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Seachion B

Assignment Diff~~e~~rential Eq.

Model 4th semester.

Q 1:

(i) $A_{m\times p} B_{p\times n} = AB_{m\times n}$

(ii) No of non zero Row in echolen Form is called Rank.

(iii) $= a=8$

(iv) $= 3$

(v) Scaler matrics.

(vi) $\log y = x - x^2 + c$ or $y = e^{x-x^2} + c$

(vii) order 1, degree $=3$.

(viii) order & degree is not def. as it's not Polynomial.

(ix) $y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$

(x) $bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

OR.

$b^2c(c-1) + ab(b-a) + ac(a-c)$

Q No 3:

$(x^2+3y^2)dx - 2xy\,dy = 0$

$y(2)=0$ given.

Sol:

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

This is homogenous differential equation to solve it, Put $y=vx$

Differentiate w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

The give differential equ become

$$v + x\frac{dv}{dx} = \frac{x^2+3(vx)^2}{2x(vx)}$$

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2x^2v} = \frac{x^2(1+3v^2)}{2x^2v} = \frac{3v^2+1}{2v}$$

$$x\frac{dv}{dx} = \frac{3v^2+1-2v^2}{2v} = \frac{v^2+1}{2v}$$

or $$\frac{2v}{v^2+1}dv = \frac{1}{x}dx$$

Integrating $\int \frac{2v}{v^2+1}dv = \int \frac{1}{x}dx$

$\ln(v^2+1) = \ln x + \ln c$

$\cancel{\ln}(v^2+1) = \cancel{\ln} x.c$

$v^2+1 = x.c$

As $y = ux \Rightarrow u = y/x$.

$$\frac{y^2}{x^2} + 1 = xc$$

$$\frac{y^2 + x^2}{x^2} = xc$$

$y^2 + x^2 = x^3 c \rightarrow$ (1) $y = 6, \; x = 2$.

so Putting the value

$$\cancel{y^2 +} \quad (6)^2 + (2)^2 = (2)^3 c$$

$$\boxed{c = 5} \rightarrow (2)$$

Put eq (2) in eq (1)

$$y^2 + x^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2 (5x - 1)$$

Taking squre on b's

$$y = \sqrt{x^2 (5x - 1)}$$

$$\boxed{y = \pm x^2 \sqrt{(5x - 1)}}$$

Q2 Part (A)

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Sol

? $$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^3 - c^2b^3) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$ab^2c^3 - ab^3c^2 - ba^2c^3 + a^3bc^2 + a^2cb^3 - a^3cb^2$$

Q#2:

B Part

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}.$$

Sol:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic of equ $\rightarrow |A - \lambda I| = 0 \rightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -3 & 3 & -1 & -1 \\ -1 & 1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Now take determinant.

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$2\lambda\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$= 0 \rightarrow B$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1.

$$\Rightarrow 3-\lambda\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\left[((3-\lambda)(2-\lambda) - (1)\right] + 1\left((-1)(2-\lambda) - (1)\right) - $$
$$-1\left((-1)(-1) - (-1)(3-\lambda)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$\Rightarrow \boxed{-\lambda + 8\lambda^2 - 18\lambda + 8} \rightarrow a$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by c_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \rightarrow (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by c_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

Put (a), (b) & c in eq (B)

$(2-\lambda)[-\lambda^3+8\lambda^2-12\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$ —

$+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda$

$-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By synthatic division we get

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\boxed{\lambda=0}$

$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

by factrozation method

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)=0$

$\boxed{\lambda=4}, \boxed{\lambda=4}$

$\boxed{\lambda_1=0, \lambda_2=2, \lambda_3=4, \lambda_4=4}$ Ans