① Estimate $\int_{0.5}^{1.3} e^{x^2}$

use trapezoidial rule a strip width of 0.2.

Sol:~

$a = 0.5$, $b = 1.3$, $\Delta x = 0.2$

Now divide the interval into 0.2 subintervals with the following end points.

$a = 0.5, 0.7, 0.9, 1.1, 1.3 = b$

$f(x_0) = f(0.5) = 1.28$

$2f(x_1) = 2f(0.7) = 3.265$

$2f(x_2) = 2f(0.9) = 4.496$

$2f(x_3) = 2f(1.1) = 6.707$

$F(x_4) = f(1.3 = 5.419$

$$\int_{0.5}^{1.3} e^{x^2} = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4)\right]$$

$$= \frac{0.2}{2}\left[1.28 + 3.265 + 4.496 + 6.707 + 5.419\right]$$

$$\boxed{\int_{0.5}^{1.3} e^{x^2} = 2.117}$$ Ans

* use Simpson's rule a strip width of 0.1

→ $\int_{0.5}^{1.3} e^{x^2}$

Solution:~

$a = 0.5, \; b = 1.3, \; \Delta x = 0.1$

$a = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 = b$

$f(x_0) = f(0.5) = 1.28$

$f(x_1) = f(0.6) = 1.433$

$f(x_2) = f(0.7) = 1.633$

$f(x_3) = f(0.8) = 1.896$

$f(x_4) = f(0.9) = 2.248$

$f(x_5) = f(1.0) = 2.718$

$f(x_6) = f(1.1) = 3.353$

$f(x_7) = f(1.2) = 4.221$

$f(x_8) = f(1.3) = 5.419$

$$\int_{0.5}^{1.3} e^{x^2} = \frac{\Delta x}{3}\Big[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + 2f(x_6) + 4f(x_7) + f(x_8)\Big]$$

$$= \frac{0.1}{3}\Big[1.28 + 4(1.433) + 2(1.633) + 4(1.896) + 2(2.248) + 4(2.718) + 2(3.353) + 4(2.221) + 5.419\Big] \; \boxed{\int_{0.5}^{1.3} e^{x^2} = 2.075}$$