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Section:- B

Subject:- Differential Equations

Semester:- 4th

Q# 1 (a)

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w}{\partial t} = \cos(x+ct)\cdot c - \sin(2x+2ct)\cdot 2c$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) - 2c\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 2c\cos(2x+2ct)\cdot 2c$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct) \quad (1)}$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)\cdot 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - \cancel{\sin}\,4\cos(2x+2ct)$$

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$$c^2 \frac{\partial^2 w}{\partial x^2} = -c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) \quad (2)$$

Combine eq (1) & eq (2)

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

Q#1 (b)

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

$$w = \tan(2x+ct)$$

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$= c \sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t} = c^2 \sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct)$$

$$= 2c^2 \sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)$$
$$\cdot\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$= 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)$$
$$\cdot\tan(2x+ct)$$

$\Rightarrow$ NOT satisfied

Q#02

Answer to the Question #02

$$F(x) = \begin{cases} x ; & -\pi < x \leq 0 \\ 2x ; & 0 \leq x \leq \pi \end{cases}$$

⇒ we have to find fouriers co-efficient, a_0, a_n & b_n

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x \cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x \cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n+2(-1)^n-2}{n^2}\right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \dfrac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases} \rightarrow (2)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{1}{\pi} \int_{-\pi}^{0} x \sin nx \, dx$$

$$+ \frac{2}{\pi} \int_{0}^{\pi} x \sin nx$$

$$= \frac{1}{\pi} \left[x\left(-\frac{\cos nx}{n} \right) - \left(-\frac{\sin nx}{n^2} \right) \right]_{-\pi}^{0} +$$

$$\frac{2}{\pi} \left[x\left(-\frac{\cos nx}{n} \right) - \left(-\frac{\sin nx}{n^2} \right) \right]_{0}^{\pi}$$

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$$b_n = \frac{1}{\pi}\left[\frac{-\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi \cos n\pi}{n}\right]$$

$$= \frac{-3 \cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the required fourier is

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$$

Q NO 3

$$y'' - 4y' + 13y = 8\sin 3x \quad \text{(1)}$$

$y(0) = 1$

$y'(0) = 2$

Associated Homogenius Equation OF (1) is

$$y'' - 4y' + 13y = 0 \rightarrow (2)$$

change (2) into Auxilary Equation :

Put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

Use quadratic Formula :

$a = 1$, $b = -4$ & $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm \sqrt{36}\, i}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$\boxed{m_1 = 2 + 3i}$$

$$\boxed{m_2 = 2 - 3i}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (A)$$

let $y_p = A\cos 3x + B \sin 3x \rightarrow (*)$

Diff wrt "x"

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again Diff w.r.t "x"

$$y_p'' = -9A\cos 3x - 9B\sin 3x$$

put in 1

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x)$$
$$+ 13(A\cos 3x + B\sin 3x) - 8\sin 3x$$

$$\Rightarrow -9A\cos 3x - 12B\cos 3x + A\,13\cos 3x$$
$$- 9B\sin 3x + 12A\sin 3x + 13B\sin 3x - 8\sin 3x$$

$$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x$$
$$= 8\sin 3x$$

Comparing co-efficients

$$\sin 3x \Rightarrow \quad 4B + 12A = 8 \rightarrow (a)$$

$$4A - 12B = 0 \Rightarrow 4A = 12B$$
$$\boxed{A = 3B} \rightarrow (b)$$

put b in a

$$4B + 12(3B) = 8$$

$$40B = 8$$

$$\boxed{B = 1/5} \rightarrow 1$$

put 1 in b

$$\boxed{A = 3/5} \rightarrow d$$

put c & d in x

$$y_p = 3/5 \cos 3x + 1/5 \sin 3x \rightarrow B$$

the General Sol is :

$$y = y_c + y_p$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + 3/5 \cos 3x + 1/5 \sin 3x \rightarrow C$$

Now we need to find the values

OF C_1 & C_2 For this

put $x=0$ & $y=1$ in (c)

$$1 = e^{x(0)}\left(C_1\cos 3(0) + C_2 \sin 3(0)\right) + \tfrac{3}{5}\cos 3(0)$$
$$+ \tfrac{1}{5}\sin 3(0)$$

$$1 = (C_1(1) + C_2(0)) + \tfrac{3}{5}(1) + \tfrac{1}{5}(0)$$

$$1 = C_1 + 3/5$$

$$\boxed{C_1 = 2/5} \longrightarrow (xx)$$

Diff (c) w.r.t "x"

$$y' = C_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) +$$
$$C_2\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right)$$
$$-6/5 \sin 3x + 3/5 \cos 3x \rightarrow (D)$$

Put

$y' = 2$, $x = 0$ in (D)

$$y' = C_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right)$$

$$+ C_2\left(2e^{2x}\sin 3x + 3e^{2x}\cos 3x\right)$$

$$-6/5 \sin 3x + 3/5 \cos 3x$$

Put $y' = 2 \quad x = 0$

$$2 = C_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right)$$

$$+ C_2\left(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)\right)$$

$$2 = C_1(2) + C_2(3) - 0 \,, + 3/5$$

$$2 = 2C_1 + 3C_2 + 3/5$$

Put $C_1 = 2/5$

$$2 = 4/5 + 3C_2 + 3/5$$

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$2 = 7/5 + 3C_2$

$3C_2 = 3/5$

$\boxed{C_2 = 3/15} \rightarrow$ ***

Put XX & XXX in C

$$y = e^{3x}\left(2/5 \cos 3x + 3/15 \sin 3x\right) + 3/5 \cos 3x + 1/5 \sin 3x$$

$$\boxed{y = 2/5\, e^{2x} \cos 3x + 3/15\, e^{2x} \sin 3x + 3/5 \cos 3x + 1/5 \sin 3x}$$

Required General Equation

Q No 4

$$(D^2 - DD')z = \cos x \cos 2y$$

The given PDE can be rewrite as

$$D(D-D')z = \cos x \cos 2y$$

in CF is given by:

$$CF = \phi_1(y) + \phi_2(y+x)$$

while its PI is given by:

$$PI = \frac{1}{(D^2 - DD')} \cdot \tfrac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= \tfrac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \tfrac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \tfrac{1}{2}\cos(x+2y) - \tfrac{1}{6}\cos(x-2y)$$

Hence the complete Solution of the given PDE is :-

$$Z = \phi_1(y) + \phi_2(y+x)$$

$$+ \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$