Midterm Paper (Summer)

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Subject # Mechanics of solid-II

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Section # B

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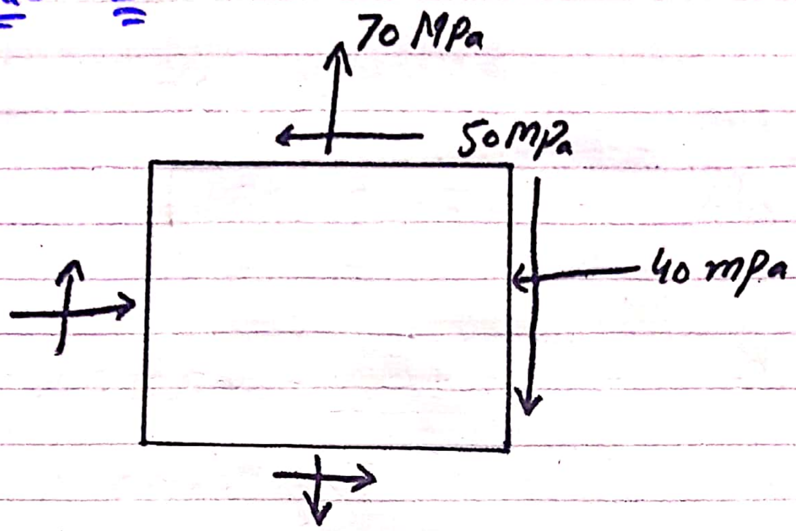
Q No 1



Sol

$\sigma_x = -40$ MPa

$\sigma_y = 70$ MPa

$\tau_{xy} = -50$ MPa

$\theta = -15^\circ \rightarrow$ due to clockwise rotation

So we have to find the transformed plane component i.e

σ_x', σ_y', τ_{xy}'

By formula

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-40 + 70}{2} + \frac{(-40 - 70)}{2} \cos 2(-15)$$

$$+ (-50) \sin 2(-15)$$

$$\sigma_{x'} = 15 + (-55)(0.866) + (-50)(-0.5)$$

$$= 15 - 47.63 + 25$$

$$\boxed{\sigma_{x'} = -7.63 \text{ MPa}}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-40+70}{2} - \frac{(-40-70)}{2}\cos 2(-15)$$

$$- (-50)\sin 2(-15)$$

$$\sigma_{y'} = 15 - (-55)(0.866) - (-50)(-0.5)$$
$$= 15 + 47.63 - 25$$

$$\boxed{\sigma_{y'} = 37.63 \text{ MPa}}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

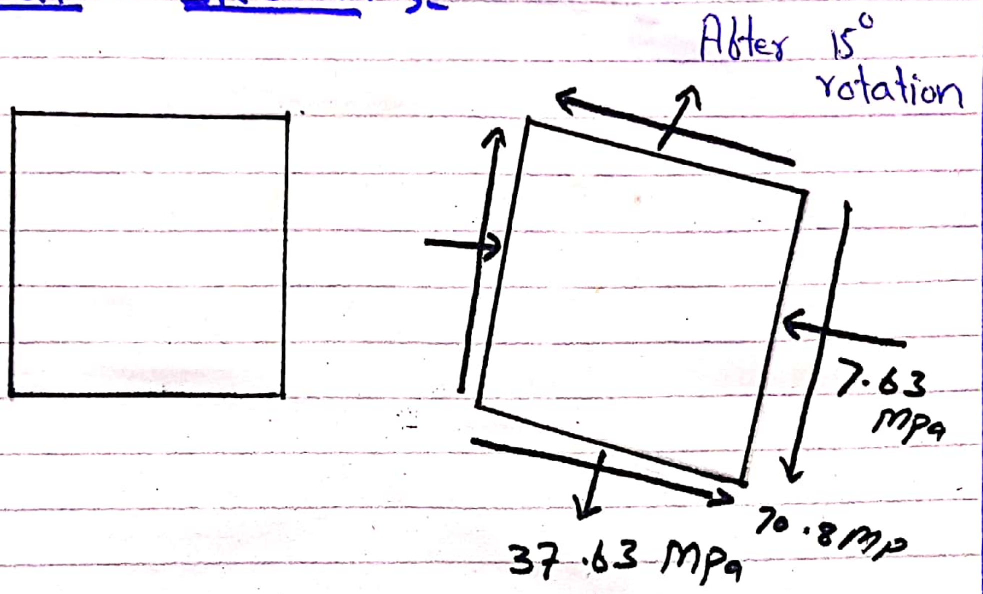
$$= -\frac{(-40-70)}{2}\sin 2(-15) + (-50)\cos 2(-15)$$

$$= -(-55)(-0.5) + (-50)(0.866)$$

$$= -27.5 - 43.3$$

$$\tau_{x'y'} = -70.8 \text{ MPa}$$

Final sketch :-



$\sigma_{x'} = -7.63$ MPa

$\sigma_{y'} = 37.63$ MPa

$\tau_{x'y'} = -70.8$ MPa

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Q No 2

Sol: First we find Principle Plane:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_p = \frac{2(-50)}{-40-70}$$

$$\tan 2\theta_p = 0.9091$$

$$2\theta_p = \tan^{-1}(0.9091)$$

$$\theta_p = \frac{42.27}{2}$$

$$\theta_p = 21.14^\circ \quad (\text{for } x\text{-axis})$$

$$2\theta_p = 42.27 + 180$$

$$\theta_p = \frac{222.27}{2}$$

$$\theta_p = 111.135^\circ \quad (\text{for } y\text{-axis})$$

Now we find Principal stress as:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-40+70}{2} + \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_1 = 89.33 \text{ Ma}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right) + (\tau_{xy})^2}$$

$$\sigma_2 = \frac{-40+70}{2} - \sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$\boxed{\sigma_2 = -59.33 \text{ MPa}}$$

Now we check which angle goes with which Principal stress.

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-40+70}{2} - \left(\frac{-40-70}{2}\right) \cos 2(21.14) + (-50) \sin 2(21.14)$$

$$\boxed{\sigma_{x_1} = -59.33 \text{ MPa}}$$

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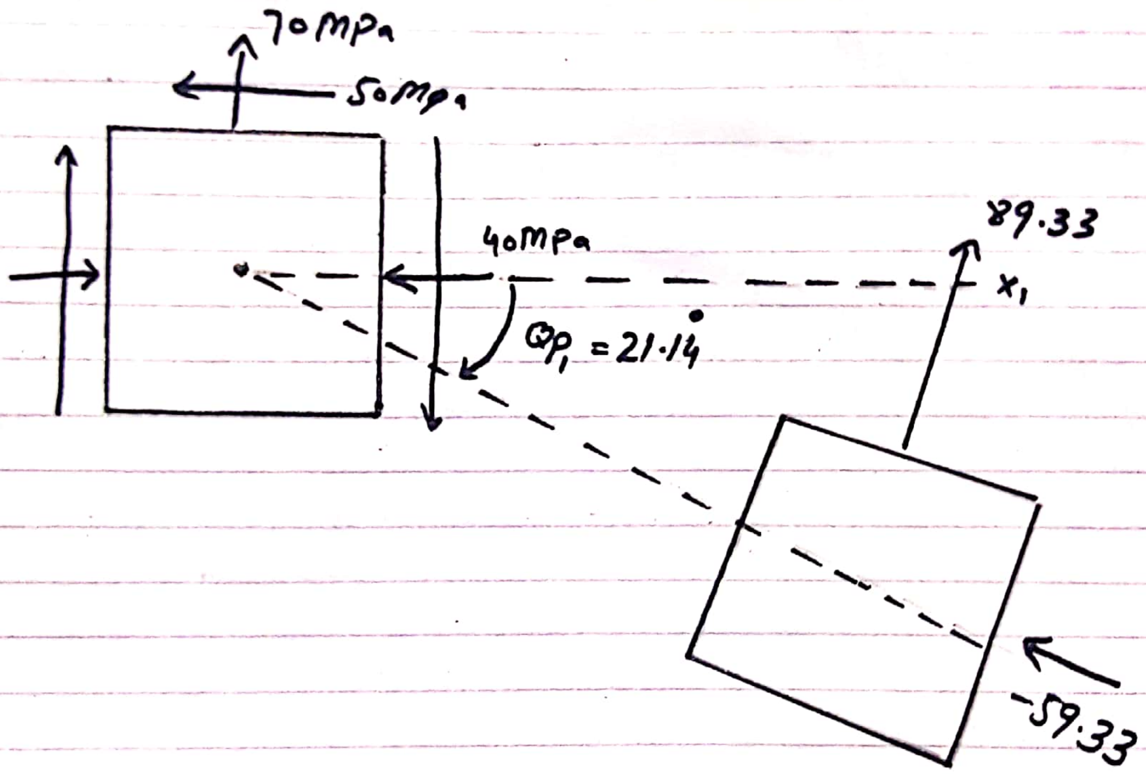
Hence,

$\sigma_1 = 89.33$ Mpa with $\theta p = 111.135$

$\sigma_2 = -59.33$ Mpa with $\theta p = 21.14°$



Q No 02 (Part b)

Given Data from Question

$\sigma_x = -40$ MPa
$\sigma_y = 70$ MPa
$\tau_{xy} = -50$ MPa

Sol:- Find orientation of maximum In plane shear stress

$$\tan 2\theta_s = \frac{-\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$= \frac{-\left(\frac{-40-70}{2}\right)}{-50}$$

$$= \frac{55}{-50} = -1.1$$

$$\tan 2\theta_s = -1.1$$

$$2\theta_s = \tan^{-1}(-1.1)$$

$$2\theta_s = -47.72^\circ$$

$$\boxed{\theta_s = -23.86^\circ}$$

$$\Rightarrow 2\theta_s = \frac{-47.72^\circ + 180}{2} = 66.14^\circ$$

(2) Finding Maximum In Plane Shear strees.

$$\tau_{max\ in\ plan} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm\sqrt{\left(\frac{-40-70}{2}\right)^2 + (-50)^2}$$

$$= \pm\sqrt{3025 + 2500}$$

$$= \pm\ 74.33\ \text{MPa}$$

(3) Find τ for corresponding θ_s

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-40-70}{2}\right)\sin 2(-23.86) + (-50)\cos 2(-23.86$$

$$= -40.645 + 33.6$$

$$\tau_{x'y'} = -7.04$$

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$\theta_{s_1} = -23.86 \qquad = -74.33$

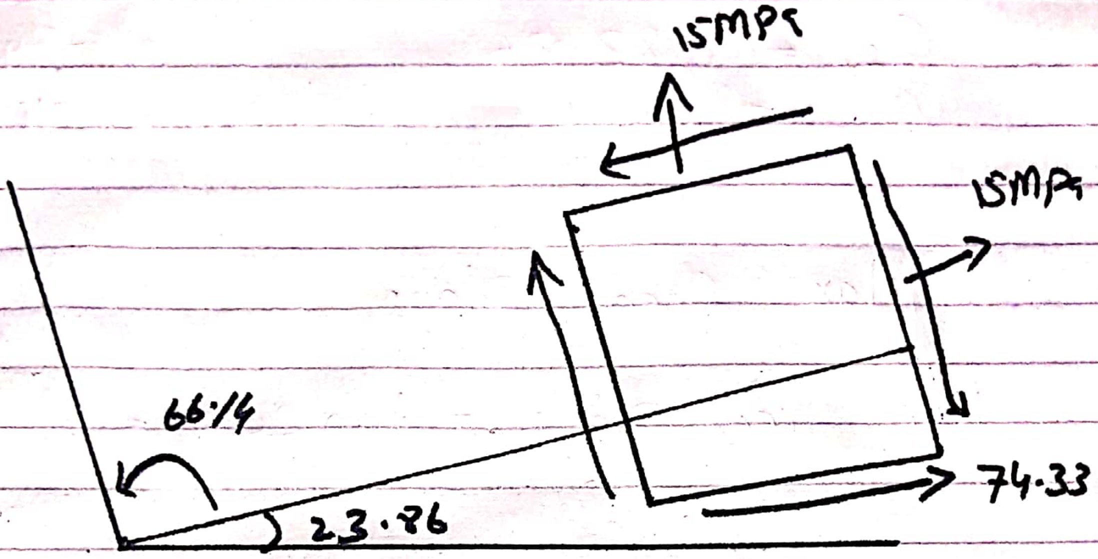
$\theta = 66.14 \qquad = 74.33$

(4) Normal stress acting on shear plane

(i) $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

$= \frac{-40 + 70}{2}$

$\boxed{\sigma_{avg} = 15 \text{ Mpa}}$


15MPa
15MPa
66.14
23.86
74.33