Course Details

Course Title — Signal & system

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Module — 4th

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Q1 Show with a help of an equation
(a) - - - - - - - result in the multiplication by jw in frequency domain.

Fourier Transform of Differention Integration of continous - Time.

Let x(t) be a continuos - time signal with a fourier transform of x(jw)

i.e

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\, e^{jwt}\, dw$$

Differentiating both side with respect to (t)

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw) \frac{d}{dt}\{e^{jwt}\} dw$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\{e^{jwt} \cdot jw\} \cdot dw$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \{jw\, x(jw)\}\, e^{jwt}\, dw$$

=>

$$\mathcal{F}\left\{\frac{d}{dt} x(t)\right\} = jw\, x(jw)$$

Result:

We concluded that if a function is differentiated in time domain. it is ~~diffe~~ multiplied by jw in frequency domain.

Q. (b) $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Produce $Y(z)$ and $Y[n]$.

Sol:

$$X[z] = 2 - 4z^{-2} + 2z^{-3}$$

$$H[z] = 3 + z^{-1} + 2z^{-2}$$

Now

$$Y(z) = H(z) * X(z)$$

$$= (3 + z^{-1} + 2z^{-2}) * (2 - 4z^{-2} + 2z^{-3})$$

$$Y(z) = 6 - 12z^{-2} + 6z^{-3} + 2z^{-1} - 4z^{-3} + 2z^{-4}$$

$$+ 4z^{-2} - 8z^{-4} + 4z^{-5}$$

$$Y(z) = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$$

And

$$Y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2]$$

$$+ 2\delta[n-3] - 6\delta[n-4] + 4\delta[n-5]$$

2 $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$

Retrieve the Fourier series for the given function.

Sol:

We know that fourier series

$\Rightarrow a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$

So we find first (a_0)

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\, dx\right.$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2} dx + \int_{0}^{\pi} \pi/2\, dx\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{\pi}{2} \Big|_{-x}^{0} + \frac{\pi}{2}\, x \Big|_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[+\frac{\pi}{2}(-\pi) + \frac{\pi^2}{2}\right]$$

So

Dc componint

$$a_0 = \frac{1}{2\pi}\left[-\frac{\pi^2}{2} + \frac{\pi^2}{2}\right] = 0$$

So

$\boxed{a = 0} \rightarrow (1)$

Then Find

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

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$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\cos nx\,dx\right.$$

$$a_n = \frac{1}{\pi}\left(-\frac{\pi}{n2}\sin nx\Big|_{-\pi}^{0} + \frac{\pi}{n2}\sin nx\Big|_{0}^{\pi}\right.$$

$$a_n = \frac{1}{\pi}\left[\frac{-\pi}{2n}(0-0)+0\right] = 0$$

Then find b_n

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\sin nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\pi}{n2}\cos nx\Big|_{-\pi}^{0} - \frac{\pi}{2}\cos nx\Big|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{\pi}{2n}(1+1) - \frac{\pi}{2n}[-1-1]\right]$$

$$b_n = \frac{1}{\pi}\left[\frac{2\pi}{2n} + \frac{2\pi}{2n}\right]$$

$$b_n = \frac{1}{n} + \frac{1}{n} = \frac{2}{n} \rightarrow (3)$$

So put in main function

$$\text{fourier Series} = 0 + \sum_{n}^{\infty}\left(0\cos nx + \frac{2}{n}\sin x\right)$$

$$b_1 = \frac{2}{1} = \boxed{2}, \quad b_2 = \frac{2}{2} = \boxed{1}$$

$$b_3 = \frac{2}{3} = \boxed{0.6} \text{ and upto so on.}$$

Q3 If $x(z) = 2z^2+2z/z^2+2z-3)$
Retrieve the $x[n]$ using inverse Z-transform method

Sol
$$x(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$x(z) = \frac{2z(z+1)}{z^2+3z-z-3}$$

$$x(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$$

$$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

OR

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{z+3} + \frac{B}{z-1}$$

xing b.s $(z+3)(z-1)$

$$2(z+1) = A(z-1) + B(z+3) \quad -(1)$$

Put $z-1=0$, $\boxed{z=1}$ -> Put in equ 1

$$2(1+1) = A(1-1) + B(1+3)$$

$$2(2) = A(0) + B(4)$$

$$4 = 4B$$

$$\frac{4}{4} = \frac{4B}{4}$$

$$\boxed{B = 1}$$

Put $z+3=0$, $\boxed{Z=-3}$ in equ ① put

$$2(-3+1) = A(-3-1) + B(-3+3)$$

$$2(-2) = A(-4) + B(0)$$

$$-4 = -4A$$

$$\frac{-4}{-4} = \frac{-4A}{-4}$$

$$\boxed{A = 1}$$

Now put A & B in equ ②

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$x(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

inverse z - Transform

$$x[n] = u[n] + 1(-1)^k$$

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Q4 Express the transfer function using the given data.

Sol:

We know that

$$\frac{Y(s)}{X(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1} B + D$$

Puting value

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \left\{ s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & -1 \\ -1 & s \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = s(s+2) + 1 \Rightarrow s^2 + 2s + 1$$

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \times \frac{1}{s^2+2s+1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix} \times \frac{1}{s^2+2s+1} \begin{bmatrix} s & +0 \\ 1 & \end{bmatrix}$$

$$H[s] = \frac{\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}}{s^2+2s+1}$$

$$H[s] = \frac{s+2}{s^2+2s+1}$$

Q5 Apply Fourier transform on the signal
- - - - - - step function.

Given

$x(t) = e^{-a|t|}$

$a > 0$

$x(j\omega) = ?$

Solution:

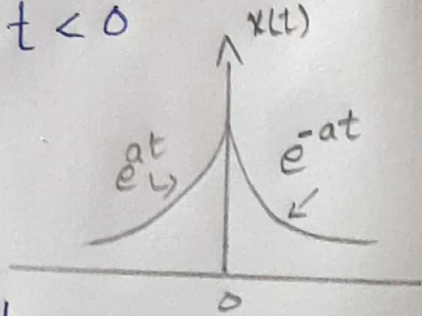
Fourier transform of the given function $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt.$$

Note:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$\therefore$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$\Rightarrow \left.\frac{e^{(a-j\omega)t}}{a-j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)}}{-(a+j\omega)}\right|_{0}^{\infty}$$

$$\Rightarrow \frac{1}{(a-jw)}\left[e^{0}-e^{-\infty}\right]-\frac{1}{(a+jw)}\left[e^{-\infty}-e^{0}\right]$$

$$\Rightarrow \frac{1}{(a-jw)}[1-0]-\frac{1}{(a+jw)}[0-1]$$

$$\Rightarrow \frac{1}{a-jw}+\frac{1}{a+jw}$$

$$\Rightarrow \frac{a+jw+a-jw}{a^2-(jw)^2}$$

$$\Rightarrow \frac{2a}{a^2+w^2}$$