ID: 11461

NAME: ASFANDYAR AWAIS

INSTRUCTOR: ENGINEER MUJTABA

COURSE: SIGNAL AND SYSTEM

Q1 (a) We know that differentiation in domain corresponds to multiplication of jw in frequency domain. From the property, we might suspect that multiplication by jt in the time domain responds roughly to differentiation in frequency domain.

As we know that

$$x(jw) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

Diff. both sides w.r.t w

$$\frac{d}{dw} x(jw) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-jwt}\, dt$$

$$\frac{d}{dw} x(jw) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$\frac{dx}{dw} j(w) = -jt\ \mathcal{F}\{x(t)\}$$

$$\Rightarrow -jt\,(x(t)) \xleftrightarrow{\mathcal{F}} \frac{d}{dt}\, x(jw)$$

②

Q1 (b) if $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Fin $y[n]$

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + 1z^{-1} + 2z^{-2}$

Now

$Y_z = H(z) * X(z)$

$= (2 - 4z^{-2} + 2z^{-3})(3 + 1z^{-1} + 2z^{-2})$

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4}$
$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$

$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

To find $y(n)$, use the delay property

~~$y[n] = 2\delta[n] + 4\delta[\ ]$~~ +

$y[n] = 6\delta[n] + 2\delta(n-1) - 8\delta[n-2]$
$+ 2\delta[n-3) - 6\delta(n-4] + 4\delta[n-5]$

Q1.

a_0 is

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi} \frac{\pi}{2}\,dx\right.$$

$$= \frac{1}{\pi}\left(-\frac{\pi}{2}\int_{-\pi}^{0} 1\cdot dx + \frac{\pi}{2}\int_{0}^{\pi} 1\cdot dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}(x)\Big|_{-\pi}^{0} + \frac{\pi}{2}(x)\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\left((0)-(-\pi)\right] + \frac{\pi}{2}\left((\pi)-(0)\right.\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi^2}{2} + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{\pi}\left[0\right]$$

$= 0.$

Cooffecient a_n

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} \frac{-\pi}{2}\cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\cos nx\,dx$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{2}\int_{-\pi}^{0}\cos nx\,dx + \frac{\pi}{2}\int_{0}^{\pi}\cos nx\,dx\right.$$

$$= \frac{1}{n\pi}\left(\frac{-\pi}{2}(\sin nx)\Big|_{-\pi}^{0} + \frac{\pi}{2}(\sin nx)\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}(\sin_n(0) - \sin_n(-\pi))\right.$$
$$+ \frac{\pi}{2}(\sin_n(\pi) - \sin_n(0))$$

$$= \frac{1}{n\pi}\left(\frac{-\pi}{2}(0) + \frac{\pi}{2}(0)\right)$$

$$= 0$$

Now b_n

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left(\int_{-\pi}^{0} \frac{-\pi}{2} \sin nx\,dx + \int_{0}^{\pi} \frac{\pi}{2} \sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{2}\left(\frac{-\cos nx}{n}\right)\Big|_{-\pi}^{0} + \frac{\pi}{2}\left(\frac{-\cos nx}{n}\right)\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left(-\frac{\pi}{2}\left(-\cos n(0) - (-\cos n(-\pi))\right)\right)$$

$$+ \frac{\pi}{2}\left(-\cos n(\pi) - (-\cos n(0))\right)\Big)$$

$$= \frac{1}{n\pi}\left(-\frac{\pi}{2}(-2) + 2\frac{\pi}{2}\right.$$

$$= \frac{1}{n\pi}\left(\frac{2\pi}{2} + \frac{2\pi}{2}\right)$$

$$= \frac{1}{n\pi}\left[\frac{4\pi}{2}\right]$$

$$= \frac{4\pi}{n2\pi} \quad \frac{1}{2n}$$

$$= \frac{1}{2n}.$$

$$= \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{2n} & \text{if } n \text{ is odd.} \end{cases}$$

$$= 0 + 0 + 0 + \cdots\cdots + \frac{1}{2}\sin x + \frac{1}{4}\sin 2x$$

$$+ \frac{1}{6}\sin 3x \cdots \cdots$$

Q. $X(z) = \frac{2z^2+2z}{z^2+2z-3}$

$X(z) = \frac{2z(z+1)}{z^2+2z-3}$

$\frac{X(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$

$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$ —(i)

$2(z+1) = A(z-1) + B(z+3)$ —(ii)

Put $z=1$ in (ii)

$2(1+1) = A(1-1) + B(1+4)$

$4 = 0 + 4B$

$B = 1.$

New Put $z=-3$ in (ii)

$2(-3+1) = A(-3-1) + B(\cancel{-3}+\cancel{3})$

$2(-2) = A(-4) + 0$

$-4 = -4A$

$A = 1$ Put in (i)

$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$

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$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Inverse z- transform

$$x[n] = u[3] + 1[-1]^k$$

Q4. Express the transfer function using the given data.

$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$

Sol: $G(s) = C(sI-A)^{-1}B + D$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$\cancel{= \begin{bmatrix} 1 & 2 \end{bmatrix} \left(\begin{bmatrix} s+2 & 1 \\ s-1 & 0 \end{bmatrix} \right.}$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 0-1 & s-0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$\cancel{= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} [0]}$

$\cancel{= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix}}$

$$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s & 2 \end{bmatrix}}{s^2+2s+1}$$

[num, dem] = ss2tf (A, B, C, D)

(A, B, C, D) = tf2ss [num, dem]

Q4.

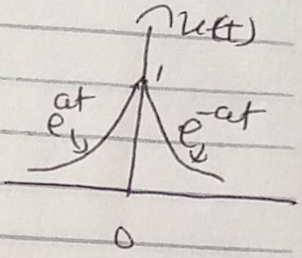
Sol The fourier transform of the given function $x(t)$ is given by.

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note:-

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$$x(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-j\omega)}\left[1-0\right] - \frac{1}{(a+j\omega)}\left(0-1\right)$$

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$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2-(j\omega)^2}$$

$$j\omega = \frac{2a}{a^2+\omega^2}$$