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Subject: probability & statistics

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Degree: BS SE

## Q1

**a) Mean and variance of binomial distribution is 4 and 9. Find n and p**

As we know

$$\text{Mean } (np) = 4 \quad \dots \text{ (i)} \quad \text{Variance } (npq) = 3 \quad \dots \text{ (ii)}$$

Dividing the LHS and RHS of equation (ii) by equation (i) we have

$$\frac{npq}{np} = \frac{3}{4}$$

$$\Rightarrow q = \frac{3}{4}$$

Therefore, we have  $p = 1 - q = 1 - \frac{3}{4} = \frac{1}{4}$

Putting the value of  $p = \frac{1}{4}$  in equation (i),

We have  $n = 16$

### B) Define critical region

A **critical region**, also known as the rejection **region**, is a set of values for the test statistic for which the null hypothesis is rejected. I.e. if the observed test statistic is in the **critical region** then we reject the null hypothesis and accept the alternative hypothesis

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### c) Write the properties of t-distribution

The **t distribution** has the following **properties**:

The mean of the **distribution** is equal to 0.

The variance is equal to  $v / (v - 2)$ , where  $v$  is the degrees of freedom (see last section) and  $v > 2$ .

The variance is always greater than 1, although it is close to 1 when there are many degrees of freedom.

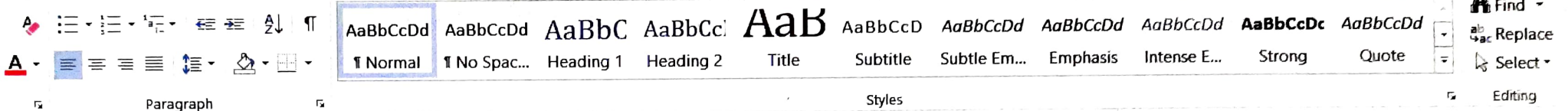
### D) Write a short note on Analysis of variance

**Analysis of variance**, or ANOVA, is a statistical method that separates observed **variance** data into different components to use for additional tests. A one-way ANOVA is used for three or more groups of data, to gain information about the relationship between the dependent and independent variables

### E) Define RBD

**RBD**: A diagram that gives the relationship between component states and the success or failure of a specified system function. The logical layout in an **RBD** can be as series system, parallel system, or a combination.

### F) Define Statistical quality control



## E) Define RBD

**RBD:** A diagram that gives the relationship between component states and the success or failure of a specified system function. The logical layout in an **RBD** can be as series system, parallel system, or a combination.

## F) Define Statistical quality control

**Statistical quality control**, the use of **statistical** methods in the monitoring and maintaining of the **quality** of products and services. One method, referred to as acceptance sampling, can be used when a decision must be made to accept or reject a group of parts or items based on the **quality** found in a sample

## G) Define the term Chance cause Assignable cause

**Chance cause:** a process that is operating with only chance causes of variation present is said to be in statistical control.

**Assignable cause** is a type of variation in which a specific activity or event can be linked to inconsistency in a system.

## H) Define traffic intensity

**traffic intensity:** A measure of the average occupancy of a facility during a specified period of time, normally a busy hour, measured in **traffic** units (erlangs) and defined as the ratio of the time during which a facility is occupied (continuously or cumulatively) to the time this facility is available for occupancy

## I) Write the characteristics of queuing theory:

A **queuing** system is specified completely by the following five basic **characteristics**: The Input Process. It expresses the mode of arrival of customers at the service facility governed by some probability law. The number of customers emanate from finite or infinite sources.

## Q2

### Part A)

$$\begin{aligned}
 E(X) &= \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x} \\
 &= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\
 &= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}
 \end{aligned}$$

since the  $x = 0$  term vanishes. Let  $y = x - 1$  and  $m = n - 1$ . Subbing  $x = y + 1$  and  $n = m + 1$  into the last sum (and using the fact that the limits  $x = 1$  and  $x = n$  correspond to  $y = 0$  and  $y = n - 1 = m$ , respectively)

$$\begin{aligned}
 E(X) &= \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y} \\
 &= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\
 &= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}
 \end{aligned}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting  $a = p$  and  $b = 1 - p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$



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so that

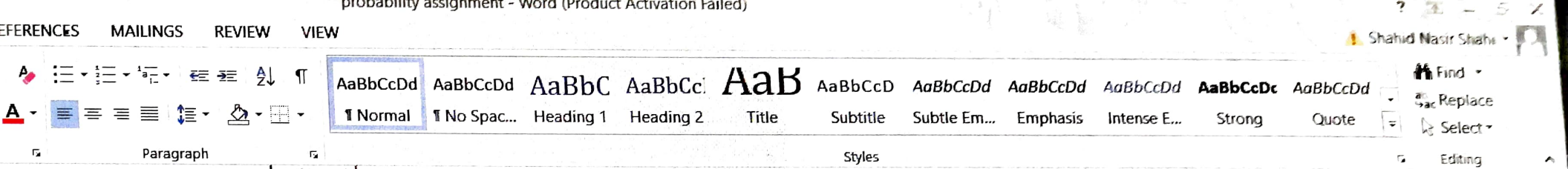
$$E(X) = np$$

Similarly, but this time using  $y = x - 2$  and  $m = n - 2$

$$\begin{aligned}
 E(X(X-1)) &= \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x} \\
 &= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\
 &= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x} \\
 &= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x} \\
 &= n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\
 &= n(n-1)p^2 (p + (1-p))^m \\
 &= n(n-1)p^2
 \end{aligned}$$

So the variance of  $X$  is

$$\begin{aligned}
 E(X^2) - E(X)^2 &= E(X(X-1)) + E(X) - E(X)^2 = n(n-1)p^2 + np - (np)^2 \\
 &= np(1-p)
 \end{aligned}$$



$$= np(1-p)$$

### Part b)

Let  $X$  denote number of cars hired out per day

Poisson distribution mean =  $m = 1.5$

$$P(X=x) = \frac{(e^{-m} (m^x))}{(x!)} = \frac{(e^{-1.5} (1.5^x))}{(x!)}$$

1)  $P$  (neither car is used):

$$P(X=0) = \frac{(e^{-1.5} (1.5^0))}{0!} = 0.2231$$

2)  $P$  (Some demand is refused) =  $P$  (Demand is more than 2 cars per days)

$$P(x > 2)$$

$$= 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

$$= 1 - \left[ \frac{(e^{-1.5} (1.5^0))}{0!} + \frac{(e^{-1.5} (1.5^1))}{1!} + \frac{(e^{-1.5} (1.5^2))}{2!} \right]$$

$$= 1 - e^{-1.5} [1 + 1.5 + \frac{2.25}{2}] = 0.1912$$

Proportion of days on which neither car is used

$$= 0.2231 = 22.31 \%$$

$$\text{Proportion of days on which some demand is refused} = 0.1912 = 19.12 \%$$



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Q3

