NAME : SULEMAN HAKEEM
I.D : 16757
SEC : B
DATE : 30th/06/2020
SEM : 4th
PAPER : Differential Equations.

Q1) The wave Equation ...?

i) $w = \sin(x+ct) + \cos(2x+2ct)$

Solution:-

diff on b/s

$$\frac{dw}{dt} = \cos(x+ct)+c - \sin(2x+2ct)+2c$$

$$\frac{d^2w}{dt^2} = -\sin(x+ct)+c^2 - \cos(2x+2ct)+4c^2 \rightarrow (i)$$

$$\frac{dw}{dx} = \cos(x+ct) - \sin(2x+2ct)+2.$$

$$\frac{d^2w}{dx^2} = -\sin(x+ct) - 4\cos(2x+2ct).$$

$$= [-\sin(x+ct) - 4\cos(x+2ct).$$

8.00 am

$$\frac{d^2w}{dt^2} = c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

8.30

9.00

$$c^2 = \frac{d^2w}{dx^2}.$$

9.30

10.00

ii) $w = \tan(2x+ct)$

10.30

11.00

SOLUTION :-

11.30

$$\frac{dw}{dt} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

Noon

12.30

$$= c\sec^2(2x+ct).$$

1.00

$$\frac{d^2w}{dt^2} = c\,2\sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct)$$

1.30

2.00

$$\frac{d^2w}{dt^2} = 2c^2\sec(2x+ct)\sec(2x+ct) \rightarrow \tan(2x+ct)$$

2.30

3.00

$$\frac{d^2w}{dt^2} = 2c^2\sec^2(2x+ct)$$

3.30

4.00

$$\frac{d^2w}{dt^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)\tan(2x+ct)\times 2.$$

4.30

5.00

6.00 pm

$$= c^2 8\sec^2(2x+ct)\tan(2x+ct)$$

$$= 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)\tan(2x+ct)$$

So it is not possible.

not satisfied.

QUESTION NO = 2.

ANSWER :-

Given function

$$F(x) = \begin{cases} x; & -\pi < x \leq 0 \\ 2x; & 0 \leq x \leq \pi \end{cases}$$

Now finding fourier- coefficient, a_0, a_n and b_n.

NOW :-

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx.$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2} \rightarrow (i)$$

Now

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} (x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x\cos nx)\,dx$$

$$a_n = \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0} + \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right) + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

5

So;

$$a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases} \longrightarrow \text{(ii)}$$

Now b_n :-

As

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x) \sin nx \, dx = \frac{1}{\pi}\int_{-\pi}^{0} x \sin nx \, dx + \frac{2}{\pi}\int_{0}^{\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n} - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

So

$$b_n = \frac{1}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi \cos n\pi}{n}\right]$$

$$= \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

6

So Required fourier Series is :

$$f(x = \frac{\pi_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)x}{(2n-1)} + 3\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

Question No = 3

Answer :-

$y'' - 4y' + 13y = 8\sin 3x, y(0) = 1$ and $y'(0) = 2$

$y'' - 4y' + 13y = 8\sin 3x \rightarrow$ (i)

Associated Homogenous Eq of (i) is

$y'' - 4y' + 13y = 0 \rightarrow$ (ii)

Changing (ii) into Auxilary equation.

put $y = m$ in (ii)

7

8.00 am

$m^2 - 4m + 13 = 0$

8.30

Using Quadratic formula

9.00

9.30 $a = 1$, $b = -4$, $c = 13$

10.00 $m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

10.30

putting values

11.00 $m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$

11.30

Noon $m = \frac{4 \mp \sqrt{16 - 52}}{2}$

12.30

1.00 $m = \frac{4 \pm \sqrt{-36}}{2}$

1.30

$m = \frac{4 \pm 6i}{2}$

2.00

2.30 $m = 2 \pm 3i$

3.00

So

3.30 $m_1 = 2 + 3i$
$m_2 = 2 - 3i$

4.00

4.30 $y_c = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) \rightarrow$ A

5.00

6.00 pm

8.00 am Let $y_p = A\cos 3x + B\sin 3x \rightarrow (x)$

8.30

9.00 <u>Diff w.r.t x.</u>

9.30 $y'_p = -3A\sin 3x + 3B\cos 3x.$

10.00 Now Again diff w.r.t to x.

10.30

11.00 $y''_p = -9A\cos 3x - 9B\sin 3x$

11.30

Noon <u>put in (i)</u>

12.30 $\Rightarrow$ So $(-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x)$

1.00 $+13(A\cos 3x + B\sin 3x) = 8\sin 3x$

1.30

$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x$

2.00 $9B\sin 3x + 12A\sin 3x + 13B\sin 3x = 8\sin 3x.$

2.30

3.00 $\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

3.30

<u>Now Comparing Coffecient.</u>

4.00

$\sin 3x \Rightarrow 4B + 12A = 8 \rightarrow (a)$

4.30

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B.$

5.00

$\Rightarrow A = 3B \rightarrow (b)$

6.00 pm

October 2018
M 1 8 15 22 29
T 2 9 16 23 30
W 3 10 17 24 31
T 4 11 18 25
F 5 12 19 26
S 6 13 20 27
S 7 14 21 28

November 2018
M 5 12 19 26
T 6 13 20 27
W 7 14 21 28
T 1 8 15 22 29
F 2 9 16 23 30
S 3 10 17 24
S 4 11 18 25

December 2018
M 31 3 10 17 24
T 4 11 18 25
W 5 12 19 26
T 6 13 20 27
F 7 14 21 28
S 1 8 15 22 29
S 2 9 16 23 30

(9)

18 Tuesday 352/013

8.00 am

Now put (b) in (a)

8.30

9.00

$$4B + 2(3B) = 8$$

9.30

$$4B + 36B = 8$$

10.00

$$40B = 8$$

10.30

$$B = 8/40$$

11.00

$$B = \frac{1}{5} \rightarrow (c)$$

11.30

Noon

Now put (c) in (b)

12.30

So $A = 3/5 \rightarrow (d)$

1.00

1.30

put (c), (d) in (x)

2.00

So;

2.30

$$y_p = \frac{B}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (B)$$

3.00

3.30

The General Solution is

4.00

4.30

$$y = y_c + y_p$$

5.00

6.00 pm

January 2019					February 2019					March 2019				
M		7	14	21	28	M		4	11	18	25	M		4	11	18	25
T	1	8	15	22	29	T		5	12	19	26	T		5	12	19	26
W	2	9	16	23	30	W		6	13	20	27	W		6	13	20	27
T	3	10	17	24	31	T		7	14	21	28	T		7	14	21	28
F	4	11	18	25		F	1	8	15	22		F	1	8	15	22	29
S	5	12	19	26		S	2	9	16	23		S	2	9	16	23	30
S	6	13	20	27		S	3	10	17	24		S	3	10	17	24	31

10

8.00 am

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (c)$$

8.30

9.00

Now we finding the values of

9.30

c_1 and c_2

10.00

So for this

10.30

put

11.00

$x = 0$ and $y = 1$ in (c)

11.30

$$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin 3(0) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0).$$

Noon

12.30

$$1 = (c_1(1) + c_2(0) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

1.00

1.30

$$1 = c_1 + \frac{3}{5}$$

2.00

2.30

$$c_1 = \frac{2}{5} \rightarrow (a)$$

3.00

3.30

Diff (c) w.r.t to x.

4.00

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x} \rightarrow \sin 3x + 3e^{2x}\cos 3x)$$

4.30

5.00

6.00 pm

11

$$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow (D)$$

<u>put</u> $y' = 2$, $x = 0$ in (D)

$$2 = \left(C_1\left(2e^{2(0)}\cos 3(0) + 3e^{2(0)}\cos 3(0)\right) + C_2\left(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)\right)\right) - \frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$$

$$2 = C_1(2) + C_2(3) - 0 + \frac{3}{5}$$

$$2 = 2C_1 + 3C_2 + \frac{3}{5}$$

<u>put</u> $C_1 = 2/5$

$$2 = \frac{4}{5} + 3C_2 + \frac{3}{5}$$

$$2 = \frac{7}{5} + 3C_2$$

$$3C_2 = 2 - \frac{7}{5}$$

$$3C_2 = \frac{3}{5}$$

$$C_2 = 3/5 \rightarrow (R)$$

January 2019 February 2019 March 2019

(12)

Emperor's Birthday (Japan)

8.00 am

put (q) and (R) in (c)

8.30

9.00 $y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$

9.30

10.00 $\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$

10.30

it is Required general Solution

11.00

11.30 QUESTION No 4

Noon ANSWER :-

12.30 $(D^2 - DD')z = \cos x \cos 2y$.

1.00 It can be rewrite as,

1.30 $D(D-D')z = \cos x \cos 2y$

2.00 In CF is given by;

2.30

3.00 $CF = \phi_1(y) + \phi_2(y+x)$

3.30 In PI is

4.00

4.30 $PI = \frac{1}{(D^2-DD')} \cdot \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$.

5.00

6.00 pm

8.00 am

8.30

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

9.00

9.30

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y).$$

10.00

So Complete Solution:-

10.30

Hence the Complete Solution of the given pDE is

11.00

11.30

$$\boxed{u = \phi_1(y) + \phi_2(y+x) + \tfrac{1}{2}\cos(x+2y) - \tfrac{1}{6}\cos(x-2y)}$$

Noon

Ans

12.30

1.00

1.30

2.00

2.30

3.00

3.30

4.00

4.30

5.00

6.00 pm