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Subject: DSP

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①

Q# 1 (a) Determine the response $y(n), n \geq 0$, of system described by the second order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$. And the initial condition are $y(-1) = y(-2) = 0$

Sol:-

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence

$$y(h)(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Subsituting this solution into the difference equation we obtain

$$\Rightarrow K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-1) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n=2$, $K(1+4+4)=2$

$K=2/9$ The total solution is

$y(n)=[C_1 2^n + C_2 n 2^n + 2/9(-1)^n]u(n)$

From the initial conditions

we obtain, $y(0)=1$, $y(1)=2$

Than

$$C_1 + 2/9 = 1$$

$$\Rightarrow C_1 = 7/9$$

$$\Rightarrow 2C_1 + 2C_2 - 2/9 = 2$$

$$\Rightarrow C_2 = 1/3$$

Q# (1)

(b) Determine the impulse response & unit step response of the systems described by difference equation

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution:-

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = 1/2, \; 1/5$ Hence

$$y_h(n) = C_1 \, 1/2^n + C_2 \, 1^n/5$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7\,y(0) = 0$$

$$\Rightarrow y(1) = 1.4$$

Hence, $C_1 + C_2 = 2$

And

$$1/2 \, C_1 + 1/5 = 1.4$$

$$1.4 = 7/5$$

$$\Rightarrow C_1 + 2/5 \, C_2 = 14/5$$

These equations yield

$C_1 = 10/3, \quad C_2 = -4/3$

$h(n) = [10/3\,(1/2)^n - 4/3\,(1/5)^n]\,u(n)$

The step response is

$$S(n) = \sum_{K=0}^{n} h(n-K)$$

$$S(n) = 10/3 \sum_{K=0}^{n} (1/2)^{n-K} - 4/3 \sum_{K=0}^{n} (1/5)^{n-K}$$

$$S(n) = 10/3\,(1/2)^n \sum_{K=0}^{n} 2^K - 4/3\,(1/5)^n \sum_{K=0}^{n} 5^K$$

$$S(n) = 10/3\,(1/2)^n (2^{n+1} - 1)\,u(n) - 1/3$$

$$(1/5)^n (5^{n+1} - 1)\,u(n)$$

Q# 2 (a)

Determine the causal signal $x(n)$ having the z transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

(Hint Take inverse z-transform using partial fraction method.

Sol:-

Taking inverse of z-Transform

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{z^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence

$$x(n) = [4(2)^n - 3 - n]\,u(n)$$

Q#2

(b) Perform the circular convolution of the following two sequences. Solve the problem step by step

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Sol:-

Eeach sequence consist of four non-zero Points For the purpose of illustrating the operations involved in circular convolution it is desired to graph each. Sequence as points on a circle, Thus the sequence $x_1(n)$ and $x_2(n)$ are graphed as illustrated we note that the sequence are graphed in a counterclock wise direction on a circle.

Now, $x_3(m)$ is obtained by circularly convalving $x_1(n)$ with $x_1(n)$ with $x_2(n)$ as specified begning with $m=0$ we have

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle. The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point Finally we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

it is easily verified that $x_2(1-n)_4$ it is simply the sequence $x_2(-n)_4$ rotated counter clockwise by one unit in the time.

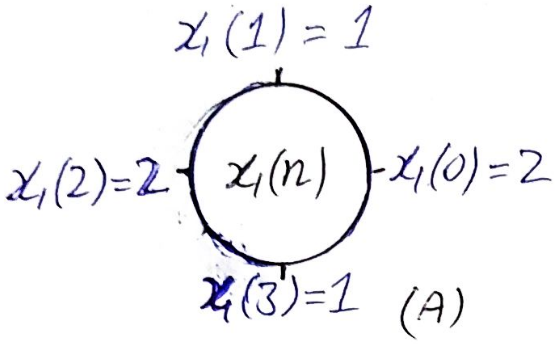
⇒ This rotated sequence multiplies $x_1(n)$ to yield the product sequence also finally we sum the values in the product sequence to obtain $x_3(1)$ Thus

$$x_3(1) = 16$$

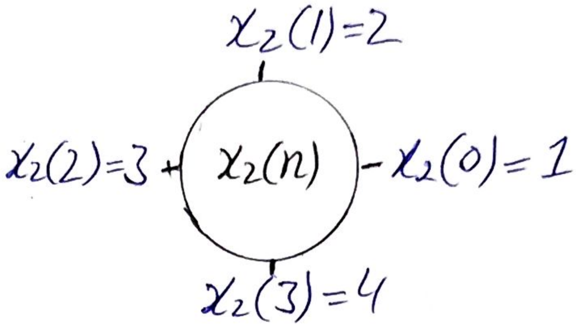
For $m=2$ we have

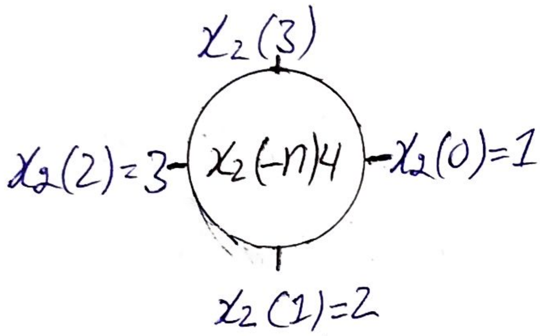
$x_3(2) \sum_{n=0}^{3} x_1(n)\, x_2\,(2-n)_4$ is the folded

Sequence.

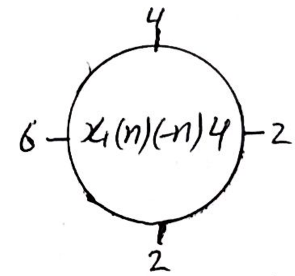


(A)

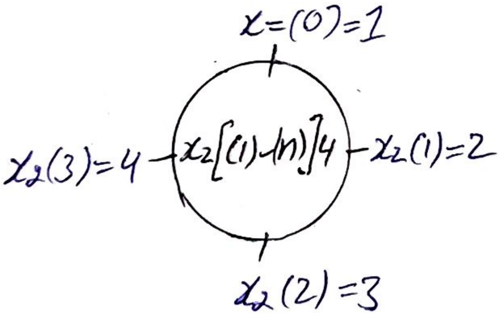




Folded Sequence (B)

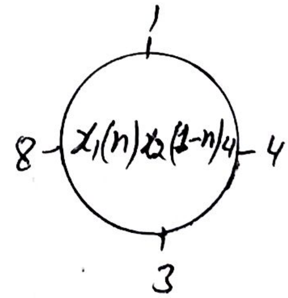


product Sequence

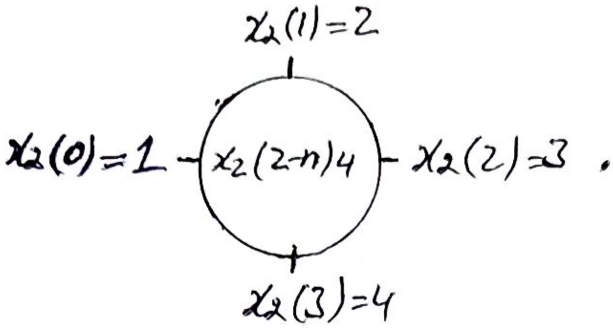


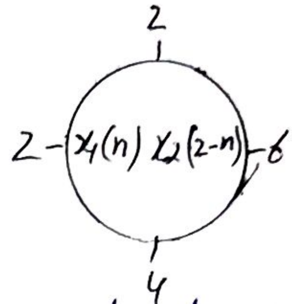
(C)

Folded sequence rotated by one unit in time



product Sequence

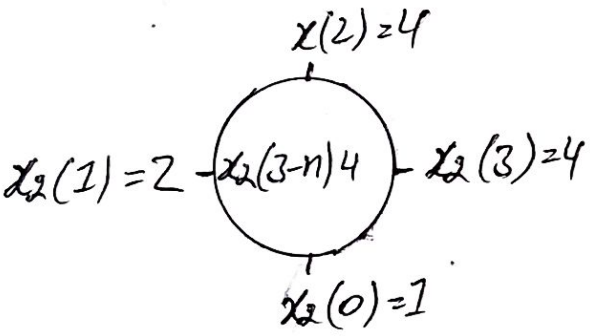


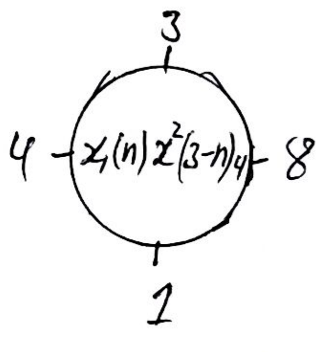


(D)

Folded Sequence rotated by two unit in time

product Sequence





(E)

=> Folded Sequence rotated by three unit in time

product Sequence

Q#3

(a) A two-pole low pass filter has the system response

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

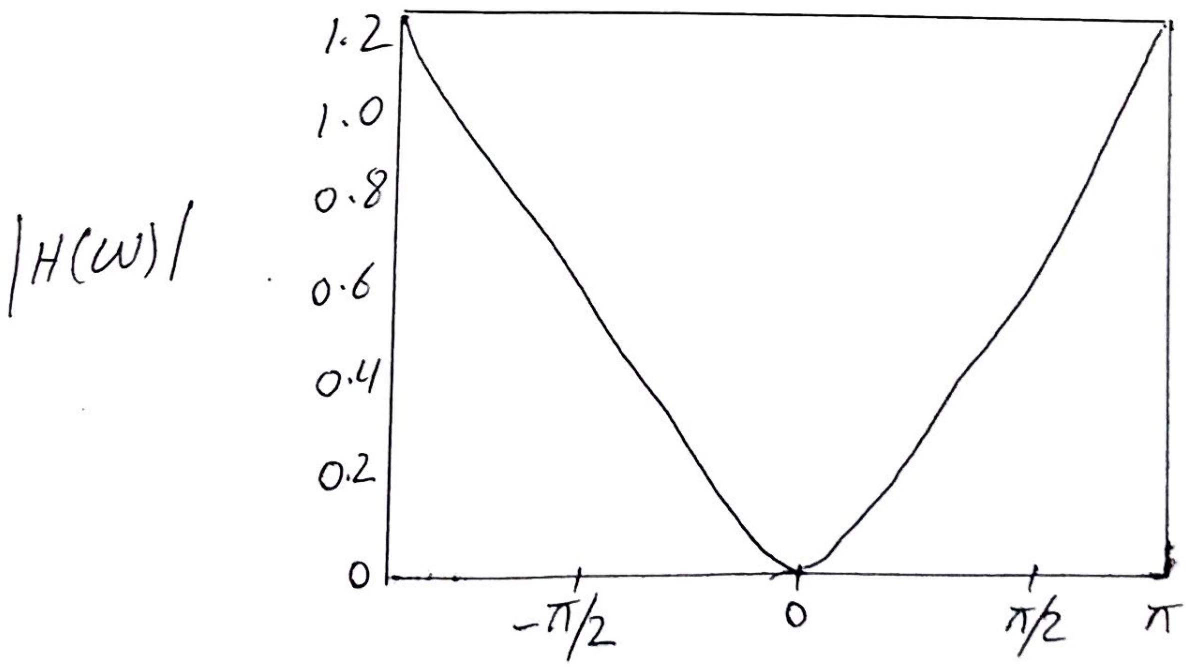
Determine the values of b_0 & p such that the frequency response $H(\omega)$ satisfy the condition $H(0)$ 1 & $|H(\pi/4)|^2 = 1/2$

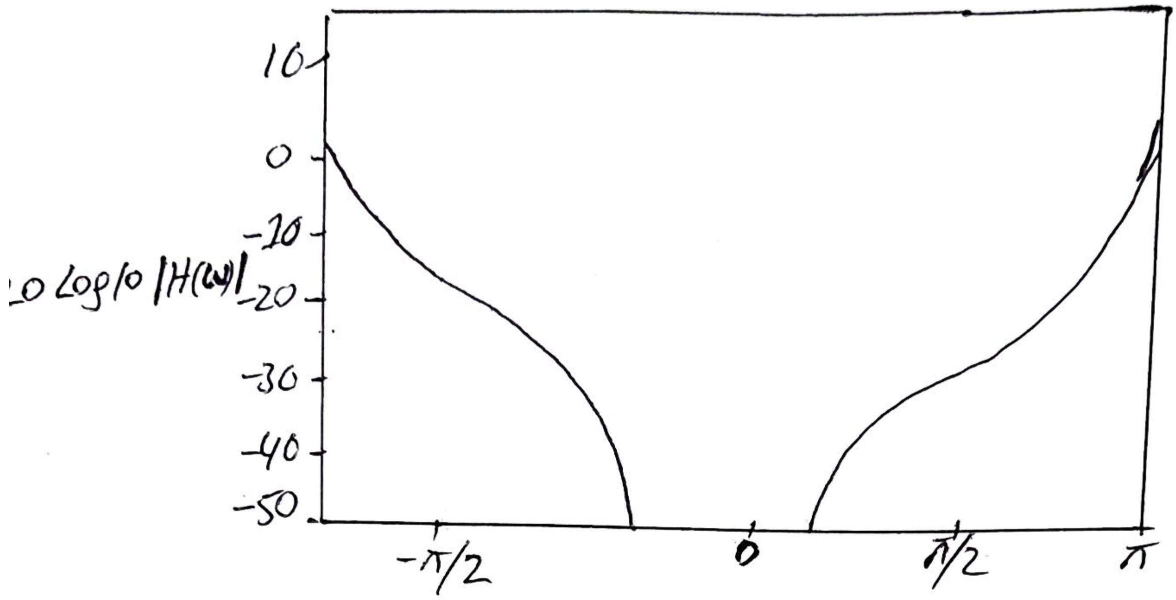
Sol:

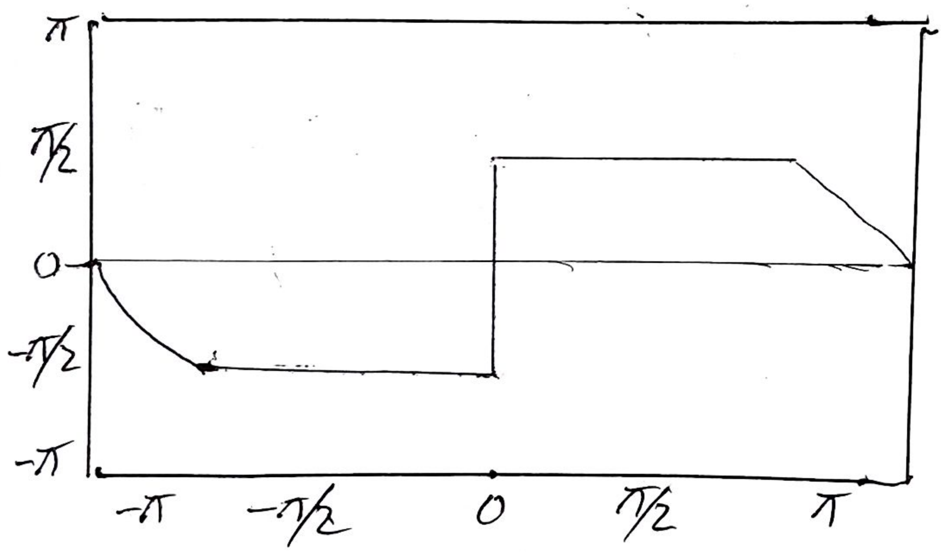
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{[1-P\cos(\pi/4)+jP\sin(\pi/4)^2]}$$

$$= \frac{(1-P)^2}{[1-P/\sqrt{2}+jP/\sqrt{2}]^2}$$

Hence

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2}$$

$$= 1/2$$

$\Rightarrow$ Equivalently

$$\sqrt{2}\,(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

The system Function for desired filter

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q#3

(b) Design a two-pole bandpass filter that has center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ & $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{3}}$ at $\omega = 4\pi/9$

Sol:-

The filter must have poles

at $P_{12} = \gamma e$

And zero at $z = 1$ And $z = -1$

Consequently. the same system function is

$$\Rightarrow \quad H(z) = G_1 \frac{(z-1)(z+1)}{(z-j\gamma)(z+j\gamma)}$$

$$H(z) = \frac{G_1 z^2 - 1}{z^2 + \gamma^2}$$

The gain factor is determined evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

$$H = (\pi/2) = G \frac{2}{1-\gamma^2} = 1$$

$$G = \frac{1-\gamma^2}{2}$$

The value of γ is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$

we have

$$[H(4\pi/9)]^2 = \frac{(1-\gamma^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+\gamma^4+2\gamma^2\cos(8\pi/9)}$$

$$= 1/2$$

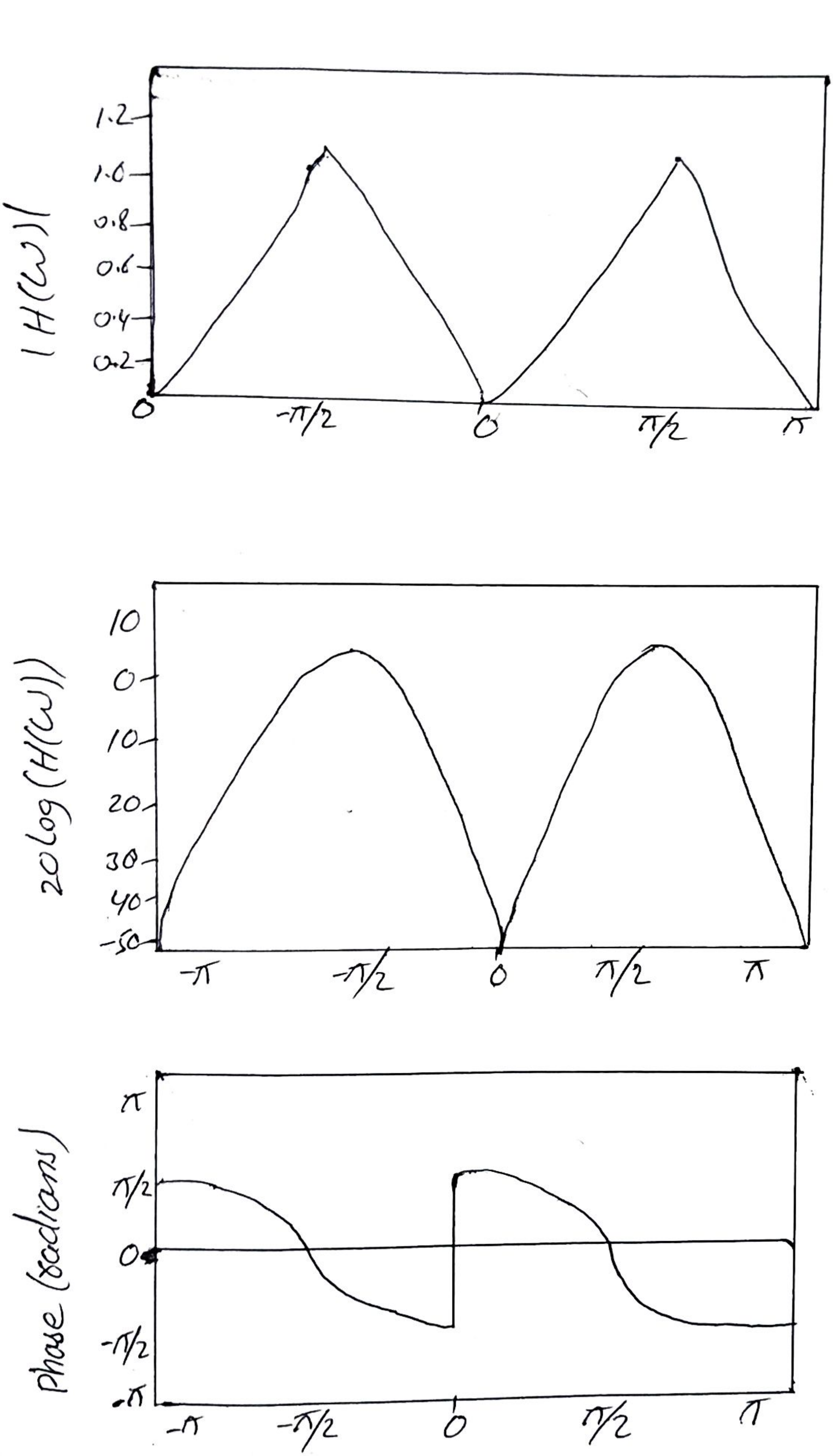
$\Rightarrow$ Equivalently

$$1.94(1-\gamma^2) = 1-1.88\gamma^2+\gamma^4$$

The value of $\gamma^2 = 0.7$ Satisfies this equation therefore the system function for desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

Its frequency response is illustrated



⇒ Magnitude & phase response of a simple bandpass filter is

$$H(z) = 0.15 \left[(1-z^{-2}) / 1+0.7z^{-2}) \right]$$