Linear Algebra
BS-SE 2nd Semester

Pray for the safety of humanity.
Pray for all the Muslims and Pakistanis wherever they are.
Pray for our university, staff and students.

Instructions:

This is an open-book take-home mid-term assignment, to be submitted till Monday, April 26th, 2020. You may consult the textbook, your notes, and any material posted on SIC. No other sources of information are allowed, including friends, classmates, materials from other classes, tutors, etc. Please write your solutions as clearly and neatly as possible. Also, show all your work, preferably with explanations for each step. If you are asked to do a problem a specific way (for example, "use the standard matrix representation. . . "), then you will receive no credit for doing it any other way. You will also receive no credit for answers without sufficient work to produce them. Attempt all questions. Answers copied will both be marked zero. Late submission will not be accepted and marked zero.

How to Submit?

1. Write your names and Ids at the top of answer sheet.

2. Scan / Take Photo of each paper and save each photo with a number. E.g. photo of paper 1 of answer sheet be saved with name 1.jpg, then 2.jpg and so on.

3. Put all answer photos in a word file by simply copy and pasting images, name the document with subject name, your name and id e.g. LA_Ali_12345.

4. You will be provided upload link on sic to submit your answers.Go to Lectures section and click on Upload Assignment and upload your answers document file in the subject.Different formats are mentioned for uploading assignment, student can choose any one of them.

5.Due date and remaining time will be shown on the same window.

Q. No. 1 Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system. Where ID3 is the 3rd digit in your ID and ID_last is the last digit of your ID in inverse e.g. if your ID is 12345 then –ID_last = -5.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\_Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Q. No. 2

(a) Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(b) Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a. $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \Pi & 0 & 0 \\ 0 & 0 & -\Pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b. $\begin{bmatrix} 1 & 0 & \Pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

c. $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

d. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Q. No. 3

(a) The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

(b) Find an echelon form for the below matrix using row operations. Where ID2 is 2nd digit in your ID e.g. if your ID is 12345 ID2 = 2, ID3=3, ID_first_last is the first and last digit of your ID i.e.15

$$\begin{bmatrix} 1 & \text{ID2} & 8 \\ 2 & 8 & -1 \\ -\text{ID3} & 0 & 0 \\ 1 & -4 & \text{ID\_First\_Last} \end{bmatrix}$$

Q No # 1

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID_{last} & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Solution.

$$\begin{bmatrix} 1 & 9 & 3 & 0 & 5 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 9 \end{bmatrix}$$

This equation is already solved, because from row fourth we know that e.g $z = 9$ So this equation is already in echelon form and cannot be reduce any more.

Q No # 2. Part (a)

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution.

(i) $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

(i) $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$

$\sim_R \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & -1 & -3 \end{bmatrix}$ $R_3 - 2R_2$

$\sim_R \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$ $R_3 + R_2$

(ii) $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

$\sim R$ $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -3 \end{bmatrix}$ $R_3 + 2R_2$

$\sim R_1$ $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$ R_3

(3) No*2 Part (b)

(a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form.

The above matrix is in echelon form because in this matrix the number of zeros increased row by row and the first non-zero entries are other than one. Therefore it is not in reduced echelon form.

(b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

The given matrix is in echelon form. Because the number of zeros in the given matrix, increased row by row and it has two zero rows, and two non-zero rows so its rank is 2.

(c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

The give matrix is in not reduced echelon form. it is in echelon form because in the first row the first non-zero entry 5 is other than one.

(d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ in Reduced echelon form.

The given matrix is not in reduced echelon form it ~~becomes~~ becomes reduced echelon form by elementry row operation $R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

Now it is in reduced echelon form.

Q No # 3 Part (a)

Difference between Echelon form & Reduced echelon form:-

Echelon form	Reduced Echelon form.
The echelon form of matrix is not unique, which means there infinite answers are possible.	Reduced row echelon form is at the other end of spectrum. it is unique, which means that row reduction will be ~~def~~ performed the same answer no matter how to perform the same row operations.

★★ Use of Reduced echelon form:-

It is used practically in ~~bta~~ balancing chemical equations

★★ Example:-

Rust is formed when there is chemical ~~reacti~~ reaction between iron and oxygen. Rust is an iron oxide ~~where~~ whose chemical formula is Fe_2O_3.

Chemical Formula:-

$$Fe + O_2 \longrightarrow Fe_2O_3$$

Balance the Equation.

$$p\,Fe + q\,O_2 \longrightarrow r\,Fe_2O_3$$

$$Fe : P = 2r$$
$$O : 2q = 3r$$

The homogenous equation become

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$ where $A \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & -3 \end{bmatrix}$

Q No # 3 Part (b).

$$\begin{bmatrix} 1 & 8 & 8 \\ 2 & 8 & -1 \\ -9 & 0 & 0 \\ 1 & -4 & 61 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & -8 & -21 \\ 1 & -4 & 61 \\ -9 & 0 & 0 \end{bmatrix} \quad R_1 - 2R_1, \quad R_3 \leftrightarrow R_4$$

$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & -8 & -21 \\ 0 & -12 & -59 \\ -9 & 0 & 0 \end{bmatrix} \quad R_3 - R_1$$

$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & -8 & -21 \\ 0 & 0 & -55 \\ -9 & 0 & 0 \end{bmatrix} \quad 2R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & -8 & -21 \\ 0 & 0 & -55 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 + 9R_1$$

Ans.