IQRA National University Peshawar

Name Amir

ID 16436

Department BS(CS)

Semester 2th

Subject Discreat sturture

~~Sumbi~~

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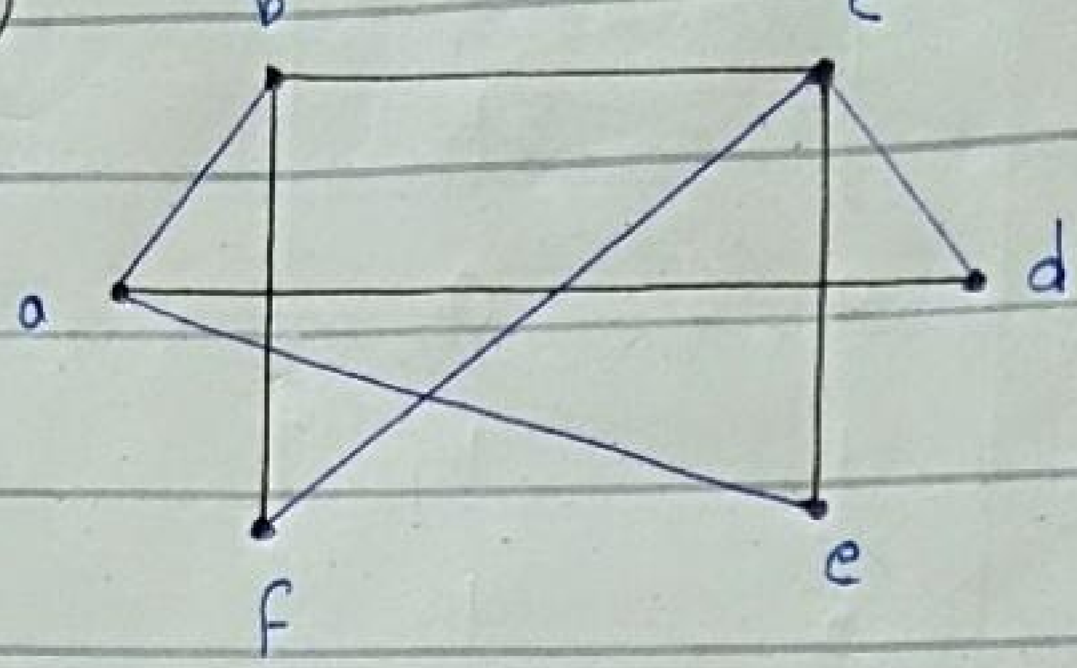
Data : 24/6/2020

Assignment : 3

Question 1

(i)



This is NOT bipartite.

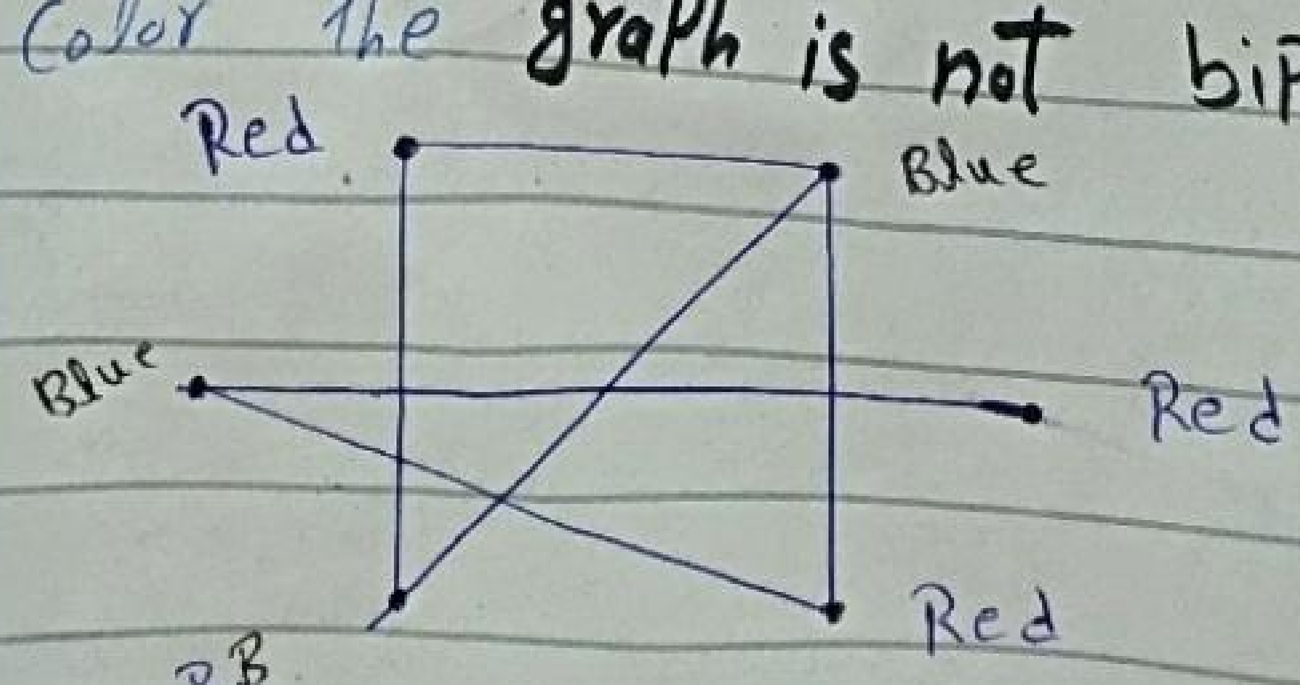
Bipartite Graph =>

A bipartite graph is a simple graph whose vertices can be partitioned into set V_1 and V_2 such that there are no edges among the vertices of $V1$ and no edges among the vertices of V_2, while there can be edges between a vertex of V_1 and vertex of V_2.

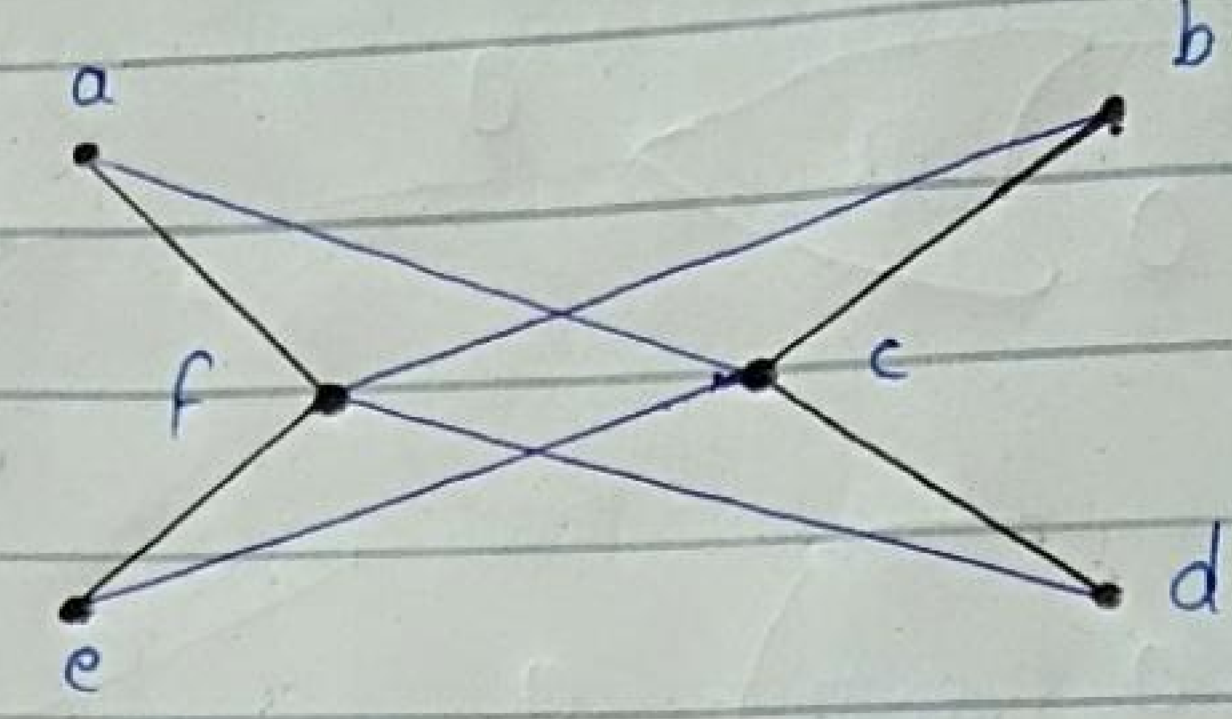
A simple graph is bipartite if and only if it possible to assign red or blue to every vertex such that no two connected vertices have the same color.

Let us assign red or blue to each vertex.

If two vertices are connected, then they Should not have the color.
we start by assigning 'Blue' to a.
• Since b is connected to the blue ~~to b, Since c is connected to the~~ red a, we assign "Red" to b.
• Since c is connected to the red b, we assign "Blue" to c.
we then note that f is connected to the red b and the blue c, which means that we cannot assign a color to f such that it differ from the color of the connect vertices. (As there are only two color. Red & blue)
Thus it is not Possible to assign red or blue to each vertex such that connect vertices do not have the same color the **graph is not** bipartite.



(ii)



Solution =)

This is biPartite.

A simPle graPh is biPartite if and only if it is Possible to assign red or blue to every vertex such that no two Connected vertices have the Same color.

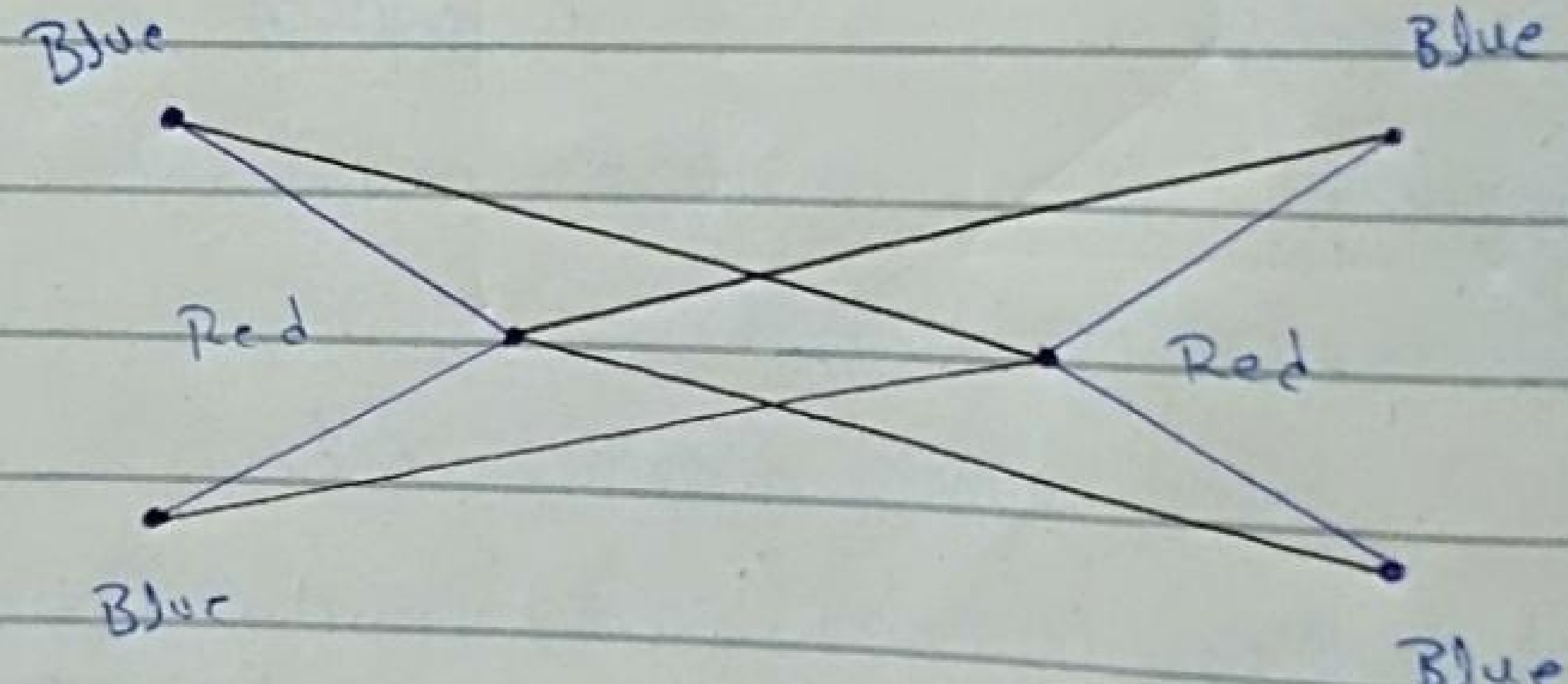
Let us assign red or blue to each vertex.

If two vertices are Connected, then Should not have the Same color. we then note that is Possible to assign red or blue to each vertex such that connected vertices do not have the Same color and thus the graph is biPartite.

Moreover, the Partitioning of the vertices and the set with the red vertices

$$V_1 = \{a, b, c, d\}$$

$$V_2 = \{e, f\}$$

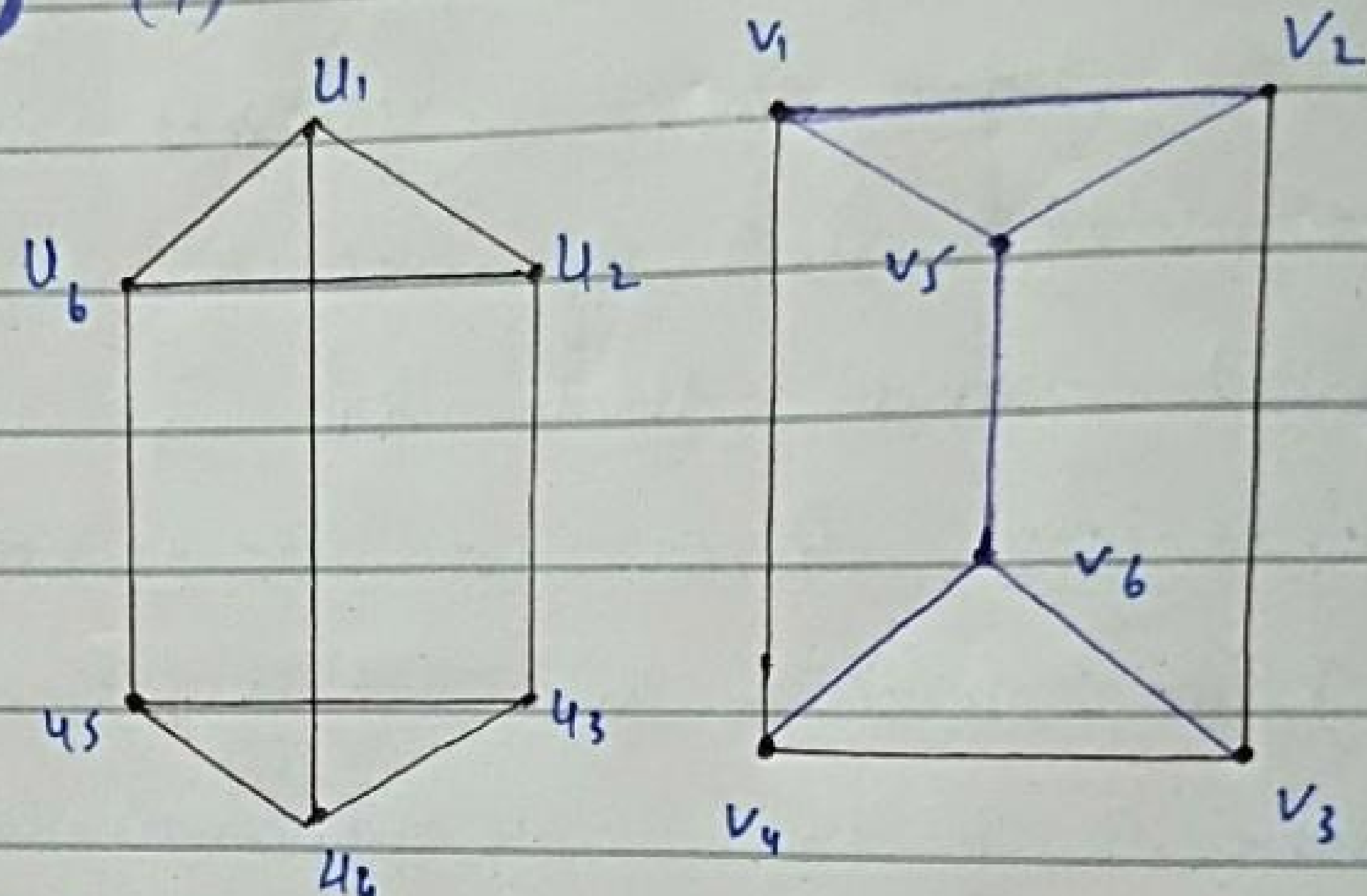


Graph is BiPartite.

Question 2

Graph is Isomorphic

(a) (i)



Solution

Let us first determine set of vertices of edge.

$V_1 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$

$E_1 = \{(u_1, u_2), (u_1, u_4), (u_1, u_6), (u_2, u_3), (u_2, u_6)$
$(u_3, u_4), (u_3, u_5), (u_4, u_5), (u_5, u_6)\}$

Let us the first determine set of vertices and set of edge right graph.

$V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$E_2 = \{(v_5, v_2), (v_5, v_6), (v_5, v_1), (v_2, v_3)$
$(v_2, v_1), (v_3, v_6), (v_3, v_4), (v_6, v_4), (v_4, v_1)\}$

By computing the two set of edge we can define the following

one to one and onto funicatio
f from V_1 to V_2.

Note: you could also use the degree of the vertice because and their image need to have same degree.

$f(U_1) = V_5$ $\quad f(U_2) = V_2$

$f(U_3) = V_3$ $\quad f(U_4) = V_6$

$f(U_5) = V_4$ $\quad f(U_6) = V_1$

f is then a funicution that make the two graph isomorphic

U_1 and U_2 are adjacent $f(U_1) = V_5$ and $f(U_2) = V_2$ are adjacent U_1 and U_4 are adjacent, while $f(U_1) = V_3$ and $f(U_4) = V_6$ are adjacent. U_1 and U_6 are adjacent while $f(U_1) = V_5$ and $f(U_6) = V_1$ ~~This is Isomorphic~~ are adjacent.

U_2 and U_3 are adjacent, while $f(U_2) = V_2$ and $f(U_3) = V_3$ are adjac

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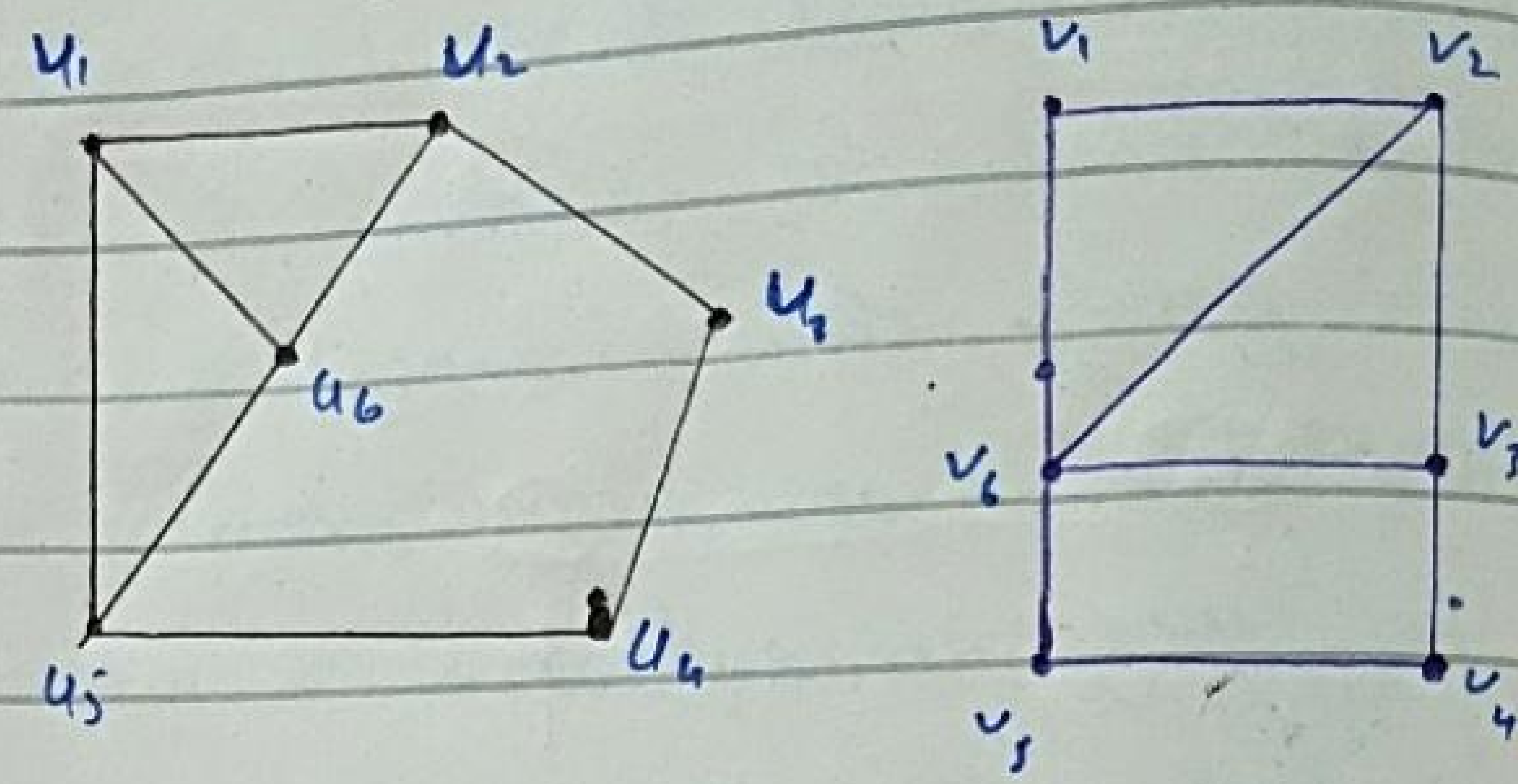
u_3 and u_4 are adjacent, while $f(u_2) = v_2$ and $f(u_6) = v_1$ are adjacent.

u_3 and u_5 are adjacent, while, v_6 and $f(u_5) = v_4$ are adjacent.

u_5 and H_6 are adjacent, while $f(u_5) = v_4$ and $f(u_6) = v_1$ are adjacent.

isomorphic.

Q=2 (b)



Let us the deter mine

$V_1 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$

$E_1 = \{(u_1, v_1), (u_2, v_4), (u_1, u_6), (u_2, v_3)$
$(v_2, v_6), (v_3, u_4) (v_3, u_5) (v_5, u_6)$

Let us first determine of vertices and set of edges

$V_2 = \{v_1, v_2, v_3, v_4, v_5, v_5\}$

$E_2 = \{u_5, v_2) (v_5, v_6), (v_2, u_3) (v_2, v_3)$
$(v_2, v_1) (v_3, v_6) (v_3, v_4) v_6, v_4)\}$

$f(u_1) = v_5$

$f(u_2) = v_2$

$f(u_3) = v_3$

$f(u_4) = v_6$

$f(u_5) = v_4$ $f(u_6) -$

f is then a funiction that make two graph ~~im~~ isomophic.

This is also isomophic

Question 3

(a)

$$= \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Solution:

Let us add all element in the two matrixes, which will represent the number of connection of a vertex to another vertex which is double the number edge.

A contains 8 ones and thus the simple graph corresponding to A contain 8 connection.

B contains 10 ones and thus the simple graph corresponding to A contain 10 connections

Since the number of connection of the two graph are not the same, the number of edges in the graph are not the same and then the Graph are not isomorphi...

b

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Solution:

Let us add all element in the two matrices, which will represent the number of connection of a vertex to another vertex.

A Contain 8 ones and thus the simple graph corresponding to A contain 8 connection. B Contain one and thus the simple graph corresponding to A contain 6 connection.

Since the number of connection of the two graph are not the same, the number of edges in the graph are not the same and then the Graph are not isomorphic.

Question 4

(a)

Euler circuit ⇒

An Euler circuit is a simple circuit has contain every edge of the graph.

A Path in a directed graph G is a sequence of edges in G.

A simple Path is a Path that does not contain the same edge more than once. A circuit is a Path that begin and ends in the same vertex.

The degree of a vertex is the number of edges that connect to the vertex.

Let us first determine the degree of every vertex in the given graph.

deg (a) = 2   deg (b) = 4

deg (c) = 2   deg (d) = 4

deg (e) = 4   deg (f) = 4

deg (g) = 2   deg (h) = 4

deg (i) = 2

A graph has an Eular circuit if and only if each of the vertices has a even degree. Since degree are even, there exist an Euler circuit.

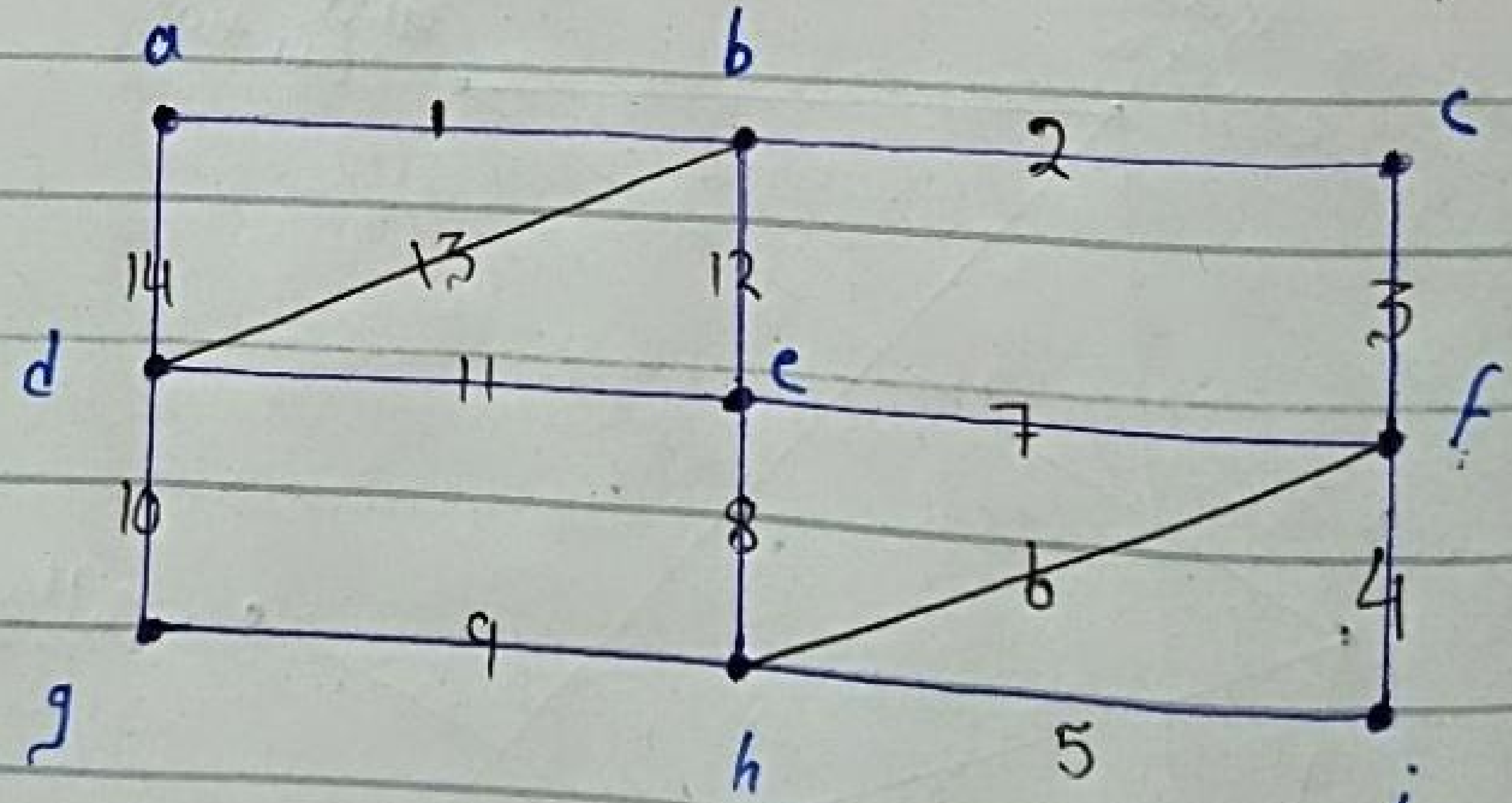
deg (g) = 2 deg (h) = 4

deg (i) = 2

A graph has Euler cirecuit if and only if each of the vertices has an even degree. Since all degree are even, there exist Euler circuit.

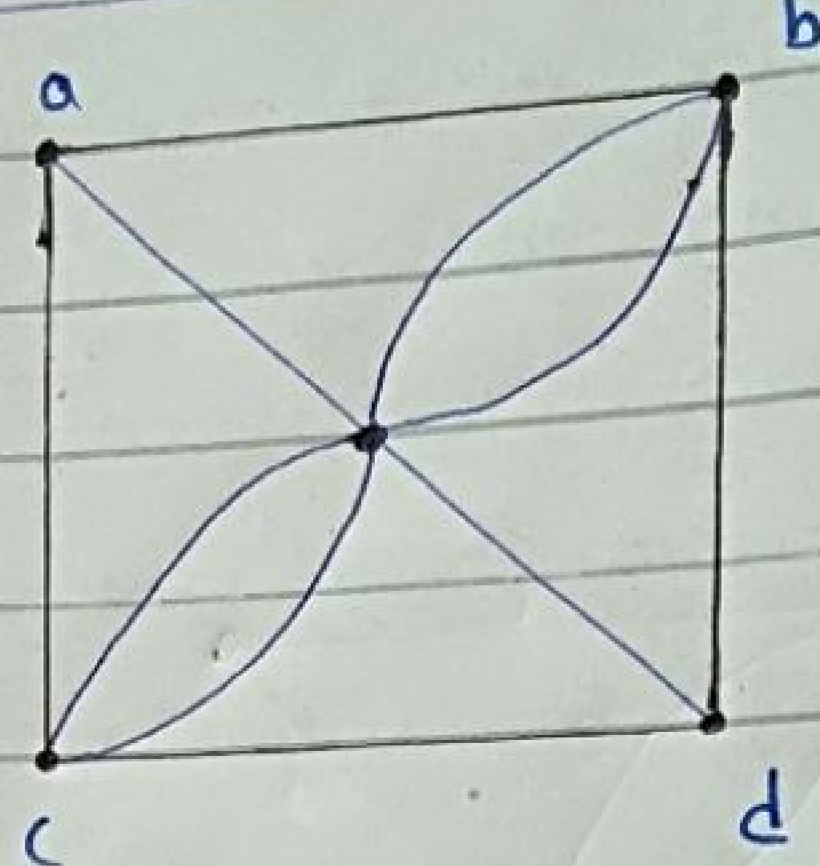
A Possible Euler circuit is

a, b, c, d, f, i, h, f, c, h, g, d, c, b, d, a



This is Euler circuit exist.

b



Solution:

Let us first determine the degree of every vertex in the given graph.

deg (a) = 3 deg (b) = 4

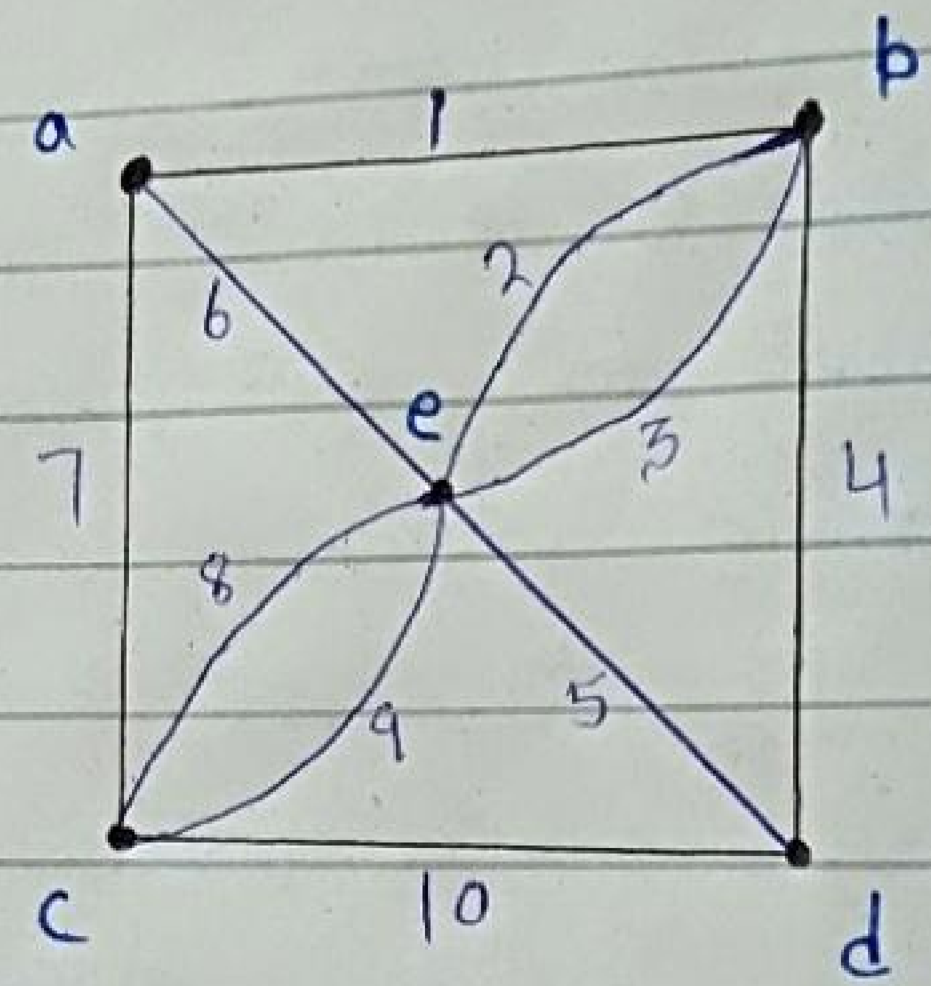
deg (c) = 4 deg (d) = 3

deg (e) = 6

A graph has an Euler circuit if and only if each of the vertices has a even degree. Since some degree are odd, there is no Ewer circuit.

A graph has an Eular Path if and only if there are exactly two vertices who have an odd degree. we note that vertices a and d have an odd degree thus an Euler Path exist.

A Possible Euler Path is
a, b, c, ~~d~~ b, d, e, a, c, e, e, d



No Euler circuit exists
Euler Path exists.

Question 5.

(a)
Hamilton circuit=)
A Hamilton circuit is a simple circuit that passes throug every vertex exactly once.
Dirac's therom A simple graph G with ~~A~~ n vertices ($n \geq 3$) has Hamilton circuit, if the degree of every vertex in G is at least $n/2$. A Path in a directed graph G is a sequence of edge in G. A simple Path is a that does not contain the same edge more than once.
A circuit is a Path that begins and in the same vertices.
The degree of a vertice is the number of edge that connect to the vertix. A cut edge in an edge is an edge that when removed from a graph, the resulting graph has more connected component than the original graph.

Let us first determine the degree of every vertex in give

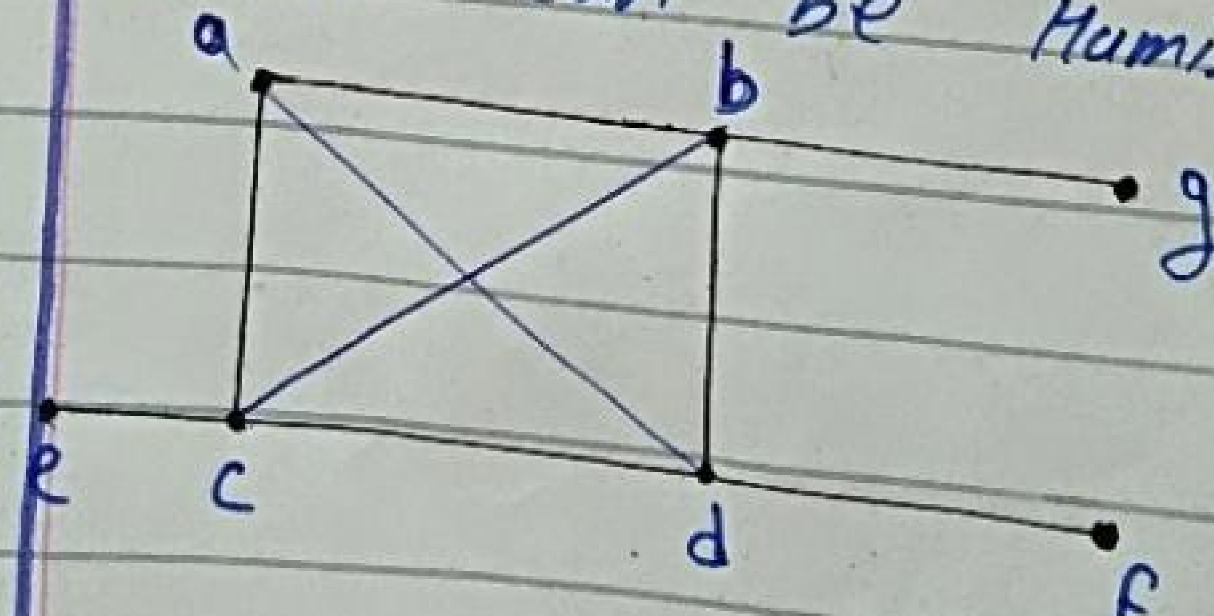
deg (a) = 3 deg (b) = 3

deg (c) = 3 deg (d) = 2

deg (e) = 4 deg (f) = 1

we then note that Dirac, therom is not satified less than $n/2$ $= 6/2 = 3$, but this does not necessorily mean that no Hamilton circuit exist.

However we do note that there is only one edge (c, f) connecting to f and than any circuit that contain f need to Pass through c twice, which mean than no circuit can be Hamilton circuit



Hamilton Circuit does not exist because f has only 1 edge connecting

(b) $Q = 5$

Let us first determine the degree of every matrin.

deg (a) = 2 deg (b) = 4

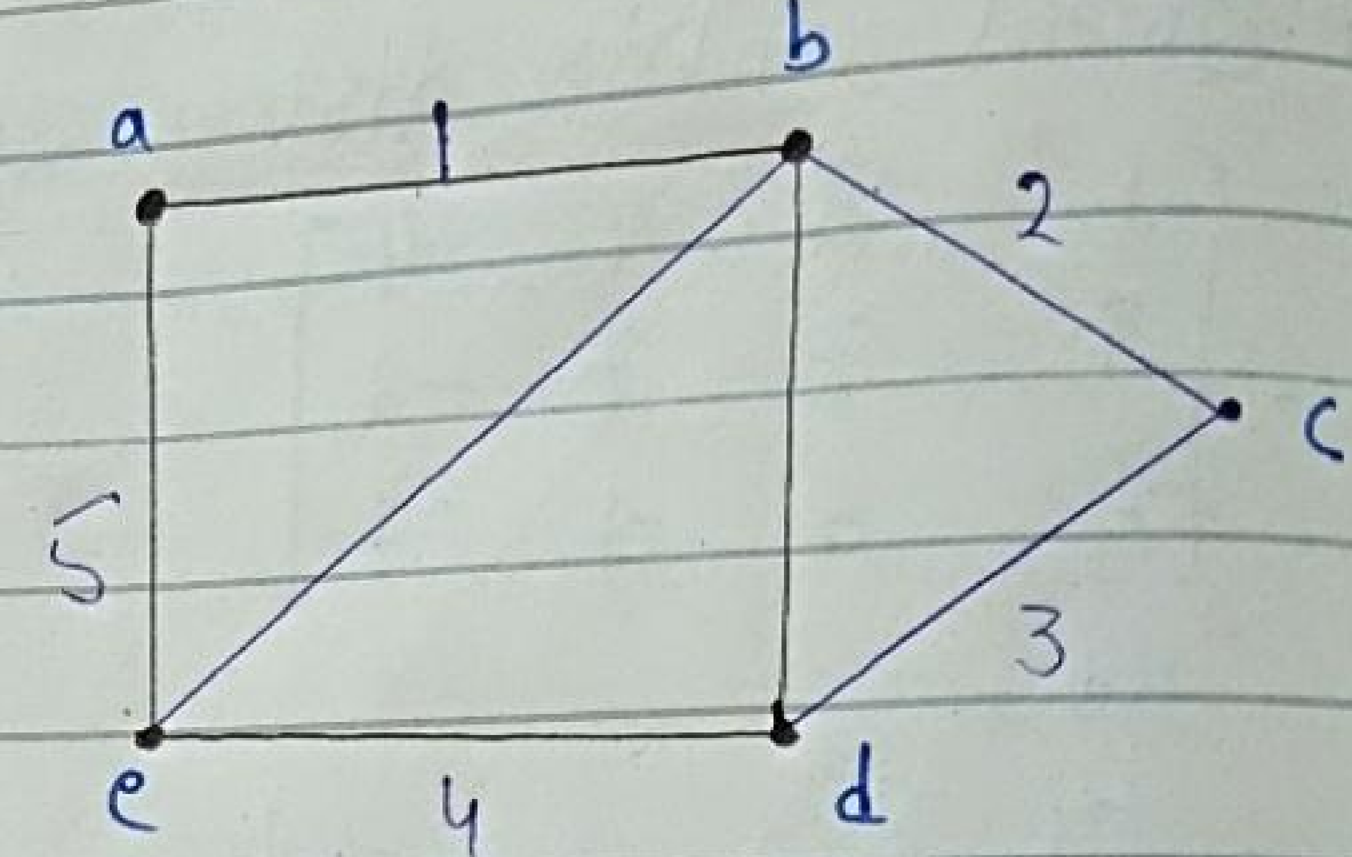
deg (c) = 2 deg (d) = 3

deg (e) = 3

we then note that Dirac's therom is not satisfied less than $n/2 = 5/2 = 2.5$) but this does not necessarily mean that no Hamilton circut enist

However, we do note that given graph graph contain the cycle C_5 and thy cycle C_5 with in the given graph form a Hamilton circuit (as the circuit will pass through all vertices exactly once.

A Possible Hamilton circuit is thus the Path C_5 : a, b c, d, e, a



This is Hamilton Circuit exist.

END