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Section :- B

Subject :- Differential equation

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QNo:1 : Part A:-

(i) $w = \sin(x+ct) + \cos(2x + 2ct)$

Sol:-

$$\frac{dw}{dt} = \cdot\cos(x+ct) + (-\sin(2x+2ct) + 2c$$

$$\frac{d^2w}{dt^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \ldots 1$$

$$\frac{dw}{dx} = \cdot\cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{d^2w}{dx^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+ct)\right]$$

$$\frac{d^2w}{dt^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

$$c^2 + \frac{d^2w}{dx^2}$$

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Q1: Part B:-

Sol: $\frac{dw}{dt} = \frac{dw}{dt}\left(\tan(2x+ct)\right)$

$= \sec^2(2x+ct)\cdot(c)$

$= c\sec^2(2x+ct)$

$\frac{d^2w}{dt^2} = c\frac{d}{dt}\left[\sec^2(2x+ct)\right]$

$= c\,\frac{d}{dt}\left(\cos(2x+ct)\right)^{-2}$

$= c\,\frac{d}{dt}\left(\cos(2x+ct)\right)^{-3}(-\sin 2x+ct)$

$= \frac{d^2w}{dt^2} = 2c^2\sec^2(2x+ct)\sin(2x+ct)$

Now w.r.t x

$$\frac{dw}{dx} = \frac{d}{dx}\left(\tan(2x+ct)\right)$$

$$= \sec^2(2x+ct)(2)$$

$$\frac{dw}{dx} = 2\sec^2(2x+ct)$$

$$\frac{d^2w}{dw^2} = 2\frac{d}{dx}\left(\sec^2(2x+ct)\right)$$

$$= 2\frac{d}{dx}\left(\cos(2x+ct)^{-2}\right)$$

$$\frac{d^2w}{d\cancel{x}^2} = 2(-2)\left(\cos(2x+ct)\right)^{-3}$$

$$(-\sin(2x+ct)(2))$$

Result:-

Hence it is not the

Solution of the wave

equation.

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Q No: 2:-

Given function to

$$F(x) = \begin{cases} x & ; -\pi < x \le 0 \\ 2x & ; 0 \le x \le \pi \end{cases}$$

we have to find the Fourier Co-efficient, a_0, a_n & b_n

Now

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left(\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}} \rightarrow (i)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

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$$a_n = \frac{1}{\pi}\left[x\left(\frac{\sin x}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So,

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases}} \rightarrow (2)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\, dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\, dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\, dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$(3) \leftarrow \boxed{b_n = \frac{1}{\pi}\left[-\frac{\pi\cos\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}}$$

So the required Fourier series is :-

$$F(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)x}{2n-1} + 3\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

QNo: 3 :-

Given differential eq is

$y'' = 4y' + 13y = 8 \sin 3x$, $y(0) = 1$

& $y'(0) = 2$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8 \sin 3x$$

$\cancel{D^2()}$ $D^2y - 4Dy + 13y = 8 \sin 3x$ | let $D = d/dx$ & $D^2 = d^2/dx^2$

$(D^2 - 4D + 13) y = 8 \sin 3x \rightarrow 1$

Now general sol of 1 is

$y = y_c + y_p \rightarrow A$

$D^2 - 4D + 13 = 0$

$D^2 - 4D + 4 + 9 = 0$

$(D-2)^2 - 9i^2 = 0$

$(D-2)^2 - (3i)^2 = 0$

$(D-2-3i)(D-2+3i) = 0$

$D-2-3i = 0$ or $D-2+3i = 0$

$D = 2+3i$ or $D = 2-3i$

is $D = 2 \pm 3i$

which are imaginary roots

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

For P.I we have

$y_P = \dfrac{8 \sin 3x}{D^2 - 4D + 13} = 8 \cdot \dfrac{\text{imaginary Part of } e^{i3x}}{D^2 - 4D + 13}$

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(8)

$$y_p = \frac{8 \cdot \text{im part of } e^{i3x}}{(3i)^2 - 4(3i) + 13}$$

$$= \frac{8 \cdot \text{im part of } e^{i3x}}{-9 - 12i + 13}$$

$$= \frac{8 \cdot \text{im part of } e^{i3x}}{4(1-3i)}$$

$$y_p = \frac{2}{(1-3i)} \cdot \frac{(1+3i)}{(1+3i)} \text{ IM Part of } e^{i3x}$$

$$y_p = \frac{(2+6i)}{10} \text{ IM of } e^{i3x}$$

$$= \frac{\cancel{2}(1+3i)}{\cancel{10}} \text{ IM}(\cos 3x + i\sin 3x)$$

$$= \frac{(1+3i)}{5} \text{ IM}(\cos 3x + i\sin 3x)$$

$$y_p = \frac{(3\cos 3x + \sin 3x)}{5}$$

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$A \Rightarrow y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) +$

$\frac{1}{5}(3\cos 3x + \sin 3x) \rightarrow 2$

$y' = 2e^{2x}(c_1 \quad 3x + c_2 \quad 3x) + e^{2x}(-3c_1 \quad 3x$

$+ 3\cos 3x) + \frac{1}{5}(-9\sin 3x + 3\cos 3x) \rightarrow 3$

For $y(0) = 1$

$2 \Rightarrow y = e^0(c_1 \cos 0 + c_2 \sin 0) + \frac{1}{5}(3\cos 0 + \sin 0)$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5 = 2/5$

For $y'(0) = 2 \Rightarrow 2 = 2e^0(c_1 \cos 0 + c_2 \sin 0)$

$+ e^0(-3c_1 \sin 0 + 3c_2 \cos 0)$

$+ 1/5(-9\sin 0 + 3\cos 0)$

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$\Rightarrow 2 = 2C_1 + 3C_2 + 3/5$

$3C_2 = 2 - 3/5 - 2\,\frac{2}{5}$ | $-\, C_1 = 2/5$

$3C_2 = \frac{10-3-4}{5} = 3/5$

$C_2 = 1/5 \cdot 3/5 = \frac{1}{5} \Rightarrow C_2 = 1/5$

$2 \Rightarrow y = e^{2x}\left(2/5 \cos 3x + 1/5 \sin 3x\right) + 1/5\,(3\cos 3x$

$+ \sin 3x)$

Ans

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Q No: 4:-

Solve:-

$(D^2 - DD')z = \cos x \cos 2y$

Sol:-

This can also be written as:

$$\frac{d^2z}{dx^2} = \frac{d^2z}{\partial x \partial y} = E \cos x \cdot \cos 2y$$

Now as we have:

Corresponding A.E is

$m^2 - m = 0$

when $D/D' = m$ so $m = 1$ $m = 1$

$C.F = \phi_1(y) + \phi_2(y + x)$

Now

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$$PI = \frac{1}{(D^2 - DD')} \cos x \cos 2y$$

$$= \frac{1}{2} \frac{1}{D^2 - DD'} \left[\cos(x+2y) + \cos(x-2y) \right]$$
I II

$$PI = \frac{1}{2} \left[\frac{1}{-1+2} \cos(x+2y) + \frac{1}{-1-2} \cos(x-2y) \right]$$

Now when $\cos(ax+by)$ replacing D^2 by $-a^2$,
D'^2 by $-b^2$ & $DD' = -ab$

$$P.I = \frac{1}{2} \left[\cos(x+2y) - \frac{1}{3} \cos(x-2y) \right]$$

The complete solution:-

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2} \cos(x+2y) - \frac{1}{6} \cos(x-2y)$$

Result:-

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2} \cos(x+2y) - \frac{1}{6} \cos(x-2y)$$

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