Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

Name:	Fayaz ullah	**Student ID:**	13854

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n , & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Name : Fayaz ullah
ID : 13854
Subject : DSP

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Answers

Question No 1

Part (a)

Sol:

Signal, $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

i):

Minimum Sampling Rate

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$f_1 = 50\,Hz$, $f_2 = 100\,Hz$, $f_3 = 100\,Hz$

So,

f_s ~~xx~~ is max (greater than f_1)

Since,

$f_1 = 50\,Hz$ is minimum sampling rate to avoid aliasing

ii):

From equation

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$= 3\cos \frac{100\pi t}{100} + 4\sin \frac{200\pi t}{100}$

$$\boxed{x_a(t) = 3\cos \pi t + 4\sin 2\pi t}$$

So, the effect of this sampling rate on the newly generated discrete time is ~~that~~ ~~there~~ will be no (Aliasing) means there will be present components in the ~~reconst~~ reconstruction of the signal and we can reconstruct the original signal.

iii): For ideal interpolation we can construct the original signal and also frequency components,

$f_1 = 50\ Hz$, $f_2 = 100\ Hz$

$$y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

The original signal is different from it because we used the sampling frequency. This distortion of the ~~signal~~ original analog signal caused by the aliasing effect.

Question 1

Part (b)

Discrete time signal

$$x(n) = \begin{cases} 0.5^n, & n > 0 \\ 0, & n < 0 \end{cases}$$

$Fs = 2\,Hz$

i):

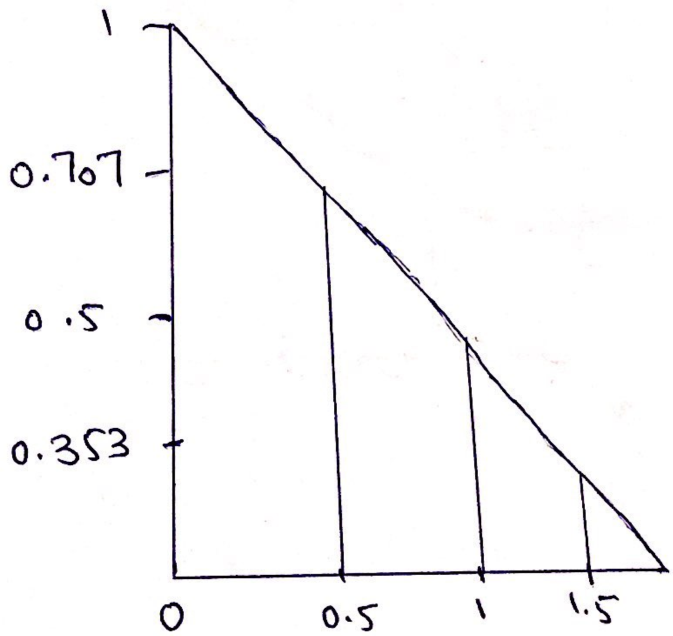
$Fs = 2\,Hz$

$fs = \frac{1}{T}$

$T = \frac{1}{fs} \Rightarrow \frac{1}{2} = 0.5$

$\boxed{T = 0.5\,sec}$

x(n)	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



ii):

$$L = 2^n$$

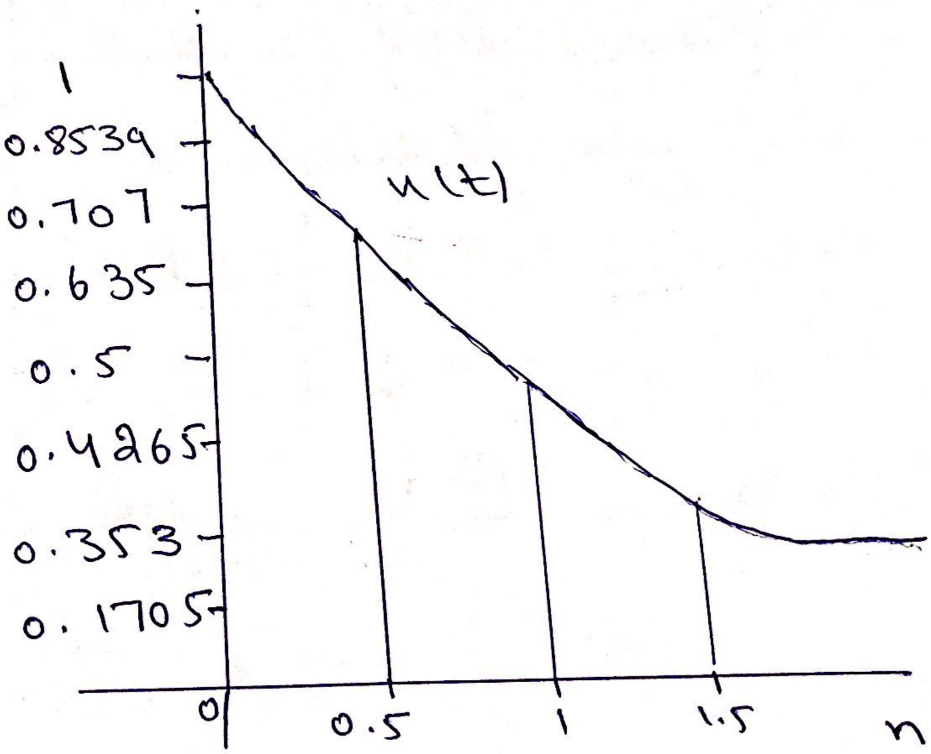
$$n = 3 \qquad \therefore n = \text{bits}$$

Quantization level

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1-0}{8} = \boxed{0.125}$$

Range of Quantization



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iii): Tabular Form

n	$x(n)$	$x_q(n)$ Truncation	$x_q(n)$ Rounding off	$x_q(n) - x(n)$
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	0.0465
2	0.707	0.7	0.7	-0.07
3	0.6035	0.6	0.6	-0.035
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	-0.0265
6	0.353	0.3	0.4	0.047
7	0.1765	0.1	0.2	0.0235

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Question 2

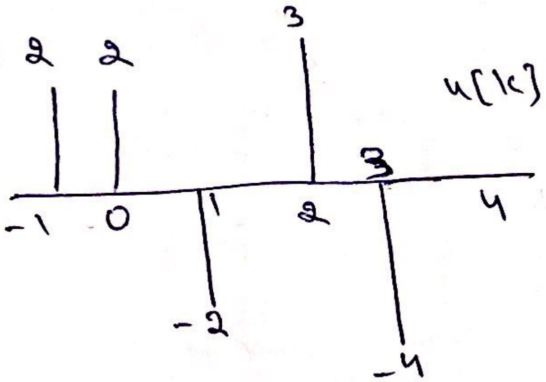
Part (a)

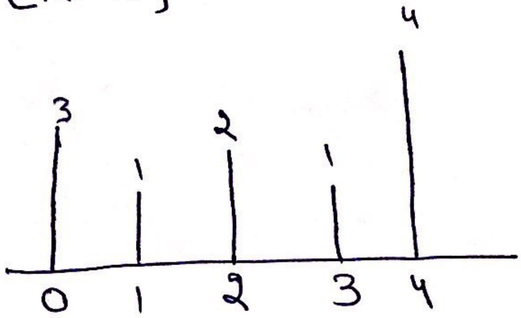
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution:

$$y[n] = \sum_{k=\infty}^{\infty} x[k]\, h[n-k]$$





h[-k] folded signal

$$y[0] = \sum_{k=-1}^{0} x[-1]h[-1] + x(0)\,h(0)$$

$$= 2*1 + (1)(3)$$

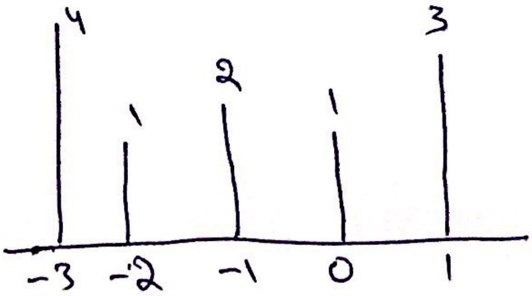
$$= 2\times 1 + 3$$

$$= 2+3$$

$$= 5$$

for $n = 1$

$h(1-k)$



$$y[1] = \sum_{k=-1}^{1} x[n]\, h[1-k]$$

$$= x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1)$$

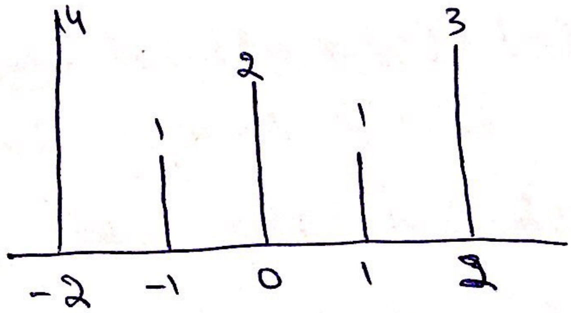
$$= (2)(2) + (1)(1) + 3(-2)$$

$$= 4 + 1 - 6$$

$$= -1$$

for $n = 2$

$h[2-k]$



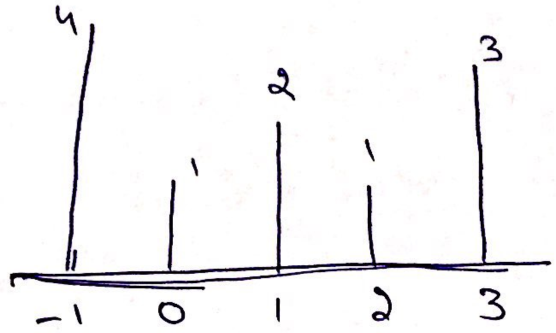
$$y[2] = \sum_{k=-1}^{2} x[n]\, h[2-k]$$

$$= x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1) + x(2)\,h(2)$$

$$= 2(1) + (1)(2)$$

for, n = 3 ⑧

h[3-k]



$$y[3] = \sum_{k=-1}^{3} x[n]\, h(3-k)$$

$= x(-1)\, h(-1) + x(0) h(0) + x(1) h(1) +$
$x(2) + x(3) h(3)$

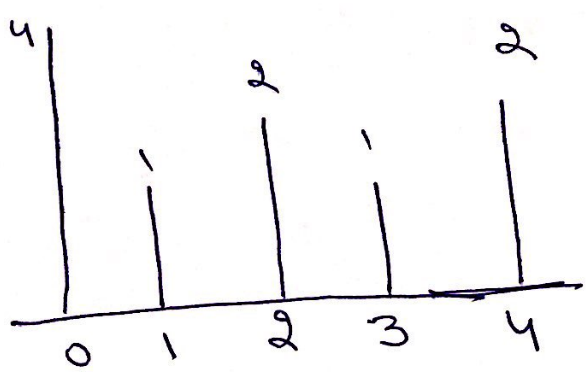
$= 2 \times 2 + (1)(1) + (-2)(2) + (3)(1) + (4)$

$= 4 + 1 - 4 + 3 - 12$

$= -8$

for n = 4

h[4-k]



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$$y[4] = \sum_{k=0}^{4} x[n]h[4-k]$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

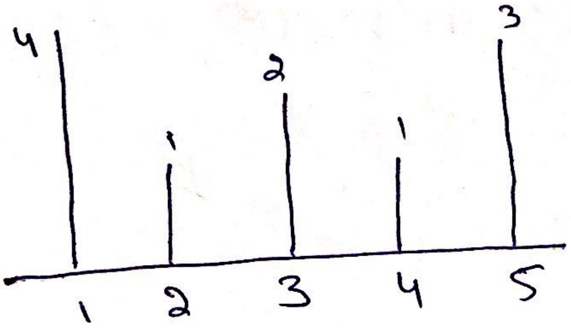
$$= (1\times 4) + (-2)(1) + (3)(2) + (-4)(1)$$

$$= 4 - 2 + 6 - 4$$

$$= 4$$

for $n = 5$

$h(5-k)$



$$y[5] = \sum_{k=1}^{5} x(n)\, h(5-k)$$

$$= x(1)h(1) + x(2)h(2) + x(3)h(3)$$

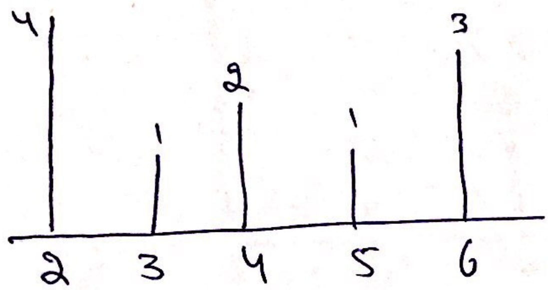
$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8 + 3 - 8$$

$$= -13$$

for $n = 6$

$h(6-k)$



$$y[6] = \sum_{k=2}^{6} x(2)h(2) + x(3)h(3)$$

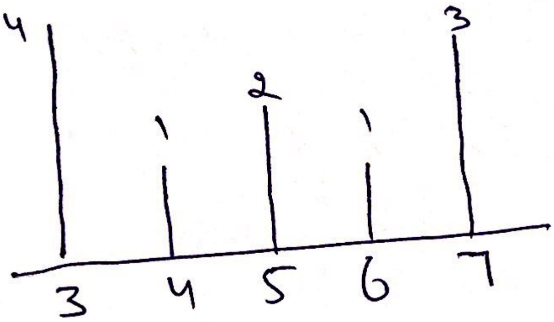
$$y[6] = (3)(4) + (1)(-4)$$

$= 12 - 4$

$= 8$

for $n=7$

$h(7-k)$

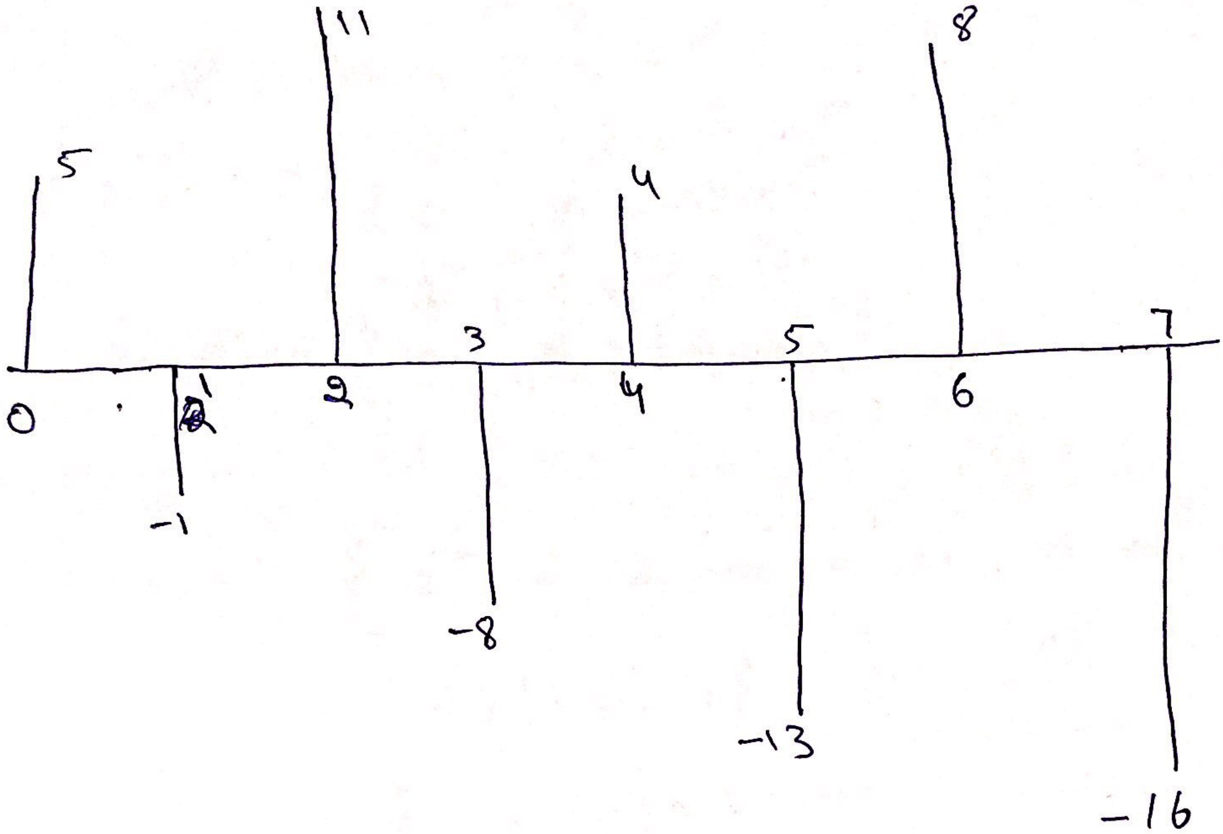


$y[7] = x(3)\,h(3)$

$= 4 \times (-4)$

$= -16$

y[n]



Question 2

(14) (12)

Part (b)

Convolution $y(n) = ?$

$$u(n) = \begin{cases} a^{n+1}, & -3 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n}, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Solution:

$$u(n) = \{\alpha^{-2}, \alpha^{-1}, \alpha, \alpha^{1}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$

$$h(n) = \{1, 2, 4, 8, 16\}$$

$$y[n] = \sum_{k=0}^{4} h(k)\, x(n-k)$$

Therefore

$$y(-2) = \alpha^{-2}$$

$$y(-1) = u(-\alpha) + u(1) = \alpha^{-2} + \alpha^{-1}$$

$$y(0) = h(u)u(-2) + h(u)u(-9) + h(u)u(\alpha)$$

$$= 9 - \alpha^{-2} + 9 - \alpha^{-2} + 9 - 9 + \alpha^{-3} + 9$$

$$y(1) = \alpha^{-2} + \alpha^{-3} + 9 + h(9)u(4-3)$$

$$= \alpha^{-2} + \alpha^{-1}$$

$$y(2) = -\alpha^{-2} + \alpha^{-1} + 1 + 2\alpha^{1} + h(2)u(2)$$

$$= \alpha^{-2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2}$$

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$$y(3) = \alpha^{-2} + \alpha^{-1} + 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3}$$

$$y(4) = \alpha^{-2} + \alpha^{-1} + 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3}.$$

$$h(4) \cdot u(4)$$

$$= \alpha^{-2} + \alpha^{-3} + 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4}$$

$$y(5) = 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{5} + 5$$

$$y(6) = 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4} + \alpha^{5} + \alpha^{6}$$

$$y(7) = 8\alpha^{3} + \alpha^{4} + \alpha^{5} + \alpha^{6}$$

$$y(8) = 16\alpha^{4} + \alpha^{5} + \alpha^{6}$$

$$y(9) = \alpha^{5} + \alpha^{6}$$

$$y(10) = \alpha^{6}$$

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Question 3

Part (a)

1:

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

In form Z-transform

$$X(z) = \sum^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

using geometric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

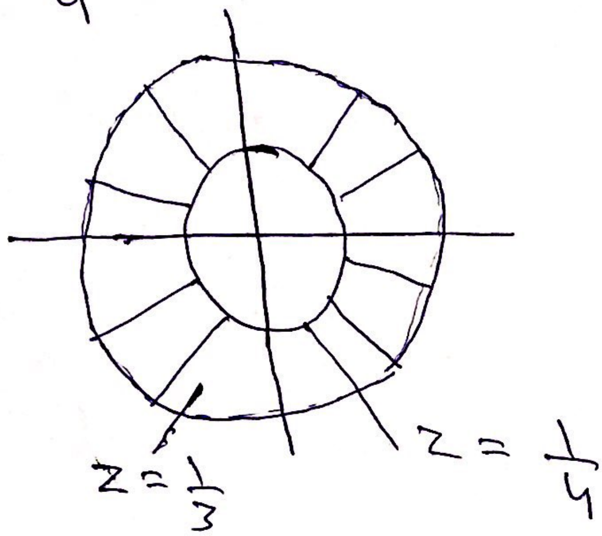
$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)} - 1$$

$$= \frac{(1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1}) - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\cancel{1} - \cancel{\frac{1}{3}z} + 1 - \cancel{\frac{1}{4}z^{-1}} - \cancel{1} + \cancel{\frac{1}{3}z} + \cancel{\frac{1}{4}z^{-1}} + \frac{1}{2}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

Hence ~~the~~ ROC is $\frac{1}{4} < |z| < 3$



(ii):

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

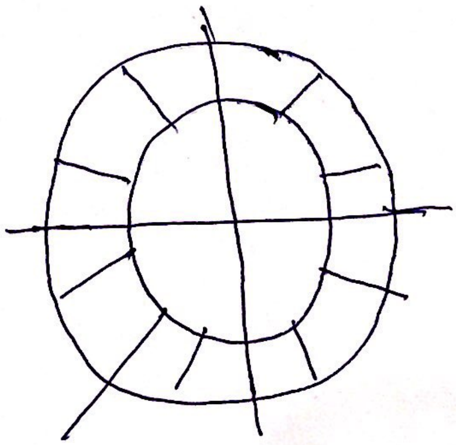
Using geometric series

$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

$$= \frac{-\frac{5}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

The ROC is $|z| > 3$



$z = 3$