} (2.15)$$

$$\frac{2\pi\xi 7}{2\pi 7} = \frac{2.15}{2\pi \times 7}$$

$$\xi = 0.04 = 4.88\%$$

(b) Natural period of un-damped vibration = ?

Solutions:-

$T_n = ?$

For T_n 7 cycle of vib are complete in time 3.57 sec

Time required to complete one cycle $\frac{3.57}{7} = T_D$

$T_D = 0.51$ sec

Now.

$\omega_D = \omega_n \sqrt{(1 - \xi^2)}$

$2\pi/\omega_D = 2\pi / \omega_n \sqrt{1 - \xi^2})$

$T_D = \frac{T_n}{\sqrt{1 - \xi^2}}$

$T_n = T_D \times \sqrt{1 - \xi^2}$

$T_D = 0.51 sec$

$T_n = 0.51 \times \sqrt{1-(0.04)^2}$

$T_n = 0.51 \times 0.99$

$T_n = 0.51 sec$

c) Stiffness of structure.

$k = \frac{60 \times \cos(60°)}{2} = 15$ k/in = 1800 lb/ft

d) Weight of tank = ?

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{w/g}} = \sqrt{\frac{k \cdot g}{W}}$

$\omega_n^2 = k \times g / W$

Also $\omega_n = 2\pi / T_n$

$\Rightarrow W = kg \left(\frac{4\pi^2}{T_n^2}\right) = kg \times \frac{T_n^2}{4\pi^2}$

$W = \frac{\frac{180000 \text{ lb}}{ft} \times \frac{32.2 \text{ ft}}{sec^2} (0.51)^2}{4\pi^2}$

$W = 74875$ lb $= 74.9k$

e Damping coefficient = ?

$C = ?$

its know that

$\zeta = \frac{c}{2m\omega_n}$

$c = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi/T_n)$

$c = 0.04 \times 4 \times \pi \left(\frac{7.4275}{22.2} \right)$

$\overline{\quad 0.51 \quad}$

$C =$ ~~3… …~~ 1455.3 lb…

(f) No of cycle to reduce displacement amplitude 7.75in to 0.5"

Solution: $j = \frac{1}{2\pi\zeta} \ln \left[\frac{v_1}{v_{j+1}} \right]$

$j = \frac{1}{2 \times \pi \times 0.04} \ln \left[\frac{7.75}{0.5} \right]$

$j = 3.97 \ln \left[\frac{7.75}{0.5} \right]$

$J = 10.88$

$J = 11$ cycle.

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Paper Earth quick